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On halving line arrangements. (English. English summary)

Kleitman and combinatorics: a celebration (Cambridge, MA, 1999).
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A halving line in a set of n distinct points in the plane (and in general position) cuts the remaining $n-2$ points in half. Let $h(n)$ be the maximum number of halving lines in a set of n points arranged in any general position. Then it follows that $h(2n+1) = 0$ $\forall n > 0$, $h(2) = 1$, $h(4) = 3$, $h(6) = 6$, $h(8) = 9$, $h(10) = 13$ [D. Eppstein, "Sets of points with many halving lines", Tech. Rep. ICS-TR-92-86, Dept. Inf. Comput. Sci., Univ. California Irvine, Irvine, CA, 1992; per bibl.; S. Felsner, Discrete Comput. Geom. **18** (1997), no. 3, 257–267; MR 98k:52031; G. Stöckl, "Gesammelte und neue Ergebnisse über extreme k-Mengen für ebene Punktmengen", Diplomarbeit, Inst. Inf., Tech. Univ. Graz, Graz, 1984; per bibl.]. and $h(12) = 18$ [A. Andrzejak et al., in *Proceedings of SCG 98: 14th ACM Symposium on Computational Geometry (Minneapolis, MN, 1998)*, 192–199, ACM, New York, 1998]. Generally it can seen that $h(n) \geq n-1$. Some bounds are also known for $h(16)$ and $h(18)$ [D. Eppstein, op. cit.]

In this paper, it is confirmed that $h(12) = 18$, and it is also shown that $h(14) = 22$. The half-line problem is also generalised to pseudolines. In that case a simple arrangement of n pseudolines (a finite collection of n simple closed curves in the projective plane that intersects pairwise in $\binom{n}{2}$ distinct points) are considered, and $\widehat{h}(n)$ is the maximum number of intersection points with $(n-2)/2$ pseudolines above them.

It is the case that $\hat{h}(n) \geq h(n)$, and the authors obtain $\widehat{h}(12)$ exactly; this gives an alternative proof that $h(12) = 18$. Further results are obtained that reduce the computational efforts in determining $\widehat{h}(n)$, and the topologically distinct simple arrangements of 10 pseudolines with a marked cell are enumerated.

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