Multicolor Ramsey Numbers Involving K_3+e and K_4-e

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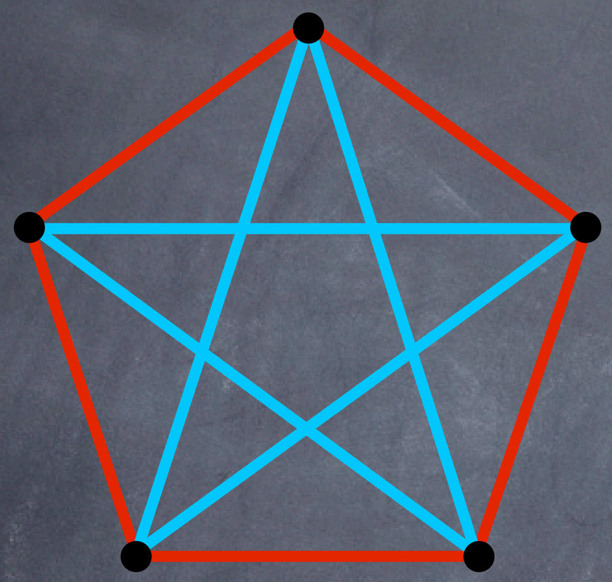
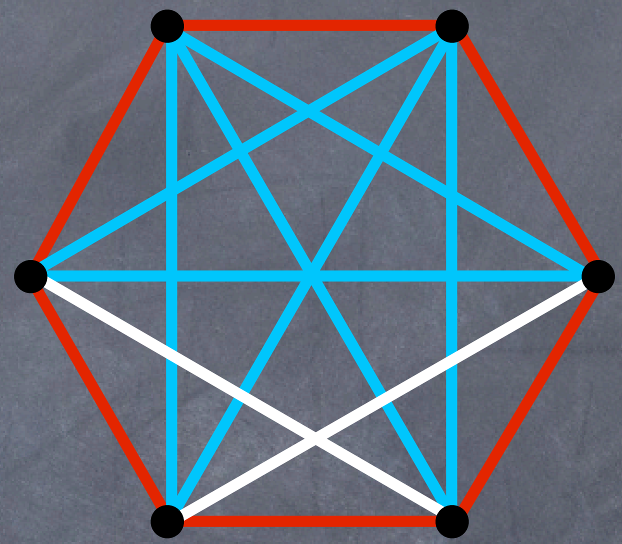
Definitions

The Ramsey number $R(G,H) = n$ iff
n = the least positive integer such that in any 2-coloring of the edges of K_n there is a monochromatic G in the first color or a monochromatic H in the second color.

Good Graph: A $(G,H;n)$-good graph is a graph on n vertices that avoids G, and avoids H in the complement.



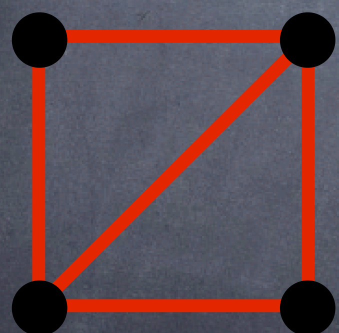
$(K_3,K_3;5)$-good graph

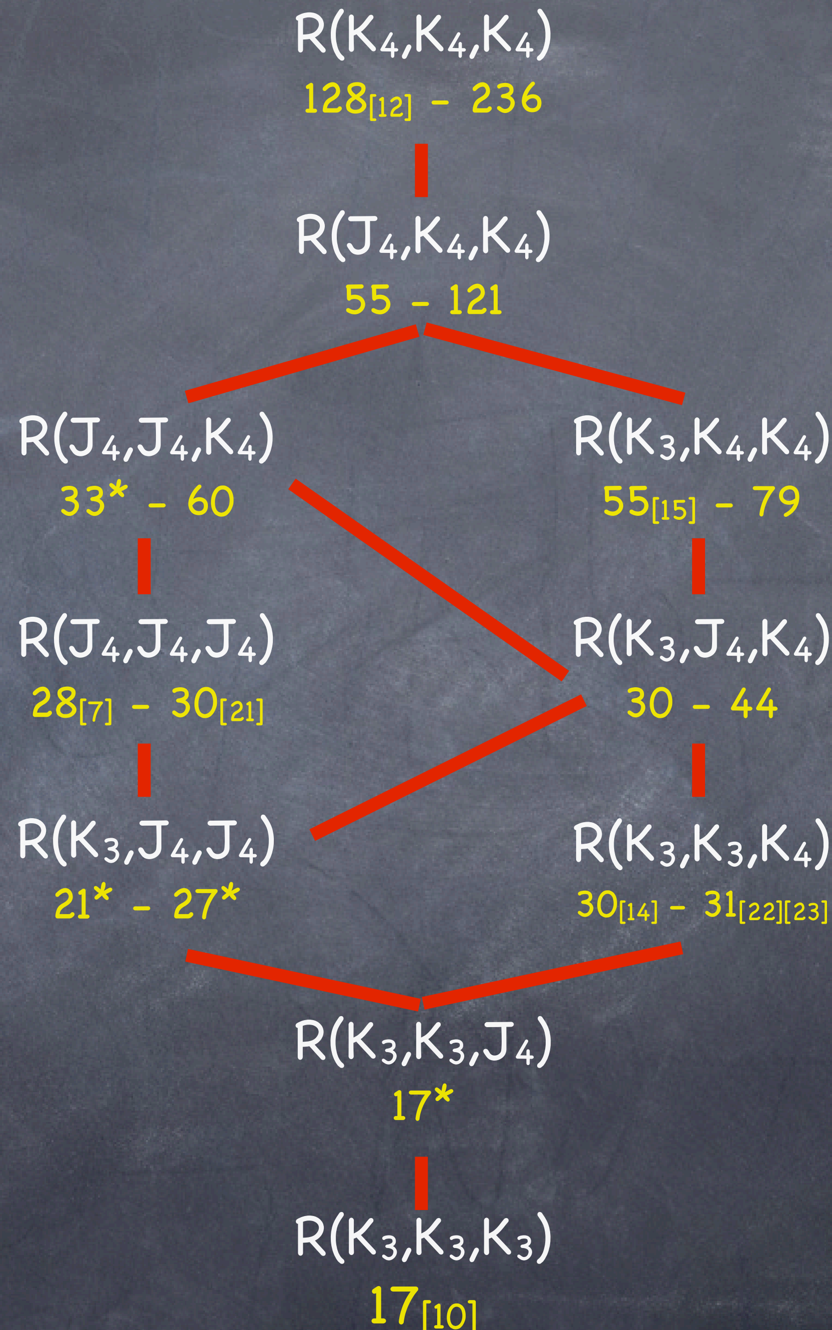


$R(K_3,K_3) \leq 6$

$J_n = K_n - e$

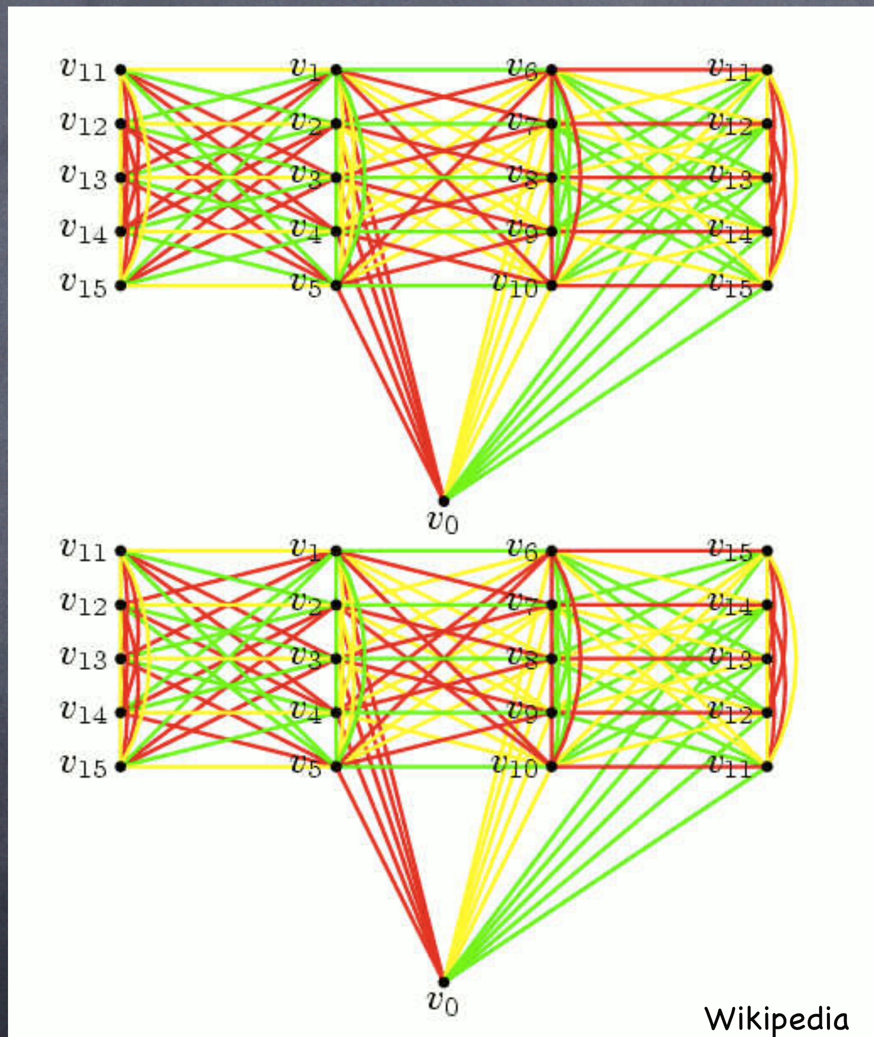
$J_4 = K_4 - e$





R(K4,K4,K4)
128[12] - 236
R(J4,K4,K4)
55 - 121
R(J4,J4,K4)
33* - 60
R(K3,K4,K4)
55[15] - 79
R(J4,J4,J4)
28[7] - 30[21]
R(K3,J4,K4)
30 - 44
R(K3,J4,J4)
21* - 27*
R(K3,K3,K4)
30[14] - 31[22][23]
R(K3,K3,J4)
17*
R(K3,K3,K3)
17[10]

(K_3,K_3,K_3;16)-colorings

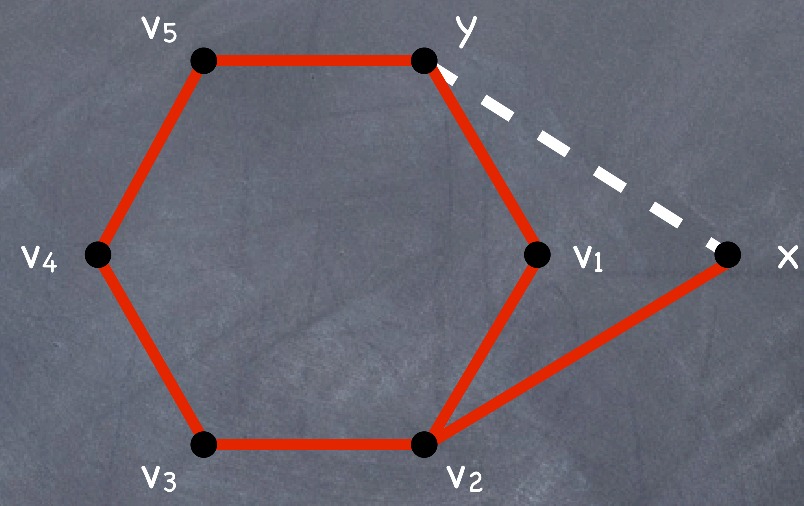


Previous Results

- $R(K_3+e,K_3+e,K_3+e) = R(K_3,K_3,K_3) = 17$
  - Yuansheng and Rowlinson, 1994
- $R(K_3+e,K_3+e,K_4) = R(K_3,K_3,K_4)$
  - Arste, Klamroth, and Mengersen, 1996

New Results

Lemma 2: $J_7 \rightarrow (K_3+e, J_4)$



Lemma 3: If m is the largest order of all splittable (J_7,K_3)-good graphs then $R(K_3,K_3,J_4) = m+1$

New Results

Theorem 1: $R(K_3,K_3,J_4) = R(K_3+e,K_3+e,J_4)$ [= 17]

Proof: Computational

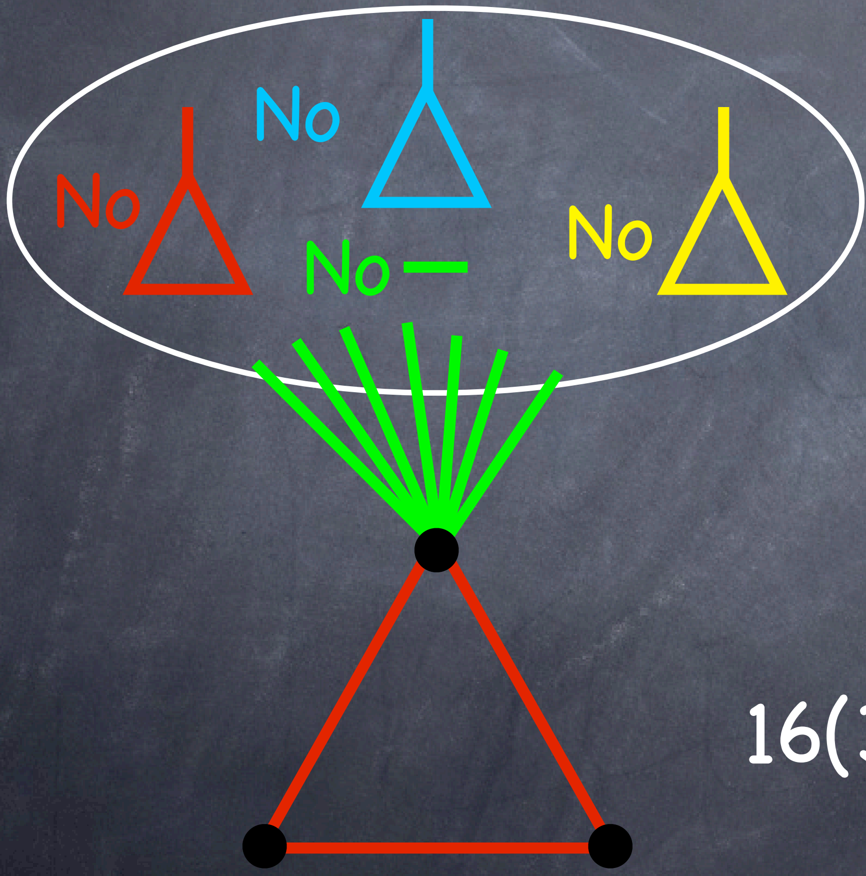
Theorem 2:

(a) If $R_4(K_3) = 51$, then $R_4(K_3+e) = R(K_3,K_3,K_3,K_3+e) = 52$

(b) If $R_4(K_3) > 51$, then $R_4(K_3+e) = R(K_3)$

If $R_4(K_3)>51$ then $R_4(K_3+e)=R_4(K_3)$

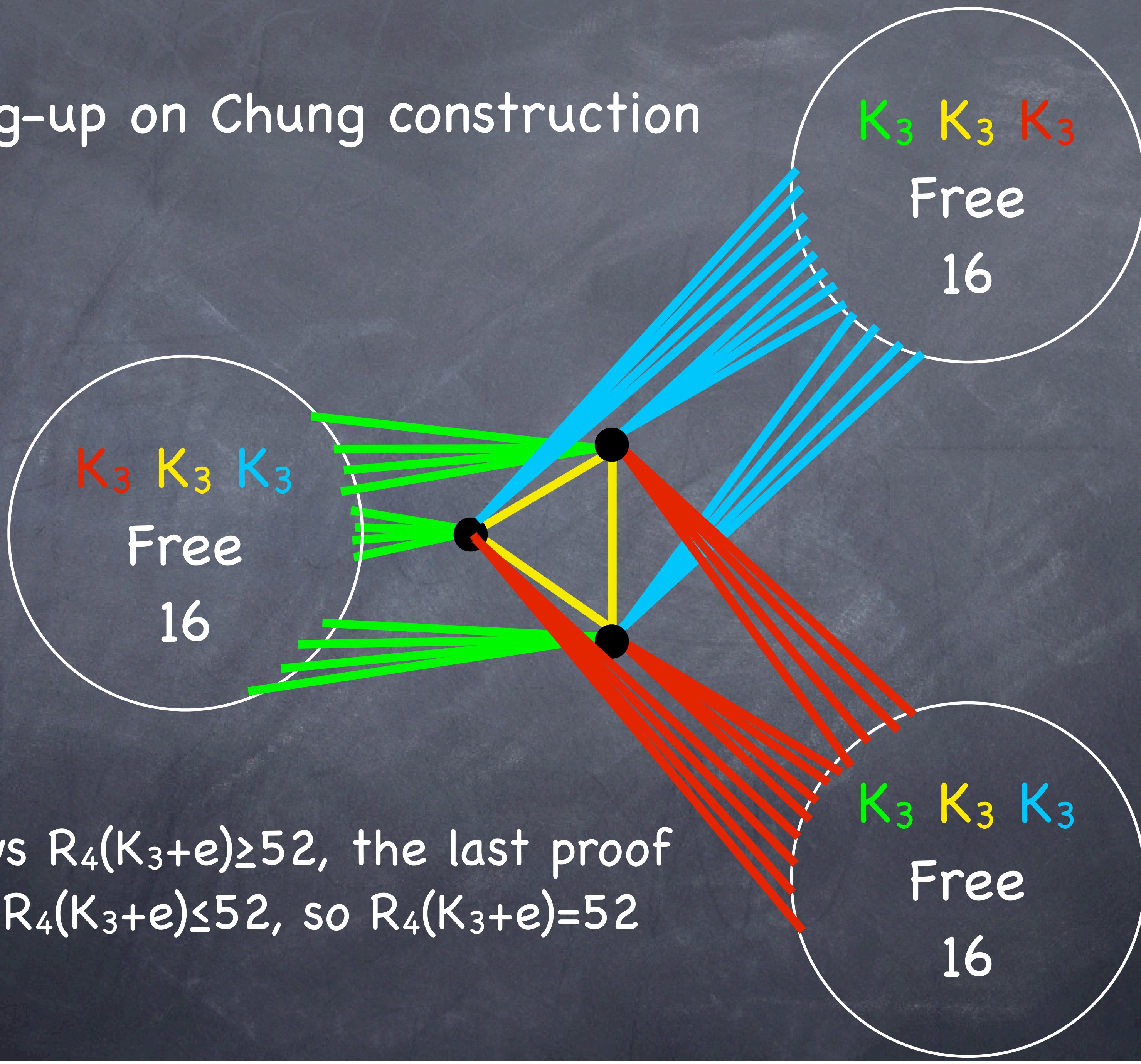
Consider any $_4(K_3+e;52)$-coloring



$(K_3+e, K_3+e, K_3+e; \leq 16)$-coloring

$16(3)+3=51$

Building-up on Chung construction



This shows $R_4(K_3+e) \geq 52$, the last proof showed $R_4(K_3+e) \leq 52$, so $R_4(K_3+e)=52$

Thanks!