The Parameters 4-(12,6,6) and Related t-Designs

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Abstract

It is shown that a 4-(12,6,6) design, if it exists, must be rigid. The intimate relationship of such a design with 4-(12,5,4) designs and 5-(12,6,3) designs is presented and exploited. In this endeavor we found: (i) 30 nonisomorphic 4-(12,5,4) designs; (ii) all cyclic 3-(11,5,6) designs; (iii) all 5-(12,6,3) designs preserved by an element of order three fixing no points and no blocks; and (iv) all 5-(12,6,3) designs preserved by an element of order two fixing 2 points.

1 Introduction

A simple $t-(v,k,\lambda)$ design is a pair $(X,\mathcal{D})$ where X is a v-element set of points and $\mathcal{D}$ is a collection of distinct k-element subsets of X called blocks such that: for all $T \subset X$, $|T| = t$, $|\{K \in \mathcal{D} : T \subset K\}| = \lambda$. For $v \leq 12$, 4-(12,6,6) is the only parameter case for which existence is unsettled. It is known that necessary conditions for the existence of a $t-(v,k,\lambda)$ design are that for each $0 \leq i \leq t$

$$\lambda \binom{v-i}{t-i} \equiv 0 \ \ (modulo \ \binom{k-i}{t-i}).$$

Given integers $0 \leq t \leq k \leq v$ the smallest positive integer λ such that these necessary conditions hold is said to be the minimum λ for the parameters t, k and v. It is usually denoted by $\underline{\lambda}$. The largest such λ is $\overline{\lambda} = \binom{v-t}{k-t}$ and it is achieved when all k-element subsets are chosen as blocks. It is now easy to see that $0 \leq \lambda \leq \overline{\lambda}$ and that $\underline{\lambda}$ divides λ. If $\lambda = 0$ or if $\lambda = \overline{\lambda}$ the design is said to be trivial. Furthermore, since whenever $(X,\mathcal{D})$ is a $t-(v,k,\lambda)$ design, then $(X, \binom{X}{k} - \mathcal{D})$ is a $t-(v,k,\overline{\lambda}-\lambda)$ design, we usually only search for $t-(v,k,\lambda)$ designs with $0 < \lambda \leq \overline{\lambda}/2$. The existence/nonexistence of $4-(12,6,\lambda)$ designs is given in Table I.

Table I

λ	Exists?	Construction
2	no	Dehon [D] and Oberschelp [O].
4	yes	5-(12,6,1) as a 4-design.
6	?	Unknown.
8	yes	5-(12,6,2) as a 4-design.
10	yes	Kreher and Radziszowski [KR2].
12	yes	5-(12,6,3) as a 4-design.
14	yes	Extension of 3-(11,5,14).

If a 4-(12,6,6) does not exist it would be the first known example when the yes's in such a table do not form an interval. This is perhaps strong evidence that a 4-(12,6,6) exists, but constructing the beast, as we shall see in the next sections, is a different matter. For the remainder of this paper $(X, \mathcal{D})$ will denote the possible 4-(12,6,6) design we search for.

2 Structure

For $I \subseteq X$, let $\lambda(I) = |\{K \in \mathcal{D} : K \supseteq I\}|$. If $0 \leq |I| = i \leq 4$, then $\lambda(I) = \lambda_i = 6\binom{12-i}{4-i}/\binom{6-i}{4-i}$. Thus $\lambda_0 = 198$, $\lambda_1 = 99$, $\lambda_2 = 45$, $\lambda_3 = 18$ and $\lambda_4 = 6$. For $S \subseteq X$, $|S| = s$, let $\alpha_i(S)$ be the number of blocks in $\mathcal{D}$ intersecting S in exactly i points. Note that $\alpha_i(S) = 0$, if $i > s$. The following equations hold:

$$\sum_{i=0}^{s} \binom{i}{j} \alpha_i(S) = \binom{s}{j} \lambda_j \tag{1}$$

for all $0 \leq j \leq 4$. For $|S| = 6$ there are only 4 solutions, A, B, C, and D to the equations in (1), and they are given in Table II. In particular α_6 is at most 1 so if a 4-(12,6,6) design exists it has no repeated blocks. Let N_A, N_B, N_C and N_D, be the number of $S \subseteq X$, $|S| = 6$ which yield solution A, B, C and D respectively. Clearly $N_A + N_D = |\mathcal{D}| = \lambda_0 = 198$.

Table II

α_0	α_1	α_2	α_3	α_4	α_5	α_6	solution
0	8	50	80	55	4	1	A
1	4	55	80	50	8	0	B
0	9	45	90	45	9	0	C
1	3	60	70	60	3	1	D

Remark: The existence of a 4-(12,6,2) is easily ruled out by using the equations in (1). In this case $\lambda = \lambda_4 = 2$, $\lambda_3 = 6$, $\lambda_2 = 15$, $\lambda_1 = 33$, $\lambda_0 = 66$ and $s = |S| = 6$. An alternating sum of these equations yields $\alpha_0 + \alpha_5 + 5\alpha_6 = 3$. But if S is a block, then $\alpha_6 = 1$ and this is a contradiction.

Let β_i be the number of 5-subsets appearing in exactly i of the blocks in $\mathcal{D}$. Note that $\beta_i = 0$ for $i \geq 6$, since α_5 is ≤ 4 for solution A and D in Table II. Counting, (1) the number of 5-element subsets, (2) the number of pairs $(F, K) \in \binom{X}{5} \times \mathcal{D}$ such that $F \subseteq K$, and (3) the number of unordered pairs of blocks intersecting in 5 elements; the following system of equations on the β_i holds:

$$\begin{aligned} \beta_0 + \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 &= \binom{12}{5} = 4\lambda_0 \; ; \\ \beta_1 + 2\beta_2 + 3\beta_3 + 4\beta_4 + 5\beta_5 &= \binom{6}{5}\lambda_0 = 6\lambda_0 \; ; \\ \beta_2 + 3\beta_3 + 6\beta_4 + 10\beta_5 &= (4N_A + 3N_D)/2 = (N_A + 3\lambda_0)/2 \; . \end{aligned}$$

Taking the linear combination with coefficients +1,-1,+1 of these 3 equations, respectively, gives

$$\beta_0 + \beta_3 + 3\beta_4 + 6\beta_5 = 4\lambda_0 - 6\lambda_0 + (N_A + 3\lambda_0)/2 = (N_A - \lambda_0)/2 \; .$$

Thus, $0 \le N_A - \lambda_0$. But also $N_A + N_D = 198 = \lambda_0$. Consequently $N_A = 198$, $N_D = 0$ and $\beta_0 + \beta_3 + 3\beta_4 + 6\beta_5 = 0$ so $\beta_0 = \beta_3 = \beta_4 = \beta_5 = 0$ and $\beta_1 = \beta_2 = 2\lambda_0 = \binom{12}{5}/2$. Let $\overline{\mathcal{D}} = \{X - K : K \in \mathcal{D}\}$ and for any $S \subseteq X$, $|S| = s$, set $\alpha(S) = [\alpha_0(S), \alpha_1(S), ..., \alpha_s(S)]$. We have the following structure theorems.

Theorem S1: *$\overline{\mathcal{D}} \cap \mathcal{D} = \emptyset$ and thus $\overline{\mathcal{D}} \cup \mathcal{D}$ is a simple 5-(12,6,3) design.*

Proof: To show that $\overline{\mathcal{D}} \cup \mathcal{D}$ is a 5-(12,6,3) design consider any 5-element set $S \subseteq X$. Let $\Delta(S) = |\{K \in \mathcal{D} : K \cap S = \emptyset\}|$. Then by inclusion-exclusion $\Delta(S) = \lambda_0 - 5\lambda_1 + 10\lambda_2 - 10\lambda_3 + 5\lambda_4 - \lambda_5(S)$, where $\lambda_5(S)$ is the number of blocks of $\mathcal{D}$ containing S. Thus the number of blocks in $\overline{\mathcal{D}} \cup \mathcal{D}$ containing S is $\Delta(S) + \lambda_5(S) = 3$ and therefore $\overline{\mathcal{D}} \cup \mathcal{D}$ is a 5-(12,6,3) design. In order for it to be simple we need $\overline{\mathcal{D}} \cap \mathcal{D} = \emptyset$. This follows since $N_A = 198$ and $N_D = 0$. So from Table II we see that $\alpha_0 = 0$. □

Theorem S2: *Let S be any 6-element subset in X. Then*

$$\alpha(S) = \begin{cases} [0,8,50,80,55,4,1] & \textit{if } S \in \mathcal{D}\,; \\ [1,4,55,80,50,8,0] & \textit{if } S \in \overline{\mathcal{D}}\,; \\ [0,9,45,90,45,9,0] & \textit{if } S \notin (\mathcal{D} \cup \overline{\mathcal{D}})\,. \end{cases}$$

Proof: This also follows from $N_A = 198$, $N_D = 0$ and Table II. □

Theorem S3:

(i) *Exactly one half of the 5-element subsets are contained in precisely one block of $\mathcal{D}$; the other half are in two. Thus $\mathcal{D} \cup \overline{\mathcal{D}}$ is a* 5-(12,6,3) *design.*

(ii) *Exactly one half of the 7-element subsets contain precisely one block of $\mathcal{D}$; the other half contain two.*

Proof: (i) follows from $\beta_1 = \beta_2 = \binom{12}{5}/2$ and (ii) is because the complement of a 5-element subset is a 7-element subset. □

Theorem S4: *Every block $K \in \mathcal{D}$ can be written uniquely as $K = F_K \cup P_K$ where*

(i) *$F_K = \{f_1^K, f_2^K, f_3^K, f_4^K\}$ and for each $i = 1,2,3,4$ there is exactly one $\hat{f}_i \in X - K$ with $K_i = (K - \{f_i^K\}) \cup \{\hat{f}_i^K\} \in \mathcal{D}$ and*

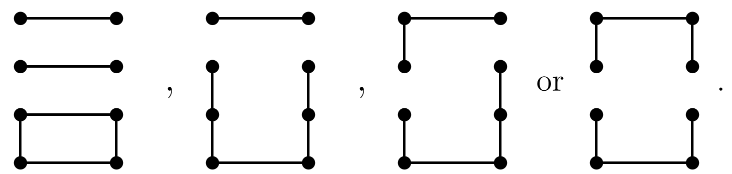
(ii) *$P_K = \{p_1^K, p_2^K\}$ and the only block containing $(K - \{p_i^K\})$ is K itself, $i = 1,2$.*

Proof: This follows from $\alpha_5(K) = 4$ and Theorem S3. □

The sets $\{F_K : K \in \mathcal{D}\}$ are called special four-sets, the pairs $\{P_K : K \in \mathcal{D}\}$ are called special pairs. Let $F \subseteq X$ be any 4-element set. We define the graph of F to be $\Gamma(F) = (V, E)$

with vertices $V = X - F$ and edges $E = \{\{v, w\} : F \cup \{v, w\} \in \mathcal{D}\}$. The graph $\Gamma(F)$ is also often called the derived design of $\mathcal{D}$ with respect to F.

Theorem S5: *For any 4-element set F, $\Gamma(F)$ is isomorphic to*



Proof: $\Gamma(F)$ has only 6 edges since $\lambda_4 = 6$. By Theorem S3(i), five element subsets are in either one or two blocks. Thus each vertex has degree 1 or 2. Also $\Gamma(F)$ has no triangles by Theorem S3(ii). The only graphs on 8 vertices satisfying this are given above. □

Theorem S6: *Let $\mathcal{F}_i = \{S \in \binom{X}{5} : |\{K \in \mathcal{D} : K \supseteq S\}| = i\}$, $i = 1, 2$. Then $(X, \mathcal{F}_1)$ and $(X, \mathcal{F}_2)$ partition $\binom{X}{5}$ into two disjoint $4-(12, 5, 4)$ designs.*

Proof: By Theorem S3 we have $|\mathcal{F}_1| = |\mathcal{F}_2| = \binom{12}{5}/2$ and that $\mathcal{F}_1 \cap \mathcal{F}_s = \emptyset$. Thus it need only be shown that $\mathcal{F}_i$, $i = 1$ or 2, is a 4-(12,5,4) design. Let $F \subseteq X$ be any 4-element set. Then by Theorem S5 the graph $\Gamma(F)$ has precisely four vertices of degree 1 and four vertices of degree 2. The four vertices of degree i correspond to four blocks in $\mathcal{F}_i$ containing F. □

3 Automorphisms

In this section $aut(\mathcal{B})$ denotes the automorphism group of the set system $\mathcal{B}$. In particular let G be the automorphism group of a possible 4-(12,6,6) design $(X, \mathcal{D})$. Keep in mind that this means G is also the automorphism group of the 4-(12,6,6) design $(X, \overline{\mathcal{D}})$ and the 5-(12,6,3) design $(X, \mathcal{D} \cup \overline{\mathcal{D}})$. The structure theorems establish:

(1) $K \in \mathcal{D}$ if and only if K contains precisely two members of $\mathcal{F}_1$ and four members of $\mathcal{F}_2$;

(2) $K \in \overline{\mathcal{D}}$ if and only if K contains precisely four members of $\mathcal{F}_1$ and two members of $\mathcal{F}_2$;

(3) $S \in \mathcal{F}_1$ if and only if S is contained in precisely one member of $\mathcal{D}$ and two members of $\overline{\mathcal{D}}$; and

(4) $S \in \mathcal{F}_2$ if and only if S is contained in precisely two members of $\mathcal{D}$ and one member of $\overline{\mathcal{D}}$.

This intimate relationship between the set systems $\mathcal{F}_1$, $\mathcal{F}_1$, $\mathcal{D}$ and $\overline{\mathcal{D}}$ implies that their automorphism groups are identical. We therefore have the following theorem.

Theorem A1: *$G = aut(\mathcal{F}_1) = aut(\mathcal{F}_2) = aut(\mathcal{D}) = aut(\overline{\mathcal{D}})$.*

We now proceed to systematically rule out possible orders of automorphisms in G. We of course need not consider automorphisms of prime order exceeding 11 since G is a permutation group on 12 symbols.

Theorem A2: *Let $F \subseteq X$ be any 4-element set. Then $G_F = \{g \in G : F^g = F\}$ is a 2-group.*

Proof: If $g \in G_F$ then $g^* = g|_{X-F}$ is an automorphism of $\Gamma(F)$. But by Theorem S5, it is easy to see $aut(\Gamma(F))$ is a 2-group. □

Theorem A3: *There are no elements of order 11 in G.*

Proof: Using a backtracking algorithm we have found that there are exactly 70 distinct (0,1)-solutions U to the system of 15 equations in 42 unknowns

$$A_{3,5}U \;=\; 6J,$$

where $A_{3,5}$ is the incidence matrix of 15 orbits of 3-sets versus 42 orbits of 5-sets for the group generated by $g = (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$. These solutions yield all distinct cyclic 3-(11,5,6) designs. If G has an element of order 11, then its derived design through the fixed point has to be one of the 70 designs obtained by the above procedure. The full automorphism group of these designs is by virtue of the element g primitive. It is therefore easily checked that their full automorphism group is cyclic of order 11. Hence any two such designs are isomorphic if and only if they are isomorphic by an element of $H = \langle g, h\rangle$ where $h = (1, 2, 4, 8, 5, 10, 9, 7, 3, 6)$. The resulting set of 7 nonisomorphic solutions is given in Table III. Theorems S3 and S6 imply that each 4-set R must be covered 1 or 2 times in the derived design. However for each of the 7 nonisomorphic cyclic 3-(11,5,6) designs a 4-set can be found that is covered 0 or at least 3 times, thus none of them extends to a 4-(12,6,6). □

Table III: The 7 nonisomorphic cyclic 3-(11,5,6) designs.

No.	*Base blocks*				
1	0 1 2 5 6	0 1 2 3 7	0 2 4 5 6	0 1 2 4 7	0 2 3 5 7
	0 1 4 6 9	0 2 3 6 10	0 2 3 4 10	0 2 5 8 9	
2	0 1 2 4 6	0 1 2 5 6	0 2 3 4 8	0 1 2 3 8	0 1 2 5 8
	0 2 3 5 9	0 2 4 5 9	0 2 3 6 10	0 2 3 5 10	
3	0 1 2 4 6	0 1 2 5 6	0 2 3 4 7	0 1 4 5 7	0 2 3 4 8
	0 1 2 3 8	0 2 4 5 8	0 2 3 5 9	0 2 3 5 10	
4	0 1 2 4 6	0 1 2 5 6	0 2 3 4 7	0 2 4 5 6	0 2 3 5 7
	0 1 4 5 7	0 1 2 3 8	0 2 3 5 8	0 2 3 6 10	
5	0 1 2 3 6	0 1 4 5 6	0 1 2 4 7	0 2 3 5 7	0 2 5 6 8
	0 2 3 4 8	0 2 4 5 9	0 2 3 6 10	0 2 3 4 10	
6	0 1 2 3 6	0 2 3 4 7	0 2 4 5 6	0 1 4 5 6	0 1 2 4 8
	0 2 3 5 8	0 1 4 5 9	0 2 3 5 10	0 2 4 6 10	
7	0 1 2 4 5	0 1 2 3 7	0 2 4 5 6	0 1 2 5 7	0 2 3 4 8
	0 2 3 5 8	0 1 4 5 8	0 2 3 5 10	0 2 4 6 10	

Theorem A4: *There are no elements of order 7 in G.*

Proof: Every element of order 7 must fix a 4-element set. This is impossible by Theorem A2. □

Theorem A5: *There are no elements of order 5 in G.*

Proof: Let $g \in G$, $|g| = 5$. Then g cannot fix 4 or more points for otherwise we violate Theorem A2. Thus we may assume without loss that $X = \{1, 2, 3, ..., 12\}$ and $g = (1, 2, 3, 4, 5)(6)(7, 8, 9, 10, 11)(12)$. Now $|\mathcal{D}| = 198 \equiv 3 \pmod 5$. So there is a $K \in \mathcal{D}$ (in fact at least three of them) such that $K^g = K$. Without loss let $K = \{1, 2, 3, 4, 5, 6\}$. But by Theorem S1 we know that $\alpha_1(K) = 8$. This is impossible given the automorphism g. $\Box$

Theorem A6: *Elements of order 3 in G fix no points and no blocks in $\mathcal{D}$ or in $\overline{\mathcal{D}}$.*

Proof: If g fixes a point then it must fix some set F of 4 points and act as an element of order 3 on the remaining points $X - F$. This is impossible by Theorem A2.

Suppose g fixes a block K. Then since by the above g fixes no points it is impossible for g to fix any block intersecting K in exactly one point. This contradicts $\alpha_1(K) = 8$. This also holds for $\overline{\mathcal{D}}$, since it is isomorphic to $\mathcal{D}$. $\Box$

We show in Theorem A7 with the aid of a computer that G contains no elements of order 3 and thus in particular 9 does not divide the order of G. This last fact can also be established by a much easier proof. If G contains an element g of order 9, then g^3 would have order 3 and fix 3 points, contrary to Theorem A6. Thus, if 9 divides the order of G then G contains a subgroup H isomorphic to $Z_3 \times Z_3$. Then there exists $a, b \in G$ such that $|a| = |b| = 3$ and $bab^2 = a$. By Theorem A6 neither a nor b can fix points. It is impossible to find such automorphisms on 12 points.

Theorem A7: *There are no elements of order 3 in G.*

Proof: If G contains an element of order 3, then we may assume by Theorem A6 that the 5-(12,6,3) design $\mathcal{D} \cup \overline{\mathcal{D}}$ is fixed by the automorphism (0,1,2)(3,4,5)(6,7,8)(9,10,11) and that it fixes no blocks. A computer search by the method described in [R] establishes that there are exactly 7 such nonisomorphic designs. These 7 designs are displayed in Table IV and it is easily checked by a backtracking algorithm that none of them can be partitioned into the required 4-(12,6,6) designs. $\Box$

Table IV

Design I. $G_I = \langle(0,5,10)(1,3,11)(2,4,9)(6,7,8),(0,3,9)(1,4,10)(2,5,11)(6,8,7)\rangle$. $|G_I| = 9$.

Orbit representatives of G_I generating 5-(12,6,3) design I.

0 1 3 6 8 4	0 1 3 6 8 9	0 1 3 6 4 10	0 1 3 6 7 11	0 1 3 6 9 11	0 1 3 8 4 9
0 1 3 8 7 9	0 1 3 8 7 11	0 1 3 8 10 11	0 1 3 4 7 10	0 1 3 4 7 11	0 1 3 4 10 11
0 1 3 9 10 11	0 1 6 8 4 7	0 1 6 8 7 11	0 1 6 8 9 11	0 1 6 4 7 9	0 1 6 4 9 11
0 1 8 4 9 11	0 1 4 7 9 11	0 1 7 9 10 11	0 3 6 8 4 11	0 3 6 4 7 11	0 3 6 4 9 11
0 3 8 4 7 11	0 3 8 9 10 11	0 3 4 9 10 11	0 6 7 9 10 11	0 8 4 9 10 11	1 3 6 8 4 7
1 3 6 8 4 11	1 3 6 8 7 9	1 3 6 4 7 9	1 3 6 4 7 10	1 3 8 4 9 11	1 3 4 7 9 11
1 3 4 9 11 5	1 3 7 9 10 11	1 6 8 9 10 11	1 8 4 7 9 11	3 6 8 4 9 11	3 6 8 9 10 11
3 6 4 9 10 11	3 4 7 9 10 11				

Design II. $G_{II} = \langle(0,5)(1,3)(2,4)(6,10)(7,11)(8,9),$
$(0,8)(1,6)(2,7)(3,10)(4,11)(5,9),(0,11,1,9,2,10)(3,8,4,6,5,7)\rangle$. $|G_{II}| = 12$.

Orbit representatives of G_{II} generating 5-(12,6,3) design II.

0 1 3 6 9 4	0 1 3 6 9 7	0 1 3 6 4 7	0 1 3 6 5 10	0 1 3 6 8 7	0 1 3 9 4 5
0 1 3 9 4 8	0 1 3 9 5 8	0 1 3 4 5 11	0 1 3 4 5 10	0 1 3 5 8 7	0 1 6 9 5 8
0 1 6 9 5 7	0 1 6 4 5 11	0 1 6 4 5 8	0 1 6 4 8 7	0 1 6 4 8 10	0 1 6 5 8 7
0 1 4 5 8 10	0 3 6 9 4 8	0 3 6 4 5 8	0 3 6 4 5 7	0 3 9 4 5 7	0 6 9 4 5 8
1 3 6 9 4 5	1 3 6 9 5 8	1 3 6 9 5 10	1 3 6 9 8 7	1 3 6 4 5 8	1 3 9 4 8 11
1 3 9 4 8 7	1 6 9 4 5 7	1 6 9 4 8 10	1 9 4 5 8 7	1 9 4 5 8 10	

Design III. $G_{III} = \langle(0,10,1)(2,9,11)(3,7,6)(4,8,5),$
$(0,2,1)(3,5,4)(6,8,7)(9,11,10)\rangle$. $|G_{III}| = 12$.

Orbit representatives of G_{III} generating 5-(12,6,3) design III.

0 1 3 6 8 4	0 1 3 6 8 9	0 1 3 6 4 10	0 1 3 6 9 11	0 1 3 8 4 10	0 1 3 8 7 9
0 1 3 4 7 9	0 1 3 9 10 11	0 1 6 8 4 7	0 1 6 8 7 9	0 1 6 8 7 10	0 1 6 4 7 11
0 1 6 4 9 10	0 1 6 4 9 11	0 1 6 7 9 10	0 1 8 4 9 10	0 1 4 7 9 10	0 1 4 7 10 11
0 1 7 9 10 11	0 1 7 9 11 5	0 3 6 8 4 11	0 3 6 8 9 10	0 3 6 4 7 9	0 3 6 4 10 11
0 3 6 7 9 10	0 3 8 4 7 9	0 6 8 4 7 10	0 6 8 4 9 10	0 6 8 4 10 11	0 6 4 7 9 11
0 6 7 9 10 11	1 3 6 8 4 7	1 3 6 8 7 10	1 3 6 8 9 10	1 3 6 4 7 9	1 3 6 4 9 10
1 3 6 7 9 10	1 3 8 7 9 5	1 8 4 7 9 10	3 6 8 4 7 10		

Design IV. $G_{IV} = \langle (0, 11, 1, 9, 2, 10)(3, 6, 4, 7, 5, 8),$
$(0, 4)(1, 3)(2, 5)(6, 11)(7, 10)(8, 9)\rangle$. $|G_{IV}| = 12$.

Orbit representatives of G_{IV} generating 5-(12,6,3) design IV.

0 1 3 6 4 9	0 1 3 6 4 5	0 1 3 6 7 8	0 1 3 4 7 9	0 1 3 4 7 5	0 1 3 4 9 8
0 1 3 7 9 5	0 1 3 7 8 5	0 1 6 4 7 9	0 1 6 4 7 11	0 1 6 4 9 5	0 1 6 4 8 5
0 1 6 7 9 8	0 1 6 7 5 11	0 1 6 9 8 5	0 1 4 7 8 5	0 1 7 9 8 5	0 3 6 4 7 8
0 3 6 4 9 11	0 3 6 7 9 5	0 3 6 9 8 11	0 3 4 7 9 8	0 3 4 9 5 11	0 3 7 9 8 11
0 6 4 7 9 8	0 6 4 7 9 5	0 6 4 9 8 11	0 6 7 9 5 11	0 4 9 8 5 11	1 3 6 4 7 8
1 3 6 4 7 5	1 3 6 4 9 8	1 3 6 4 9 11	1 3 6 7 9 8	1 3 6 7 9 5	1 3 6 9 8 5
1 3 4 9 8 5	1 6 7 9 8 11	1 4 7 9 8 5			

Design V. $G_V = \langle (0, 10, 6)(2, 3, 9)(5, 7, 11), (0, 5, 8)(1, 3, 6)(2, 4, 7)(9, 10, 11),$
$(0, 1, 3, 4, 7, 8)(2, 10, 5, 9, 6, 11)\rangle$. $|G_V| = 36$.

Orbit representatives of G_V generating 5-(12,6,3) design V.

0 1 3 6 4 9	0 1 3 6 9 5	0 1 3 4 9 10	0 1 3 4 9 8	0 1 3 4 5 10	0 1 3 4 10 8
0 1 6 4 9 5	0 1 6 4 5 10	0 3 6 4 9 10	1 3 6 4 9 8	1 3 6 4 5 8	1 3 6 9 5 8

Design VI. $G_{VI} = \langle (0, 4, 10)(2, 9, 8)(5, 7, 11), (0, 3, 5, 6, 8, 1)(2, 11, 4, 9, 7, 10)\rangle$.
$|G_{VI}| = 36$.

Orbit representatives of G_{VI} generating 5-(12,6,3) design VI.

0 1 3 6 4 7	0 1 3 4 7 9	0 1 3 7 5 11	0 1 6 4 7 9	0 1 4 7 9 8	0 3 6 4 7 8
0 3 6 4 9 8	1 3 6 4 7 8	1 3 6 4 7 5	1 3 6 4 9 8	1 3 4 7 9 8	

Design VII. $G_{VII} = \langle (0, 4, 11)(3, 8, 10)(5, 7, 9), (0, 7, 10)(1, 3, 6)(2, 11, 5)(4, 9, 8)\rangle$.
$|G_{VII}| = 144$.

Orbit representatives of G_{VII} generating 5-(12,6,3) design VII.

0 1 3 6 8 4	0 1 6 8 4 7	0 1 6 4 7 11	0 3 6 8 4 11	1 3 6 8 4 7

Theorem A8: *Elements of order 2 in G fix two points and exactly ten blocks.*

Proof: Let $fix(g) = |\{x \in X : x^g = x\}|$ be the number of fixed points of g. There are three cases.

Case 1 $fix(g) \in \{4, 6, 8\}$

In this case we may assume without loss of generality that $g = (5, 6)(7, 8)(9, 10)(11, 12)$, $(7, 8)(9, 10)(11, 12)$ or $(9, 10)(11, 12)$. For each possibility consider the 5-set $S = \{2, 3, 4, 5, 6\}$. By Theorem S3(i) there are exactly 3 blocks in $\mathcal{D} \cup \overline{\mathcal{D}}$ containing S. At least one of them must therefore be fixed by g, as $|g| = 2$. Hence we may assume (interchanging the roles of $\mathcal{D}$ and $\overline{\mathcal{D}}$ if necessary), that $K = \{1, 2, 3, 4, 5, 6\} \in \mathcal{D}$. This is up to relabeling the only way to construct a block containing S that is fixed by g. Let $F_K = \{f_1^K, f_2^K, f_3^K, f_4^K\} \subseteq K$ be the special 4-set of K as defined in Theorem S4. Then by examining the structure of the automorphism g we may assume $(f_1^K)^g = f_1^K = 1$ and $\hat{f}_1^K = 11$. This implies that $K' = \{11, 2, 3, 4, 5, 6\} \in \mathcal{D}$ and $K'' = \{12, 2, 3, 4, 5, 6\} \in \mathcal{D}$ Now we have three blocks K, K' and K'' each containing the same 5-element set S, contrary to Theorem S3(i).

Case 2 $fix(g) = 10$

Here without loss $g = (11, 12)$. Let $K \in \mathcal{D}$ be any block containing the pair 11,12, and write $K = F_K \cup P_K$, the decomposition into special sets, and set $A = K - \{11, 12\}$. If $11 \in F_K$, then there is $x \in X - K$ such that $K' = ((K - \{11\}) \cup \{x\}) \in \mathcal{D}$. So, $K'' = (K')^g$ $= ((K - \{12\}) \cup \{x\}) \in \mathcal{D}$ is also a block. But now $\Gamma(A)$ contains the edges $\{11, 12\}$, $\{11, x\}$ and $\{12, x\}$, a triangle. This is contrary to Theorem S5. The same result holds for 12. That is $P_K = \{11, 12\}$ is a special pair in every block that contains it.

Now let $\{x, y\} \in X - \{11, 12\}$ and set $H = \{x, y, 11, 12\}$. Thus for every edge $\{a, b\} \in \Gamma(H)$ we have $\{a, b, x, y, 11, 12\} \in \mathcal{D}$, and $\{11, 12\}$ is the special pair in this block. But this implies a and b both have degree 2 in $\Gamma(H)$. The edge $\{a, b\}$ was arbitrary so every vertex has degree 2, contradicting Theorem S5.

Case 3 $fix(g) \in \{0, 2\}$

Let $X = A_1 \cup A_2 \cup ... \cup A_6$ be any partition of X into parts all of size 2. A subset $S \subseteq X$ will be of type $\mathit{type}(S) = [a_0, a_1, a_2]$ just when $a_i = |\{j : |A_j \cap S| = i\}|$. Let $T_{[a_0,a_1,a_2]} = \{S \subseteq X : \mathit{type}(S) = [a_0, a_1, a_2]\}$ and set $b_i = |\{K \in \mathcal{D} : \mathit{type}(K) = [i, 6 - 2i, i]\}|$. The following equations hold:

$$\begin{aligned}
b_0 + b_1 + b_2 + b_3 &= 198 \\
b_2 + 3b_3 &= \sum_{F \in T_{[4,0,2]}} |\{K \in \mathcal{D} : K \supseteq F\}| = 6\binom{6}{2} = 90 \\
6b_1 + 10b_2 + 12b_3 &= \sum_{F \in T_{[3,2,1]}} |\{K \in \mathcal{D} : K \supseteq F\}| = 6\binom{6}{4}2^4 = 1440 \\
15b_0 + 9b_1 + 4b_2 &= \sum_{F \in T_{[2,4,0]}} |\{K \in \mathcal{D} : K \supseteq F\}| = 6\binom{6}{4}2^4 = 1440
\end{aligned}$$

The general solution to these equations is given by

$$[b_0, b_1, b_2, b_3] = [18 - b_3, 90 + 3b_3, 90 - 3b_3, b_3].$$

However, $0 \leq b_3 \leq \binom{6}{3}/2 = 10$ since whenever $K \in \mathcal{D}$ we have $\overline{K} \notin \mathcal{D}$. Thus $b_0 \geq 8$. But if the partition $A_1, A_2, ..., A_6$ is given by the six 2-cycles in a supposed automorphism g of order 2 fixing no points, then $b_0 = 0$. For if not then there would be a block $K \in \mathcal{D}$ with $K^g \cap K = \emptyset$. Hence there can be no such automorphism. We therefor conclude that an automorphisms g of order 2 must fix two points. Let $A_1 = \{x, y\}$ be the set of fixed points of g, and let $A_2, A_3, ..., A_6$ be the five sets of pairs given by the 2-cycles of g. The equations and solution given above must also hold for this partition. If S is any 5-element subset of X

fixed by g, then there is some block K in the 5-(12,6,3) design $\mathcal{D} \cup \overline{\mathcal{D}}$ that contains S and is also fixed by g. Hence, either K or $\overline{K}$ is a member of $\mathcal{D}$. We may therefore assume that at least half of the 6-element subsets fixed by g are blocks in $\mathcal{D}$. It follows that exactly ten of the 6-element subsets fixed by g are blocks in the design $\mathcal{D}$. □

Theorem A9: *The order of G is not divisible by 4.*

Proof: By Theorem A8 it is easy to see that G cannot contain an element of order 4. Thus any subgroup of G of order 4 must be generated by two commuting elements a and b of order 2. This is impossible without contradicting Theorem A8. □

Applying the above Theorems we now know that if a 4-(12,6,6) design $(X, \mathcal{D})$ exists, then either $|G| = 1$ or G contains exactly one nontrivial automorphism: a permutation of order 2 fixing 2 points and 10 blocks. Furthermore we also know that $\mathcal{D} \cup \overline{\mathcal{D}}$ is a 5-(12,6,3) design. Of course the automorphism group of $\mathcal{D} \cup \overline{\mathcal{D}}$ may contain additional automorphisms besides this permutation of order 2 fixing 2 points and 10 blocks. Using the same backtracking algorithm that established Theorem A7, we ran a complete search for all 5-(12,6,3) designs whose automorphism group contained an automorphism of order 2 fixing two points. There are exactly 6 such designs and we present them in Table V. The fact that $\lambda = 3$ and $t = 5$ for these designs made this search feasible. Each of these designs were checked to see if they could be split into 4-(12,6,6) designs $\mathcal{D}$ and $\overline{\mathcal{D}}$. Theorem S5, the automorphism and the backtracking algorithm were the principle tools used to do this splitting. We found after running searches to completion that none of the 5-(12,6,3) designs split. We conclude:

Theorem A10: *If a 4-(12,6,6) design exists, then it must be rigid.*

Table V: The 6 nonisomorphic 5-(12,6,3) designs fixed by $(0,1)(2,3)(4,5)(6,7)(8,9)$

Design 1. $H_1 = \langle (0,1)(2,3)(4,5)(6,7)(8,9), (2,8)(3,9)(10,11), (0,1)(3,9)(4,7),$
$(0,5,4)(2,10,9)(3,8,11)(1,6,7) \rangle$, $|H_1| = 48$

Orbit representatives of H_1

0 1 2 3 4 8	0 1 2 3 4 5	0 1 2 4 5 6	1 2 3 4 5 6	0 1 4 5 6 7	0 1 2 5 6 8	1 2 3 4 5 8
0 2 3 5 6 8	0 1 2 3 8 9	0 1 2 4 7 8	0 2 4 5 7 8	2 3 4 5 8 9	0 1 2 3 4 10	2 3 5 6 8 9
2 3 4 5 6 7	0 2 3 4 6 10	2 3 4 7 8 10	0 2 3 5 8 10	0 2 3 8 9 10	2 3 4 5 8 10	2 3 8 9 10 11

Design 2. $H_2 = \langle (0,1)(2,3)(4,5)(6,7)(8,9), (10,11)(4,8)(5,9), (1,6)(2,3)(4,8),$
$(0,1,2)(3,7,6)(10,8,5)(4,9,11) \rangle$, $|H_2| = 48$

Orbit representatives of H_2

0 2 3 4 7 8	0 2 3 4 5 8	0 1 2 3 4 5	0 1 2 3 4 6	0 1 3 4 5 6	0 1 2 3 6 7	0 1 2 4 5 8
1 2 3 4 6 8	1 2 4 5 6 8	0 1 2 4 7 8	0 1 4 5 6 7	0 1 4 5 8 9	1 4 5 6 8 9	2 3 4 5 8 9
0 1 4 5 6 10	0 4 5 7 8 10	2 4 5 8 9 10	0 2 4 5 8 10	0 3 4 5 6 10	1 3 4 5 8 10	4 5 8 9 10 11

Table V (continued)

Design 3. $H_3 = \langle\ (0,1)(2,3)(4,5)(6,7)(8,9),\ (2,8)(3,9)(10,11),$
$(2,8)(1,7)(4,5)(10,11)\ \rangle$, $|H_3| = 16$

Orbit representatives of H_3

1 2 3 4 7 8	0 1 2 3 4 6	0 1 2 3 4 5	0 1 3 4 5 6	0 1 2 3 4 8	0 1 2 3 6 7	0 1 4 5 6 7
0 2 3 4 5 8	0 1 2 4 6 8	0 2 4 5 6 8	0 1 2 6 7 8	0 1 2 3 6 10	0 1 2 3 8 9	1 2 4 5 7 8
2 3 4 5 8 9	0 2 3 6 8 9	0 1 2 4 5 10	0 2 4 5 6 10	0 1 2 4 6 10	0 2 3 5 6 10	0 1 4 6 7 10
0 1 2 5 8 10	1 2 3 4 8 10	1 2 4 5 8 10	0 1 2 7 8 10	0 2 3 6 8 10	2 3 4 8 9 10	0 1 4 5 10 11
1 2 4 6 7 10	2 3 4 5 6 10	0 2 3 7 8 10	1 2 3 5 6 10	0 2 3 5 8 10	0 1 2 4 10 11	0 1 2 3 10 11
0 2 3 4 10 11	0 1 6 7 10 11	0 1 2 6 10 11	2 3 4 5 10 11	0 4 5 6 10 11	0 3 4 6 10 11	0 2 4 8 10 11
1 2 3 8 10 11	0 2 6 8 10 11	2 4 5 8 10 11	2 3 8 9 10 11	1 2 7 8 10 11		

Design 4. $H_4 = \langle(0,1)(2,3)(4,5)(6,7)(8,9),(2,3)(4,7)(1,9),(11,10)(4,7)(6,5)\rangle$, $|H_4| = 16$

Orbit representatives of H_4

0 1 2 3 4 5	0 1 2 4 5 6	0 2 3 4 5 6	0 1 4 5 6 7	0 1 2 3 4 8	2 3 4 5 6 7	0 1 3 4 5 8
0 1 2 5 6 8	0 2 3 5 6 8	0 2 3 4 7 8	0 4 5 6 7 8	0 2 4 5 7 8	0 1 2 3 8 9	0 1 5 6 8 9
0 1 4 5 8 9	0 1 3 5 6 10	0 2 3 4 5 10	0 1 4 5 6 10	1 2 3 5 6 10	0 2 4 5 6 10	2 4 5 6 7 10
0 1 2 4 8 10	0 2 3 5 8 10	0 1 3 4 6 10	0 2 4 5 8 10	0 1 4 7 8 10	0 4 5 6 8 10	0 1 2 3 6 10
0 1 2 8 9 10	0 2 4 6 7 10	0 1 2 3 10 11	0 1 5 7 8 10	0 1 3 5 8 10	0 1 3 4 10 11	1 2 4 5 10 11
0 1 4 5 10 11	2 3 4 5 10 11	0 2 5 6 10 11	2 3 5 6 10 11	1 4 5 6 10 11	4 5 6 7 10 11	0 2 4 8 10 11
0 2 3 8 10 11	0 5 6 8 10 11	0 1 5 8 10 11	0 4 7 8 10 11	0 1 8 9 10 11		

Design 5. $H_5 = \langle(0,1)(2,3)(4,5)(6,7)(8,9),(0,5,9)(2,3,10)(1,8,4)(11,6,7)\rangle$, $|H_5| = 6$

Orbit representatives of H_5

0 2 3 4 6 8	0 2 3 4 6 7	0 1 2 4 5 6	0 1 2 3 4 5	0 1 2 3 5 6	0 1 2 3 6 7	1 2 4 5 6 7
0 1 4 5 6 7	2 3 4 5 6 7	0 1 3 4 6 8	1 2 3 4 6 8	0 1 4 5 6 8	0 2 3 5 6 8	0 2 4 5 6 8
1 2 3 4 5 8	1 3 4 5 6 8	0 1 2 6 7 8	0 3 4 6 7 8	1 2 4 6 7 8	0 1 2 3 5 8	0 1 3 5 7 8
0 1 2 4 7 8	0 2 3 5 7 8	0 1 3 4 7 8	1 3 5 6 7 8	2 3 4 5 7 8	1 2 4 5 7 8	0 2 4 5 7 8
1 2 3 6 7 8	0 1 5 6 7 8	0 2 5 6 7 8	0 1 2 3 8 9	3 4 5 6 7 8	0 1 3 4 8 9	0 2 3 4 6 10
1 2 5 6 8 9	2 3 4 5 8 9	0 1 4 5 8 9	2 3 5 6 8 9	0 2 3 4 5 10	0 1 6 7 8 9	2 3 6 7 8 9
0 1 3 4 6 10	0 1 2 3 6 10	0 2 3 6 8 9	0 1 3 5 6 10	1 3 4 5 6 10	0 1 4 6 7 10	1 3 4 6 7 10
1 2 4 6 7 10	2 3 4 6 7 10	1 2 3 4 8 10	1 2 3 6 8 10	1 5 6 7 8 10	2 3 5 6 8 10	0 2 3 7 8 10
1 3 5 6 8 9	1 3 6 7 8 10	2 6 7 8 9 10	2 4 6 7 8 10	0 2 4 6 7 11	0 2 6 7 8 10	0 4 6 7 8 10
0 1 2 6 7 11	1 3 4 6 7 11	0 4 5 6 7 11	0 3 5 6 8 10	0 1 3 6 8 9	4 5 6 7 8 10	4 5 6 7 8 9
3 4 6 7 8 9	1 4 6 7 8 9	0 3 6 7 8 10	2 3 6 7 8 11	0 3 6 7 8 11	2 5 6 7 8 11	1 4 6 7 8 11
2 3 6 7 10 11						

Design 6. $H_6 = \langle (0,1)(2,3)(4,5)(6,7)(8,9) \rangle$, $|H_6| = 2$

Orbit representatives of H_6

0 1 2 3 4 5	0 2 3 4 5 6	0 1 2 3 5 6	0 1 2 3 6 7	0 1 3 4 6 7	0 2 4 5 6 7	0 1 4 5 6 7
2 3 4 5 6 7	0 1 2 4 5 8	0 1 2 4 6 8	0 1 3 4 6 8	0 1 3 5 6 8	1 2 3 4 6 8	0 2 3 5 6 8
1 3 4 5 6 8	0 2 4 5 6 8	0 1 3 5 7 8	0 2 4 6 7 8	0 3 4 6 7 8	1 2 4 6 7 8	1 2 3 4 5 8
0 1 2 3 7 8	0 3 4 5 7 8	0 2 3 5 7 8	0 2 3 4 7 8	0 1 4 5 7 8	1 2 4 5 7 8	1 2 3 6 7 8
0 1 5 6 7 8	3 4 5 6 7 8	1 2 5 6 7 8	2 3 5 6 7 8	0 2 3 4 8 9	0 1 2 3 8 9	0 1 3 4 8 9
0 1 4 5 8 9	0 1 2 6 8 9	2 3 4 5 8 9	0 2 3 6 8 10	0 1 4 5 6 10	0 2 6 7 8 9	1 3 5 6 8 9
2 3 4 6 8 9	0 3 4 6 8 9	0 2 5 6 8 9	0 3 5 6 8 9	2 4 5 6 8 9	1 4 5 6 8 9	0 1 6 7 8 9
2 3 6 7 8 9	1 4 6 7 8 9	0 1 2 3 4 10	4 5 6 7 8 9	0 2 3 4 6 10	0 1 2 4 6 10	1 2 3 5 6 10
1 3 4 6 7 10	1 2 4 5 6 10	1 3 4 5 6 10	0 3 4 5 6 10	0 1 2 6 7 10	1 2 4 6 7 10	2 3 4 6 7 10
0 1 2 4 8 10	0 1 3 4 8 10	0 1 2 5 8 10	0 2 3 5 8 10	1 3 4 5 8 10	0 3 4 5 8 10	2 3 4 5 8 10
0 1 3 6 8 10	1 3 4 7 8 10	0 4 5 6 8 10	2 3 4 6 8 10	1 2 3 6 8 10	1 2 5 6 8 10	0 1 5 6 8 10
1 2 3 7 8 10	2 4 5 6 8 10	0 1 2 7 8 10	0 1 4 7 8 10	0 2 6 7 8 10	1 3 5 7 8 10	2 3 4 7 8 10
0 2 5 7 8 10	2 4 5 7 8 10	4 5 6 7 8 10	1 4 6 7 8 10	0 3 6 7 8 10	0 4 6 7 8 10	1 5 6 7 8 10
3 5 6 7 8 10	0 2 4 8 9 10	0 2 3 8 9 10	1 2 4 8 9 10	0 4 5 8 9 10	0 1 6 8 9 10	0 1 2 6 8 11
1 2 4 5 6 11	1 2 6 8 9 10	1 4 6 8 9 10	0 4 6 8 9 10	3 5 6 8 9 10	3 4 6 8 9 10	2 5 6 8 9 10
2 6 7 8 9 10	0 1 2 4 5 11	0 1 3 5 6 11	0 2 3 4 6 11	0 1 3 4 6 11	1 2 3 4 6 11	0 1 2 5 6 11
0 2 3 6 7 11	2 3 4 5 6 11	0 2 4 6 7 11	1 2 4 6 7 11	0 2 3 4 8 11	0 4 5 6 7 11	0 1 2 3 8 11
0 1 3 5 8 11	1 2 3 4 8 11	1 2 3 5 8 11	0 2 4 5 8 11	0 3 4 5 8 11	0 1 2 6 10 11	1 2 5 6 8 11
0 1 4 6 8 11	0 2 3 6 8 11	0 4 5 6 8 11	2 3 5 6 8 11	1 4 5 6 8 11	1 3 4 7 8 11	0 1 2 7 8 11
3 4 5 6 8 11	0 1 4 7 8 11	1 3 5 7 8 11	2 3 4 7 8 11	0 2 5 7 8 11	1 4 5 7 8 11	2 4 5 7 8 11
0 1 6 7 8 11	3 4 6 7 8 11	0 3 6 7 8 11	1 3 6 7 8 11	2 4 6 7 8 11	0 5 6 7 8 11	2 5 6 7 8 11
0 1 4 8 9 11	1 2 4 8 9 11	2 4 5 8 9 11	1 3 4 8 9 11	2 4 6 8 9 11	2 3 6 8 9 11	0 3 6 8 9 11
1 2 6 8 9 11	1 3 6 8 9 11	0 4 6 8 9 11	1 5 6 8 9 11	0 1 2 3 10 11	4 6 7 8 9 11	0 1 4 5 10 11
1 2 3 4 10 11	0 2 4 5 10 11	2 3 4 5 10 11	0 3 4 6 10 11	0 1 4 6 10 11	1 2 3 6 10 11	0 2 5 6 10 11
0 3 5 6 10 11	1 3 5 6 10 11	2 3 6 7 10 11	2 4 5 6 10 11	0 1 6 7 10 11	3 4 6 7 10 11	0 4 6 7 10 11
4 5 6 7 10 11	0 1 3 8 10 11	0 3 4 8 10 11	0 2 4 8 10 11	1 2 4 8 10 11	0 1 5 8 10 11	1 2 5 8 10 11
2 3 5 8 10 11	1 4 5 8 10 11	0 2 6 8 10 11	1 3 6 8 10 11	0 3 7 8 10 11	1 4 6 8 10 11	3 4 6 8 10 11
2 4 6 8 10 11	0 5 6 8 10 11	3 5 6 8 10 11	1 2 7 8 10 11	0 4 7 8 10 11	2 3 7 8 10 11	0 5 7 8 10 11
3 5 7 8 10 11	4 5 7 8 10 11	0 1 8 9 10 11	2 6 7 8 10 11	1 6 7 8 10 11	4 5 8 9 10 11	0 2 8 9 10 11
2 3 8 9 10 11	3 4 8 9 10 11	0 6 8 9 10 11	5 6 8 9 10 11	6 7 8 9 10 11		

4 The known 4-(12,5,4) designs.

In this section we list the known 4-(12,5,4) designs. Because of Theorem S6, it was anticipated that an existing 4-(12,5,4) might lead directly to the construction of a 4-(12,6,6). In particular, if $(X, \mathcal{B})$ is a 4-(12,5,4) design let $\mathcal{D}_i = \{S \in \binom{X}{6} : |\{F \in \mathcal{B} : F \subseteq S\}| = i\}$ $i = 0, , 1, ..., 6$. Then $(X, \mathcal{D}_2)$, (or $(X, \mathcal{D}_4)$), might just be a 4-(12,6,6). In several cases (see our list below) $\mathcal{D}_2$ had exactly the right number, namely 198, of 6-sets to temporarily boost our enthusiasm that a 4-(12,6,6) would arise in this fashion. Unfortunately, we did not find a 4-(12,6,6) via this process. In view of Theorem A10, we know now that many of these 4-(12,5,4)'s could not yield a 4-(12,6,6), but it is still of independent interest to list what is known concerning 4-(12,5,4)'s. For each known 4-(12,5,4) we give generators of its automorphism group G and the size of this group. We also give the vector V, where $V[i] = |\mathcal{D}_i|$, $i = 0, 1, ..., 6$. Finally, we used a graph-isomorphism program, (due to Brendan McKay) to aid in determining nonisomorphism between our 4-(12,5,4)'s.

A.E. Brouwer in [B] reports that Design 1 was known to R.H.F. Denniston.

Design 1. $G_1 = \langle (0,1,2,3,4)(6,7,8,9,10), (0,5)(2,3)(6,11)(8,9), (1,2,4,3)(7,8,10,9) \rangle$.
$|G_1| = 120$. $V_1 = $ (7,0,135,640,135,0,7).

Orbit representatives of G_1 generating 4-(12,5,4) design 1.

0 1 2 3 4	0 1 3 4 8	0 1 6 8 10	0 6 7 8 9	0 1 2 6 7	0 1 3 6 8	0 1 8 9 10

Designs 2 through 16 were found by the authors and are apparently new.

Design 2. $G_2 = \langle (0,1,2,3,4)(6,7,8,9,10), (0,5)(2,3)(6,11)(8,9) \rangle$.
$|G_2| = 60$. $V_2 = $ (11,6,75,740,75,6,11).

Orbit representatives of G_2 generating 4-(12,5,4) design 2.

0 1 2 3 4	0 1 3 7 8	0 1 6 7 9	0 3 4 7 8	0 1 3 4 6	0 1 3 8 10
0 1 7 8 9	0 6 7 8 10	0 1 3 6 7	0 1 4 7 8	0 1 7 8 10	0 7 8 9 10

Design 3. $G_3 = \langle (0,1,2,3,4,5,6,7,8,9,10) \rangle$.
$|G_3| = 11$. $V_3 = $ (0,0,198,528,198,0,0).

Orbit representatives of G_3 generating 4-(12,5,4) design 3.

0 1 2 3 4	0 1 2 6 9	0 1 3 6 8	0 1 4 9 11	0 1 2 3 5	0 1 2 7 8
0 1 3 6 11	0 1 5 7 9	0 1 2 3 7	0 1 2 7 11	0 1 3 7 8	0 1 5 7 11
0 1 2 4 6	0 1 2 8 9	0 1 3 7 9	0 1 5 9 11	0 1 2 4 11	0 1 2 8 11
0 1 3 8 11	0 1 6 8 11	0 1 2 5 8	0 1 3 4 6	0 1 4 5 8	0 1 6 9 11
0 1 2 5 9	0 1 3 4 9	0 1 4 6 9	0 1 7 9 11	0 1 2 5 11	0 1 3 5 11
0 1 4 7 9	0 2 4 6 8	0 1 2 6 7	0 1 3 6 7	0 1 4 8 11	0 2 4 8 11

Design 4. Complement of Design 3. $G_4 = G_3$ and $V_4 = V_3$ as Design 3.

Design 5. $G_5 = \langle (0,1,2,3,4,5,6,7,8,9,10) \rangle$.
$|G_5| = 11$. $V_5 = $ (0,0,198,528,198,0,0).

Orbit representatives of G_5 generating 4-(12,5,4) design 5.

0 1 2 3 4	0 1 2 6 9	0 1 3 5 11	0 1 4 7 9	0 1 2 3 5	0 1 2 7 8
0 1 3 6 8	0 1 4 7 11	0 1 2 3 7	0 1 2 7 11	0 1 3 6 11	0 1 4 9 11
0 1 2 4 5	0 1 2 8 11	0 1 3 7 8	0 1 5 7 9	0 1 2 4 8	0 1 2 9 11
0 1 3 7 11	0 1 5 8 11	0 1 2 5 9	0 1 3 4 6	0 1 3 9 11	0 1 5 9 11
0 1 2 5 11	0 1 3 4 9	0 1 4 5 8	0 1 6 8 11	0 1 2 6 7	0 1 3 5 6
0 1 4 6 9	0 2 4 6 8	0 1 2 6 8	0 1 3 5 8	0 1 4 6 11	0 2 4 6 11

Design 6. Complement of design 5. $G_6 = G_5$ and $V_6 = V_5$ as Design 5.

Design 7. $G_7 = \langle (0,1,2,3,4,5)(6,7,8,9,10,11) \rangle$.
$|G_7| = 6$. $V_7 = $ (1,6,165,580,165,6,1).

Orbit representatives of G_7 generating 4-(12,5,4) design 7.

0 1 2 3 7	0 1 2 3 9	0 1 2 3 10	0 1 2 3 11	0 1 2 4 6	0 1 2 4 7
0 1 2 4 8	0 1 2 4 9	0 1 2 6 8	0 1 2 6 11	0 1 2 7 8	0 1 2 7 11
0 1 2 9 10	0 1 2 10 11	0 1 3 4 6	0 1 3 4 7	0 1 3 6 8	0 1 3 6 10
0 1 3 7 11	0 1 3 8 9	0 1 3 8 10	0 1 3 9 11	0 1 4 6 9	0 1 4 6 11
0 1 4 7 10	0 1 4 8 10	0 1 4 8 11	0 1 4 10 11	0 1 6 7 8	0 1 6 7 9
0 1 6 7 10	0 1 6 8 11	0 1 6 9 10	0 1 6 9 11	0 1 7 8 9	0 1 7 8 10
0 1 7 9 10	0 1 7 9 11	0 1 8 9 11	0 1 8 10 11	0 2 4 6 7	0 2 4 6 9
0 2 6 7 9	0 2 6 7 11	0 2 6 8 9	0 2 6 8 10	0 2 6 8 11	0 2 6 10 11
0 2 7 8 10	0 2 7 9 10	0 2 7 9 11	0 2 8 9 10	0 2 8 9 11	0 2 9 10 11
0 3 6 7 8	0 3 6 7 10	0 3 6 7 11	0 3 6 8 9	0 3 6 10 11	0 3 7 8 11
0 6 7 10 11	0 6 8 9 10	0 6 9 10 11	0 7 8 9 11	0 7 8 10 11	6 7 8 9 10

Design 8. Complement of design 7. $G_8 = G_7$ and $V_8 = V_7$.

Design 9. $G_9 = \langle (0, 1, 2, 3, 4, 5)(6, 7, 8, 9, 10, 11) \rangle$.
$|G_9| = 6$. $V_9 =$ (1,0,189,544,189,0,1).

Orbit representatives of G_9 generating 4-(12,5,4) design 9.

0 1 2 3 6	0 1 2 3 9	0 1 2 3 10	0 1 2 3 11	0 1 2 4 6	0 1 2 4 7
0 1 2 4 8	0 1 2 4 9	0 1 2 6 7	0 1 2 6 10	0 1 2 7 9	0 1 2 7 11
0 1 2 8 10	0 1 2 8 11	0 1 3 4 7	0 1 3 4 8	0 1 3 6 7	0 1 3 6 8
0 1 3 7 9	0 1 3 8 9	0 1 3 9 10	0 1 3 10 11	0 1 4 6 9	0 1 4 6 10
0 1 4 6 11	0 1 4 7 10	0 1 4 8 11	0 1 4 9 11	0 1 6 7 8	0 1 6 7 11
0 1 6 8 9	0 1 6 9 10	0 1 6 9 11	0 1 7 8 9	0 1 7 8 10	0 1 7 8 11
0 1 7 10 11	0 1 8 9 10	0 1 8 10 11	0 1 9 10 11	0 2 4 6 7	0 2 4 6 11
0 2 6 7 9	0 2 6 7 10	0 2 6 8 9	0 2 6 8 10	0 2 6 9 11	0 2 6 10 11
0 2 7 8 10	0 2 7 9 10	0 2 7 9 11	0 2 8 9 11	0 2 8 10 11	0 2 9 10 11
0 3 6 7 9	0 3 6 7 11	0 3 6 8 10	0 3 6 8 11	0 3 6 10 11	0 3 7 8 10
0 6 7 8 10	0 6 7 8 11	0 6 8 9 11	0 6 9 10 11	0 7 9 10 11	6 7 8 9 10

Design 10. Complement of design 9. $G_{10} = G_9$ and $V_{10} = V_9$.

Design 11. $G_{11} = \langle (0, 1, 2, 3, 4, 5)(6, 7, 8, 9, 10, 11) \rangle$.
$|G_{11}| = 6$. $V_{11} =$ (1,0,189,544,189,0,1).

Orbit representatives of G_{11} generating 4-(12,5,4) design 11.

0 1 2 3 6	0 1 2 3 8	0 1 2 3 9	0 1 2 3 10	0 1 2 4 6	0 1 2 4 7
0 1 2 4 10	0 1 2 4 11	0 1 2 6 9	0 1 2 6 11	0 1 2 7 8	0 1 2 7 9
0 1 2 8 10	0 1 2 10 11	0 1 3 4 6	0 1 3 4 8	0 1 3 6 7	0 1 3 7 8
0 1 3 7 10	0 1 3 7 11	0 1 3 8 11	0 1 3 9 10	0 1 4 6 10	0 1 4 7 9
0 1 4 7 11	0 1 4 8 9	0 1 4 8 10	0 1 4 9 11	0 1 6 7 10	0 1 6 7 11
0 1 6 8 9	0 1 6 8 10	0 1 6 8 11	0 1 6 9 10	0 1 6 9 11	0 1 7 8 9
0 1 7 8 10	0 1 7 10 11	0 1 8 9 11	0 1 9 10 11	0 2 4 6 9	0 2 4 6 11
0 2 6 7 8	0 2 6 7 10	0 2 6 7 11	0 2 6 8 9	0 2 6 8 10	0 2 7 8 11
0 2 7 9 10	0 2 7 9 11	0 2 8 9 10	0 2 8 9 11	0 2 8 10 11	0 2 9 10 11
0 3 6 7 9	0 3 6 7 11	0 3 6 8 9	0 3 6 8 11	0 3 6 10 11	0 3 7 8 10
0 6 7 8 11	0 6 7 9 10	0 6 8 10 11	0 7 8 9 10	0 7 9 10 11	6 7 8 9 10

Design 12. Complement of design 11. $G_{12} = G_{11}$ and $V_{12} = V_{11}$.

Design 13. $G_{13} = \langle (0, 1, 2)(3, 4, 5)(6, 7, 8)(9, 10, 11), (0, 1)(3, 4)(6, 7)(9, 10) \rangle$.
$|G_{13}| = 6$. $V_{13} =$ (1,18,117,652,117,18,1).

Orbit representatives of G_{13} generating 4-(12,5,4) design 13.

0 1 2 3 7	0 1 2 3 10	0 1 2 6 10	0 1 3 4 6	0 1 3 4 8	0 1 3 4 11
0 1 3 5 8	0 1 3 5 9	0 1 3 5 10	0 1 3 5 11	0 1 3 6 7	0 1 3 6 9
0 1 3 6 10	0 1 3 7 10	0 1 3 8 9	0 1 3 9 11	0 1 5 6 7	0 1 5 6 8
0 1 5 6 11	0 1 5 9 10	0 1 6 7 8	0 1 6 8 9	0 1 6 8 11	0 1 6 9 10
0 1 6 10 11	0 1 8 9 11	0 1 9 10 11	0 3 4 5 6	0 3 4 5 7	0 3 4 5 9
0 3 4 6 8	0 3 4 6 10	0 3 4 7 11	0 3 4 8 9	0 3 4 9 10	0 3 4 10 11
0 3 6 7 8	0 3 6 7 11	0 3 6 9 10	0 3 7 8 9	0 3 7 8 10	0 3 7 9 10
0 3 7 10 11	0 4 5 6 9	0 4 5 6 10	0 4 5 7 8	0 4 5 7 10	0 4 6 7 10
0 4 6 7 11	0 4 6 8 9	0 4 6 8 11	0 4 6 9 11	0 4 7 8 9	0 4 7 8 11
0 4 7 9 10	0 4 7 9 11	0 4 8 9 10	0 4 8 10 11	0 4 9 10 11	0 6 7 8 9
0 6 7 9 11	0 6 7 10 11	3 4 5 6 9	3 4 5 9 10	3 4 6 7 8	3 4 6 7 9
3 4 6 8 10	3 4 6 8 11	3 4 6 9 11	3 4 6 10 11	3 4 8 9 10	3 6 7 8 9
3 6 7 9 10	3 7 9 10 11	6 7 8 9 10	6 7 9 10 11		

Design 14. $G_{14} = \langle (0,1,2)(3,4,5)(6,7,8)(9,10,11), (0,1)(3,4)(6,7)(9,10) \rangle$.
$|G_{14}| = 6$. $V_{14} =$ (4,12,114,664,114,12,4).

Orbit representatives of G_{14} generating 4-(12,5,4) design 14.

0 1 2 3 7	0 1 2 3 10	0 1 2 6 10	0 1 3 4 6	0 1 3 4 8	0 1 3 4 11
0 1 3 5 8	0 1 3 5 9	0 1 3 5 10	0 1 3 5 11	0 1 3 6 7	0 1 3 6 9
0 1 3 6 10	0 1 3 7 10	0 1 3 8 9	0 1 3 9 11	0 1 5 6 7	0 1 5 6 8
0 1 5 6 11	0 1 5 9 10	0 1 6 7 8	0 1 6 8 9	0 1 6 8 11	0 1 6 9 10
0 1 6 10 11	0 1 8 9 11	0 1 9 10 11	0 3 4 5 6	0 3 4 5 7	0 3 4 5 9
0 3 4 6 8	0 3 4 6 10	0 3 4 7 11	0 3 4 8 9	0 3 4 9 10	0 3 4 10 11
0 3 6 7 8	0 3 6 7 11	0 3 6 9 10	0 3 7 8 9	0 3 7 8 10	0 3 7 9 10
0 3 7 10 11	0 4 5 6 9	0 4 5 6 10	0 4 5 7 8	0 4 5 7 10	0 4 6 7 10
0 4 6 7 11	0 4 6 8 9	0 4 6 8 11	0 4 6 9 11	0 4 7 8 9	0 4 7 8 11
0 4 7 9 10	0 4 7 9 11	0 4 8 9 10	0 4 8 10 11	0 4 9 10 11	0 6 7 8 9
0 6 7 9 11	0 6 7 10 11	3 4 5 6 9	3 4 5 9 10	3 4 6 7 8	3 4 6 7 9
3 4 6 8 10	3 4 6 8 11	3 4 6 9 11	3 4 6 10 11	3 4 8 9 10	3 6 7 8 9
3 6 7 9 10	3 7 9 10 11	6 7 8 9 10	6 7 9 10 11		

Design 15. $G_{15} = \langle (0,1,2,3,4)(6,7,8,9,10) \rangle$.
$|G_{15}| = 5$. $V_{15} =$ (1,11,145,610,145,11,1).

Orbit representatives of G_{15} generating 4-(12,5,4) design 15.

0 1 2 3 5	0 1 2 3 8	0 1 2 3 9	0 1 2 3 10	0 1 2 5 7	0 1 2 5 10
0 1 2 6 7	0 1 2 6 9	0 1 2 6 10	0 1 2 6 11	0 1 2 7 8	0 1 2 8 11
0 1 2 9 11	0 1 2 10 11	0 1 3 5 9	0 1 3 5 11	0 1 3 6 8	0 1 3 6 9
0 1 3 6 10	0 1 3 7 9	0 1 3 7 10	0 1 3 7 11	0 1 3 8 11	0 1 3 10 11
0 1 5 6 8	0 1 5 6 10	0 1 5 6 11	0 1 5 7 8	0 1 5 7 10	0 1 5 7 11
0 1 5 8 9	0 1 5 8 10	0 1 5 9 11	0 1 6 7 9	0 1 6 7 11	0 1 6 8 9
0 1 6 8 11	0 1 7 8 9	0 1 7 8 10	0 1 7 9 10	0 1 8 9 10	0 1 9 10 11
0 2 5 6 7	0 2 5 6 9	0 2 5 6 10	0 2 5 7 9	0 2 5 7 11	0 2 5 8 9
0 2 5 8 10	0 2 5 8 11	0 2 5 9 10	0 2 6 7 8	0 2 6 7 10	0 2 6 8 11
0 2 6 10 11	0 2 7 8 9	0 2 7 9 10	0 2 7 10 11	0 2 8 9 10	0 2 8 9 11
0 5 6 7 10	0 5 6 8 9	0 5 6 8 10	0 5 6 9 11	0 5 7 8 11	0 5 7 9 11
0 5 8 10 11	0 5 9 10 11	0 6 7 8 11	0 6 7 9 10	0 6 7 9 11	0 6 8 9 10
0 6 8 10 11	0 6 9 10 11	0 7 8 9 11	0 7 8 10 11	0 7 9 10 11	5 6 7 8 9
5 6 7 8 11	6 7 8 9 10				

Design 16. $G_{16} = \langle (0,1,2,3,4)(6,7,8,9,10) \rangle$.
$|G_{16}| = 5$. $V_{16} =$ (1,6,165,580,165,6,1).

Orbit representatives of G_{16} generating 4-(12,5,4) design 16.

0 1 2 3 4	0 1 2 3 5	0 1 2 3 8	0 1 2 3 9	0 1 2 5 7	0 1 2 5 10
0 1 2 6 7	0 1 2 6 9	0 1 2 6 10	0 1 2 6 11	0 1 2 7 8	0 1 2 8 11
0 1 2 9 10	0 1 2 9 11	0 1 2 10 11	0 1 3 5 9	0 1 3 5 11	0 1 3 6 8
0 1 3 6 9	0 1 3 6 10	0 1 3 7 9	0 1 3 7 10	0 1 3 7 11	0 1 3 8 10
0 1 3 8 11	0 1 3 10 11	0 1 5 6 8	0 1 5 6 10	0 1 5 6 11	0 1 5 7 8
0 1 5 7 10	0 1 5 7 11	0 1 5 8 9	0 1 5 8 10	0 1 5 9 11	0 1 6 7 9
0 1 6 7 11	0 1 6 8 9	0 1 6 8 11	0 1 7 8 9	0 1 7 8 10	0 1 7 9 10
0 1 9 10 11	0 2 5 6 7	0 2 5 6 9	0 2 5 6 10	0 2 5 7 9	0 2 5 7 11
0 2 5 8 9	0 2 5 8 10	0 2 5 8 11	0 2 5 9 10	0 2 6 7 8	0 2 6 7 10
0 2 6 8 11	0 2 6 10 11	0 2 7 8 9	0 2 7 10 11	0 2 8 9 10	0 2 8 9 11
0 5 6 7 10	0 5 6 8 9	0 5 6 8 10	0 5 6 9 11	0 5 7 8 11	0 5 7 9 11
0 5 8 10 11	0 5 9 10 11	0 6 7 8 11	0 6 7 9 10	0 6 7 9 11	0 6 8 9 10
0 6 8 10 11	0 6 9 10 11	0 7 8 9 10	0 7 8 9 11	0 7 8 10 11	0 7 9 10 11
5 6 7 8 9	5 6 7 8 11				

In Table VI we list the starting blocks of one of the 6-(14,7,4) designs found in [KR1] because the following fourteen designs arise either by taking doubly derived deigns of this 6-(14,7,4) design or by taking complements of these doubly derived designs. Each starting block generates 13 blocks using the cyclic automorphism $(0,1,2,3,4,5,6,7,8,9,10,11,12)(13)$. In this list of 4-(12,5,4)'s we will let KR-Design (i,j) be the doubly derived design using points i and j. Observe that because of the above cyclic automorphism we may assume without loss that $i = 0$ and $j \in \{1,2,3,4,5,6,13\}$. This gives rise to designs 17 through 30 each has the identity group as an automorphism group and each has $V_i =$ (0,0,198,528,198,0,0), for $17 \leq i \leq 30$.

Design 17. KR-Design (0,13).
Design 18. Complement of Design 17.
Design 19. KR-Design (0,1).
Design 20. Complement of Design 19.
Design 21. KR-Design (0,2).
Design 22. Complement of Design 21.
Design 23. KR-Design (0,3).
Design 24. Complement of Design 23.
Design 25. KR-Design (0,4).
Design 26. Complement of Design 25.
Design 27. KR-Design (0,5).
Design 28. Complement of Design 27.
Design 29. KR-Design (0,6).
Design 30. Complement of Design 29.

Table VI: Starting blocks of a 6-(14,7,4) design.

0 1 2 3 4 5 13	0 2 3 4 5 8 13	0 1 3 5 7 8 13	4 7 8 9 10 11 12	5 6 7 9 10 11 12	1 4 5 9 10 11 12
0 1 4 5 6 9 13	0 3 5 6 7 9 13	0 3 6 7 8 9 13	1 4 7 8 10 11 12	2 5 6 7 10 11 12	2 5 6 8 9 11 12
0 1 2 3 4 6 13	0 1 2 3 6 8 13	0 2 3 5 7 8 13	2 7 8 9 10 11 12	4 6 7 9 10 11 12	2 3 5 9 10 11 12
0 2 4 5 6 9 13	0 2 3 4 8 9 13	0 1 2 4 7 10 13	1 2 7 8 10 11 12	3 4 6 7 10 11 12	2 4 6 8 9 11 12
0 1 2 4 5 6 13	0 1 3 4 6 8 13	0 2 4 5 7 8 13	1 7 8 9 10 11 12	2 6 7 9 10 11 12	1 3 5 9 10 11 12
0 1 2 4 7 9 13	0 1 3 5 8 9 13	0 2 3 4 7 10 13	4 5 6 8 10 11 12	1 3 6 7 10 11 12	1 4 6 8 9 11 12
0 1 2 3 5 7 13	0 1 2 5 6 8 13	0 1 2 6 7 8 13	5 6 8 9 10 11 12	3 5 7 9 10 11 12	1 2 5 9 10 11 12
0 1 3 4 7 9 13	0 2 3 5 8 9 13	0 1 2 5 7 10 13	1 5 6 8 10 11 12	1 2 6 7 10 11 12	2 4 5 8 9 11 12
0 1 3 4 6 7 13	0 1 3 5 6 8 13	0 2 5 6 7 8 13	3 6 8 9 10 11 12	1 5 7 9 10 11 12	2 3 4 9 10 11 12
0 1 2 5 7 9 13	0 1 4 5 8 9 13	0 1 4 5 7 10 13	1 4 6 8 10 11 12	2 4 5 7 10 11 12	1 4 5 8 9 11 12
0 2 3 4 6 7 13	0 1 4 5 6 8 13	0 4 5 6 7 8 13	2 6 8 9 10 11 12	1 4 7 9 10 11 12	1 2 4 9 10 11 12
0 1 3 5 7 9 13	0 1 2 6 8 9 13	0 2 4 5 7 10 13	2 3 6 8 10 11 12	1 4 5 7 10 11 12	1 3 5 8 9 11 12
0 1 3 5 6 7 13	0 3 4 5 6 8 13	0 1 3 4 5 9 13	1 6 8 9 10 11 12	1 3 7 9 10 11 12	4 6 7 8 10 11 12
0 3 4 5 7 9 13	0 2 4 6 8 9 13	0 1 4 6 7 10 13	1 3 6 8 10 11 12	2 3 5 7 10 11 12	1 3 4 8 9 11 12
0 2 3 5 6 7 13	0 1 2 3 7 8 13	0 2 3 4 5 9 13	4 5 8 9 10 11 12	3 4 6 9 10 11 12	3 6 7 8 10 11 12
0 2 3 6 7 9 13	0 3 4 6 8 9 13	0 1 3 5 8 10 13	3 4 5 8 10 11 12	1 3 5 6 10 11 12	1 4 6 7 9 11 12
0 1 4 5 6 7 13	0 1 2 4 7 8 13	0 1 2 3 6 9 13	3 5 8 9 10 11 12	2 3 6 9 10 11 12	3 5 7 8 10 11 12
0 1 4 6 7 9 13	0 2 5 6 8 9 13	0 2 4 5 8 10 13	2 4 5 8 10 11 12	1 2 5 6 10 11 12	2 3 6 7 9 11 12
0 2 4 5 6 7 13	0 1 3 4 7 8 13	0 2 3 4 6 9 13	3 4 8 9 10 11 12	1 2 6 9 10 11 12	2 5 7 8 10 11 12
0 3 4 6 7 9 13	0 3 5 6 8 9 13	0 2 3 6 8 10 13	1 3 5 8 10 11 12	1 3 4 6 10 11 12	2 4 5 7 9 11 12
0 1 2 4 5 8 13	0 2 3 4 7 8 13	0 1 2 5 6 9 13	1 2 8 9 10 11 12	2 4 5 9 10 11 12	2 4 7 8 10 11 12
0 2 5 6 7 9 13	0 1 4 7 8 9 13	0 2 4 6 8 10 13	2 3 4 8 10 11 12	1 2 4 6 10 11 12	2 3 5 7 9 11 12

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