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Search algorithm for Ramsey graphs by union of group orbits.

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Summary: "An algorithm for the construction of Ramsey graphs with a given automorphism group G is presented. To find a graph on n vertices with no clique of k vertices, K_k, and no independent set of l vertices, $\overline{K_l}$, $k, l \leq n$, with an automorphism group G, a Boolean formula α based on the G-orbits of k-subsets and l-subsets of vertices is constructed from incidence matrices belonging to G. This Boolean formula is satisfiable if and only if the desired graph exists, and each satisfying assignment to α specifies a set of orbits of pairs of vertices whose union gives the edges of such a graph. Finding these assignments is basically equivalent to the conversion of α from CNF to DNF (conjunctive to disjunctive normal form). Though the latter problem is NP-hard, we present an 'efficient' method to do the conversion for the formulas that appear in this particular problem. When G is taken to be the dihedral group D_n for $n \leq 101$, this method matches all of the previously known cyclic Ramsey graphs, as reported by F. R. K. Chung and C. M. Grinstead [same journal 7 (1983), no. 1, 25–37; MR 84d:05126], in dramatically smaller computer time when compared to the time required by an exhaustive search. Five new lower bounds for the classical Ramsey numbers are established: $R(4,7) \geq 47$, $R(4,8) \geq 52$, $R(4,9) \geq 69$, $R(5,7) \geq 76$ and $R(5,8) \geq 94$. Also, some previously known cyclic graphs are shown to be unique up to isomorphism."