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Subgraph counting identities and Ramsey numbers. (English. English summary)

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Progress continues on the search for small Ramsey numbers $R(m,n)$. For graphs J,G, let $s(J,G)$ be the number of induced subgraphs of G that are isomorphic to J. For v a vertex of G, let G_v^+ be the subgraph induced by v and its neighbors and G_v^- be the subgraph induced by all other vertices. For $0\le j\le i$, $T_{j,i}$ consists of K_i and another vertex joined to j vertices of K_i. An interesting identity, previously unknown for $m\ge 4$, expresses $\sum_{v\in VG} s(K_m,G_v^-)$ in terms of $s(K_1,G_v^+)$, $s(K_{m-1},G_v^+)$, $s(K_m,G_v^+)$, and $s(T_{m-1,j},G_v^+)$, $1\le j\le m-2$. Armed with this, other results stemming from their previous work, and some distributed computation, the authors lower their previous upper bound for $R(5,5)$ from 50 to 49 and provide considerable evidence that in fact the current lower bound of 43 is the true value. Also, the upper bound for $R(4,6)$ is lowered from 43 to 41, and some evidence is offered that the true value is 35.

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