Some Small Ramsey Numbers

Stanisław P. Radziszowski
Department of Computer Science
Rochester Institute of Technology
Rochester, NY 14623, spr@cs.rit.edu
http://www.cs.rit.edu/~spr

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ABSTRACT: We present data which, to the best of our knowledge, includes all known nontrivial values and bounds for specific graph, hypergraph and multicolor Ramsey numbers, where the avoided graphs are complete or complete without one edge. Many results pertaining to other more studied cases are also presented. We give references to all cited bounds and values, as well as to previous similar compilations. We do not attempt complete coverage of asymptotic behavior of Ramsey numbers, but rather we concentrate on their specific values.

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1. Scope and Notation

There is vast literature on Ramsey type problems starting in 1930 with the original paper of Ramsey [Ram]. Graham, Rothschild and Spencer in their book [GRS] present an exciting development of Ramsey Theory. The subject has grown amazingly, in particular with regard to asymptotic bounds for various types of Ramsey numbers (see the survey papers [GrRö, Neš, ChGra2, Ros2]), but the progress on evaluating the basic numbers themselves has been unsatisfactory for a long time. In the last three decades, however, considerable progress has been obtained in this area, mostly by employing computer algorithms. The few known exact values and several bounds for different numbers are scattered among many technical papers. This compilation is a fast source of references for the best results known for specific numbers. It is not supposed to serve as a source of definitions or theorems, but these can be easily accessed via the references gathered here.

Ramsey Theory studies conditions when a combinatorial object contains necessarily some smaller given objects. The role of Ramsey numbers is to quantify some of the general existential theorems in Ramsey Theory.

Let $G_1, G_2, \ldots, G_m$ be graphs or s-uniform hypergraphs (s is the number of vertices in each edge). $R(G_1, G_2, \ldots, G_m; s)$ denotes the m-color **Ramsey number** for s-uniform graphs/hypergraphs, avoiding G_i in color i for $1 \le i \le m$. It is defined as the least integer n such that, in any coloring with m colors of the s-subsets of a set of n elements, for some i the s-subsets of color i contain a sub-(hyper)graph isomorphic to G_i (not necessarily induced). The value of $R(G_1, G_2, \ldots, G_m; s)$ is fixed under permutations of the first m arguments. If $s = 2$ (standard graphs) then s can be omitted. If G_i is a complete graph K_k, then we may write k instead of G_i, and if $G_i = G$ for all i we may use the abbreviation $R_m(G; s)$ or $R_m(G)$. For $s = 2$, $K_k - e$ denotes a K_k without one edge, and for $s = 3$, $K_k - t$ denotes a K_k without one triangle (hyperedge).

The graph nG is formed by n disjoint copies of G, $G \cup H$ stands for vertex disjoint union of graphs, and the **join** $G + H$ is obtained by adding all of the edges between vertices of G and H to $G \cup H$. P_i is a **path** on i vertices, C_i is a **cycle** of length i, and W_i is a **wheel** with $i-1$ spokes, i.e. a graph formed by some vertex x, connected to all vertices of the cycle C_{i-1} (thus $W_i = K_1 + C_{i-1}$). $K_{n,m}$ is a complete n by m bipartite graph, in particular $K_{1,n}$ is a **star** graph. The **book** graph $B_i = K_2 + \bar{K}_i = K_1 + K_{1,i}$ has $i+2$ vertices, and can be seen as i triangular pages attached to a single edge. The **fan** graph F_n is defined by $F_n = K_1 + nK_2$. For a graph G, $n(G)$ and $e(G)$ denote the number of vertices and edges, respectively, and $\delta(G)$ and $\Delta(G)$ minimum and maximum degree of G. Finally, $\chi(G)$ denotes the chromatic number of G. In general, we follow the notation used by West [West].

Section 2 contains the data for the classical two color Ramsey numbers $R(k, l)$ for complete graphs, section 3 for the much studied two color cases of $K_n - e$, K_3, $K_{m,n}$, and section 4 for numbers involving cycles. Section 5 lists other often studied two color cases for general graphs. The multicolor and hypergraph cases are gathered in sections 6 and 7, respectively. Finally, section 8 gives pointers to cumulative data and to other surveys.

2. Classical Two-Color Ramsey Numbers

2.1. Values and bounds for $R(k, l)$, $k \leq 10$, $l \leq 15$

k \ l	3	4	5	6	7	8	9	10	11	12	13	14	15
3	6	9	14	18	23	28	36	40 42	47 50	52 59	59 68	66 77	73 87
4		18	25	36 41	49 61	58 84	73 115	92 149	98 191	128 238	133 291	141 349	153 417
5			43 49	58 87	80 143	101 216	126 316	144 442	171 633	191 848	213 1138	239 1461	265 1878
6				102 165	113 298	132 495	169 780	179 1171	253 1804	263 2566	317 3703	 5033	401 6911
7					205 540	217 1031	241 1713	289 2826	405 4553	417 6954	511 10578	 15263	 22112
8						282 1870	317 3583	 6090	 10630	 16944	817 27485	 41525	861 63609
9							565 6588	581 12677	 22325	 38832	 64864		
10								798 23556	 45881	 81123			1265

Table I. Known nontrivial values and bounds for two color Ramsey numbers $R(k, l) = R(k, l; 2)$.

k \ l	4	5	6	7	8	9	10	11	12	13	14	15
3	GG	GG	Kéry	Ka2 GrY	GR MZ	Ka2 GR	Ex5 GoR1	Ex20 GoR1	Ex12 Les	Piw1 GoR1	Ex8 GoR1	WW GoR1
4	GG	Ka1 MR4	Ex19 MR5	Ex3 Mac	Ex20 Mac	Ex16 Mac	HaKr1 Mac	Ex17 Spe4	SLL Spe4	2.3.e Spe4	XXR Spe4	XXR Spe4
5		Ex4 MR5	Ex9 HZ1	CaET Spe4	HaKr1 Spe4	Ex17 Mac	Ex17 Mac	Gerb HW+	Gerb HW+	Gerb HW+	Gerb HW+	Ex16 HW+
6			Ka1 Mac	Ex16 Mac	XSR2 Mac	XXER Mac	Ex16 Mac	XXR HW+	XSR2 HW+	XXER HW+	 HW+	2.3.h HW+
7				She2 Mac	XSR2 Mac	XSR2 HZ1	2.3.h Mac	XXER HW+	XSR2 HW+	XXR HW+	 HW+	 HW+
8					BR Mac	XXER Ea1	 HZ1	 HW+	 HW+	XXER HW+	 HW+	2.3.h HW+
9						She2 ShZ1	XSR2 Ea1	 HW+	 HW+	 HW+		
10							She2 Shi2	 HW+	 HW+			2.3.h

References for Table I;
HW+ abbreviates HWSYZH, as enhanced by Boza [Boza5], see 2.1.m.

We split the data into the table of values and a table with corresponding references. In Table I, known exact values appear as centered entries, lower bounds as top entries, and upper bounds as bottom entries. For some of the exact values two references are given when the lower and upper bound credits are different.

(a) The task of proving $R(3,3) \leq 6$ was the second problem in Part I of the William Lowell Putnam Mathematical Competition held in March 1953 [Bush].

(b) Greenwood and Gleason [GG] established the initial values $R(3,4) = 9$, $R(3,5) = 14$ and $R(4,4) = 18$ in 1955.

(c) Kéry [Kéry] proved that $R(3,6) = 18$ in 1964, but only in 2007 an elementary and self-contained proof of this result appeared in English [Car].

(d) All of the critical graphs for the numbers $R(k,l)$ (graphs on $R(k,l)-1$ vertices without K_k and without K_l in the complement) are known for $k=3$ and $l=3$, 4, 5 [Kéry], 6 [Ka2], 7 [RK2, MZ], 8 [BrGS] and 9 [GoR1], and there are 1, 3, 1, 7, 191, 477142, and 1 of them, respectively. All $(3,k)$-graphs, for $k \leq 6$, were enumerated in [RK2], and all (4,4)-graphs in [MR2]. There exists a unique critical graph for $R(4,4)$ [Ka2]. There are 350904 known critical graphs for $R(4,5)$ [MR4], but there might be more of them.

(e) In [MR5], strong evidence is given for the conjecture that $R(5,5) = 43$ and that there exist exactly 656 critical graphs on 42 vertices.

(f) The graphs constructed by Exoo in [Ex9, Ex12-Ex20], and some others, are available electronically from http://ginger.indstate.edu/ge/RAMSEY. Fujita [Fuj1] maintains a website with some lower bound constructions; in particular, it presents the bound $R(4,8) \geq 58$ obtained independently from Exoo.

(g) *Cyclic* (or *circular*) graphs are often used for Ramsey graph constructions. Several cyclic graphs establishing lower bounds were given in the Ph.D. dissertation by J.G. Kalbfleisch in 1966, and many others were published in the next few decades (see [RK1]). Harborth and Krause [HaKr1] presented all best lower bounds up to 102 from cyclic graphs avoiding complete graphs. In particular, no lower bound in Table I can be improved with a cyclic graph on less than 102 vertices, except possibly for $R(3,k)$ for $k \geq 13$. See also item 2.3.k and section 5.16 [HaKr1]. Several best lower bounds from *distance colorings*, a slightly more general concept than circular graphs, are presented in [HaKr2].

(h) The claim that $R(5,5) = 50$ posted on the web [Stone] is in error, and despite being shown to be incorrect more than once, this value is still being cited by some authors. The bound $R(3,13) \geq 60$ [XieZ] cited in the 1995 version of this survey was shown to be incorrect in [Piw1]. Another incorrect construction for $R(3,10) \geq 41$ was described in [DuHu].

(i) There are really only two general upper bound inequalities useful for small parameters, namely 2.3.a and 2.3.b. Stronger upper bounds for specific parameters were difficult to obtain, and they often involved massive computations, like those for the cases of (3,8) [MZ], (3,10) [GoR1], (4,5) [MR4], (4,6) and (5,5) [MR5]. The bound $R(6,6) \leq 166$, only 1 more than the best known [Mac], is an easy consequence of a theorem in [Walk]

(2.3.b) and $R(4,6) \le 41$.

(j) T. Spencer [Spe4], Mackey [Mac], and Huang and Zhang [HZ1], using the bounds for minimum and maximum number of edges in (4,5) Ramsey graphs listed in [MR3, MR5], were able to establish new upper bounds for several higher Ramsey numbers, improving on all of the previous longstanding best results by Giraud [Gi3, Gi5, Gi6].

(k) Only some of the higher bounds implied by 2.3.* are shown, and more similar bounds could be derived. In general, we show bounds beyond the contiguous small values if they improve on results previously reported in this survey or published elsewhere. Some easy upper bounds implied by 2.3.a are marked as [Ea1].

(l) In 2009, we have recomputed the upper bounds in Table I marked [HZ1] using the method from the paper [HZ1], because the bounds there relied on an overly optimistic personal communication from T. Spencer. Further refinements of this method are studied in [HZ2, ShZ1, Shi2]. The paper [Shi2] subsumes the main results of the manuscripts [ShZ1, Shi2].

(m) In 2013, Boza [Boza5] using the method of [HWSYZH], which is abbreviated as HW+ in Table I, computed the bounds marked HW+ by starting from better upper bounds for smaller parameters. Most of the currently shown bounds are thus better than those originally listed in [HWSYZH, HZ2]. Five upper bounds not shown in Table I can be obtained similarly but they are larger than 10^5.

2.2. Bounds for $R(k,l)$, higher parameters

k \ l	15	16	17	18	19	20	21	22	23
3	73 WW 87 GoR1	82 Ex21 98 Back1	92 W1+ 109 Back1	99 Ex16 121 Les	106 W1+ 132 Back2	111 Ex16 145 Les	122 W1+ 158 Les	131 W2+ 171 Back2	139 XWCS 185 Back2
4	153 XXR	164 Gerb	200 Lia+	205 2.3.e	213 2.3.g	234 Ex16	242 SLZL	314 LSLW	
5	265 Ex16	289 2.3.h	388 XSR2	396 2.3.g	411 XSR2	424 XSR2	441 2.3.h	485 2.3.h	521 2.3.h
6	401 2.3.h	434 SLLL	548 SLLL	614 SLLL	710 SLLL	878 SLLL		1070 SLLL	
7		609 2.3.h	711 2.3.g	797 2.3.h	908 SLLL		1214 SLLL		
8	861 2.3.h		961 XSR2	1045 2.3.g	1236 2.3.g		1617 2.3.h		

Table IIa. Known bounds for higher two-color Ramsey numbers $R(k,l)$, with references.
Lower and upper bounds are given for $k=3$, only lower bounds for $k \ge 4$;
Lia+, W1+ and W2+ abbreviate LiaWXS, WWY1 and WSLX2, respectively.

l / k	24	25	26	27	28	29	30	31
3	143 W1+	154 W2+	159 W1+	167 W1+	173 W2+	184 W2+	190 W2+	199 W2+

l / k	32	33	34	35	36	37	38	39	40
3	214 W2+	218 Ch+	226 Ch+	231 Ch+	239 Ch+	244 Ch+	256 Ch+		

Table IIb. Known lower bounds for higher Ramsey numbers $R(3,l)$ for $l \geq 24$; W1+, W2+ and Ch+ abbreviate WSLX1, WSLX2 and ChWXSL, respectively.

k	11	12	13	14	15	16	17
lower bound	1597	1639	2557	2989	5485	5605	8917
reference	2.2.c	XSR2	2.2.c	2.2.c	2.2.c	2.2.c	LSL
k	18	19	20	21	22	23	24
lower bound	11005	17885	19069	27077	29941		
reference	LSL	LSL	Lia+	Lia+	Lia+		

Table IIc. Known lower bounds for diagonal Ramsey numbers $R(k,k)$ for $k \geq 11$; Lia+ abreviates LiaWXCS, see also 2.2.c below.

(a) The upper bounds in Tables I and IIa marked [GoR1, Les, Back1] were obtained mainly by deriving lower bounds for several cases of $e(3,k,n)$, which denotes the minimum number of edges in n-vertex triangle-free graphs with independence number less than k. The study of $e(3,k,n)$ was also the main tool for the results obtained in [GrY, GR, RK2, RK3, GoR2].

(b) *Ramsey Calculus* [Back1], is an extensive manuscript by Backelin, which, among other goals, addresses the derivation of $e(3,k,n)$ and the corresponding realisers while avoiding reliance on computer assisted results as far as possible. It achieves the derivation of several lower bounds for $e(3,k+1,n)$ better than those in [GoR1, RK3, RK4] for n close to and above $13k/4$.

(c) The construction by Mathon [Mat] and Shearer [She2] (see also items 2.3.i, 6.2.k and 6.2.l), using the data obtained by Shearer [She4] for primes up to 7000, implies the lower bounds in Table IIc marked 2.2.c. The first two bounds credited in Table IIc to [LSL] also follow similarly from the data in [She4]. The same approach does not improve on the bound $R(12,12) \geq 1639$ [XSR2]. The bounds in [Lia+] were obtained by extending data for Payley graphs beyond [Sha4].

(d) The lower bounds marked [XXR], [XXER], [XSR2], 2.3.e and 2.3.h need not be cyclic. Several of the Cayley colorings from [Ex16] are also non-cyclic. All other lower bounds listed in Table IIab were obtained by construction of cyclic graphs.

(e) The graphs establishing lower bounds marked 2.3.g can be constructed by using appropriately chosen graphs G and H with a common m-vertex induced subgraph, similarly as it was done in several cases in [XXR].

(f) Yu [Yu2] constructed a special class of triangle-free cyclic graphs establishing several lower bounds for $R(3,k)$, for $k \geq 61$. All of these bounds can be improved by the inequalities in 2.3.c and data from Tables I and II.

(g) Unpublished bound $R(4,22) \geq 314$ [LSLW] improves over 282 given in [SL]. [LSLW] includes also $R(4,25) \geq 458$. Not yet published bounds $R(3,23) \geq 139$ [XWCS] and $R(4,17) \geq 200$ [LiaWXS] improve over 137 and 182 obtained in [WSLX2] and [LSS1], respectively.

(h) Two special cases which improve on bounds listed in earlier revisions: $R(9,17) \geq 1411$ is given in [XXR] and $R(10,15) \geq 1265$ can be obtained using 2.3.h.

(i) One can expect that the lower bounds in Table II are weaker than those in Table I, especially smaller ones, in the sense that some of them should not be that hard to improve, in contrast to the bounds in Table I.

2.3. General results on $R(k,l)$

(a) $R(k,l) \leq R(k-1,l)+R(k,l-1)$, with strict inequality when both terms on the right hand side are even [GG]. There are obvious generalizations of this inequality for avoiding graphs other than complete.

(b) $R(k,k) \leq 4R(k,k-2)+2$ [Walk].

(c) Explicit construction for $R(3,3k+1) \geq 4R(3,k+1)-3$, for all $k \geq 2$ [CleDa], explicit construction for $R(3,4k+1) \geq 6R(3,k+1)-5$, for all $k \geq 1$ [ChCD].

(d) Explicit triangle-free graphs with independence k on $\Omega(k^{3/2})$ vertices [Alon2, CPR]. For other constructive results in relation to $R(3,k)$ see [BBH1, BBH2, Fra1, Fra2, FrLo, GoR1, Gri, KlaM1, Loc, RK2, RK3, RK4, Stat, Yu1]. See also 2.3.(3) and 2.3.(4) below.

(e) The study of bounds for the difference between consecutive Ramsey numbers was initiated in [BEFS], where the bound $R(k,l) \geq R(k,l-1) + 2k-3$, for $k,l \geq 3$, was established by a construction. In 1980, Erdős and Sós (cf. [Erd2,ChGra2]) asked: If we set $\Delta_{k,l} = R(k,l) - R(k,l-1)$, then is it true that $\Delta_{k,k+1}/k \to \infty$ as $k \to \infty$? Only easy bounds on $\Delta_{k,l}$ are known, in particular $3 \leq \Delta_{3,l} \leq l$ for $k = 3$. For some discussion of the latter see [XSR2, GoR2].

(f) By taking a disjoint union of two critical graphs one can easily see that $R(k,p) \geq s$ and $R(k,q) \geq t$ imply $R(k,p+q-1) \geq s+t-1$. Xu and Xie [XX1] improved this construction to yield better general lower bounds, in particular $R(k,p+q-1) \geq s+t+k-3$.

(g) For $2 \leq p \leq q$ and $3 \leq k$, if (k,p)-graph G and (k,q)-graph H have a common induced subgraph on m vertices without K_{k-1}, then $R(k,p+q-1) > n(G)+n(H)+m$. In particular, this implies the bounds $R(k,p+q-1) \geq R(k,p)+R(k,q)+k-3$ and $R(k,p+q-1) \geq R(k,p)+R(k,q)+p-2$ [XX1, XXR], with further small improvements in some cases, such as using the term $k-2$ instead of $k-3$ in the previous bound [XSR2].

(h) $R(2k-1, l) \geq 4R(k, l-1) - 3$ for $l \geq 5$ and $k \geq 2$, and in particular for $k=3$ we have $R(5, l) \geq 4R(3, l-1) - 3$ [XXER].

(i) If the quadratic residues Paley graph Q_p of prime order $p = 4t+1$ contains no K_k, then $R(k, k) \geq p+1$ and $R(k+1, k+1) \geq 2p+3$ [She2, Mat]. Data for larger p was obtained in [LSL]. See also 3.1.c, and items 6.2.k and 6.2.l for similar multicolor results.

(j) Study of Ramsey numbers for large disjoint unions of graphs [Bu1, Bu9], in particular $R(nK_k, nK_l) = n(k+l-1) + R(K_{k-1}, K_{l-1}) - 2$, for n large enough [Bu8].

(k) $R(k, l) \geq L(k, l) + 1$, where $L(k, l)$ is the maximal order of any cyclic (k, l)−graph. A compilation of many best cyclic bounds was presented in [HaKr1].

(l) The graphs critical for $R(k, l)$ are $(k-1)$−vertex connected and $(2k-4)$−edge connected, for $k, l \geq 3$ [BePi]. This was improved to vertex connectivity k for $k \geq 5$ and $l \geq 3$ in [XSR2].

(m) All Ramsey-critical (k, l)−graphs are Hamiltonian for $k \geq l-1 \geq 1$ and $k \geq 3$, except when $(k, l) = (3, 2)$ [XSR2].

(n) Two-color lower bounds can be obtained by using items 6.2.m, 6.2.n and 6.2.o with $r=2$. Some generalizations of these were obtained in [ZLLS].

In the last seven items (1)-(7) of this section we only briefly mention some pointers to the literature dealing with asymptotics of Ramsey numbers. This survey was designed mostly for small, finite, and combinatorial results, but still we wish to give the reader some useful and representative references to more traditional papers studying the infinite.

(1) In 1947, Erdős gave a simple probabilistic proof that $R(k, k) \geq ck2^{k/2}$ [Erd1]. Spencer [Spe1] improved the constant c to $\sqrt{2}/e$. More probabilistic asymptotic lower bounds for other Ramsey numbers were obtained in [Spe1, Spe2, AlPu].

(2) The limit of $R(k, k)^{1/k}$, if it exists, is between $\sqrt{2}$ and 4 [GRS, GrRö, ChGra2].

(3) In 1995, Kim obtained a breakthrough result by proving that $R(3, k) = \Theta(k^2/\log k)$ [Kim]. The best known lower and upper bounds constants are 1/4 [BohK2] and 1 (implicit in [She1]), respectively. An independent proof of the lower bound constant 1/4 and a conjecture that it is the best possible are presented in [FizGM].

(4) Other asymptotic and general results on triangle-free graphs in relation to $R(3, k)$ can be found in [Boh, AlBK, AjKS, Alon2, CleDa, ChCD, CPR, Gri, FrLo, Loc, She1, She3].

(5) Explicit constructions yielded the lower bounds $R(4, k) \geq \Omega(k^{8/5})$, $R(5, k) \geq \Omega(k^{5/3})$ and $R(6, k) \geq \Omega(k^2)$ [KosPR]. For the same cases of k classical probabilistic arguments give $\Omega((k/\log k)^{5/2})$, $\Omega((k/\log k)^3)$ and $\Omega((k/\log k)^{7/2})$, respectively [Spe2]. These were improved to $\Omega(k^{5/2}/(\log k)^2)$, $\Omega(k^3/(\log k)^{8/3})$ and $\Omega(k^{7/2}/(\log k)^{13/4})$, respectively, in [Boh, BohK1], and in general to $R(s, t) = \Omega(t^{(s+1)/2}/(\log t)^{(s^2-s-4)/(2s-4)})$, for fixed s and large t [BohK1].

(6) Explicit construction of a graph with clique and independence k on $2^{c \log^2 k / \log\log k}$ vertices was presented by Frankl and Wilson [FraWi], and further constructions by Chung [Chu3] and Grolmusz [Grol1, Grol2]. In 2012, the best explicit construction for large k by Barak et al. [BarRSW] improved over [FraWi] by giving such a graph on $2^{2^{(\log\log k)^c}}$ vertices for some $c > 1$, or equivalently, on n vertices, where $\log\log n = (\log\log k)^c$. Explicit constructions such as these are usually weaker than known probabilistic results.

(7) In 2010, Conlon [Con1] obtained the best until now upper bound for the diagonal case:

$$R(k+1, k+1) \leq \binom{2k}{k} k^{-c \log k / \log\log k}$$

Other asymptotic bounds can be found, for example, in [Chu3, McS, Boh, BohK1] (lower bound) and [Tho] (upper bound), and for many other bounds in the general case of $R(k, l)$ consult [Spe2, GRS, GrRö, Chu4, ChGra2, LiRZ1, AlPu, Kriv].

3. Two Colors: $K_n - e$, K_3, $K_{m,n}$

3.1. Dropping one edge from complete graph

This section contains known values and nontrivial bounds for the two color case when the avoided graphs are complete or have the form $K_k - e$, but not both are complete.

G \ H	K_3-e	K_4-e	K_5-e	K_6-e	K_7-e	K_8-e	K_9-e	$K_{10}-e$	$K_{11}-e$
K_3-e	3	5	7	9	11	13	15	17	19
K_3	5	7	11	17	21	25	31	37	42 45
K_4-e	5	10	13	17	28	29 38	34	41	
K_4	7	11	19	30 33	37 52	75	105	139	184
K_5-e	7	13	22	31 39	40 66				
K_5	9	16	30 34	43 67	112	183	277	409	581
K_6-e	9	17	31 39	45 70	59 135				
K_6	11	21	37 53	110	205	373	621	1007	1544
K_7-e	11	28	40 66	59 135	251				
K_7	13	28 30	51 83	193	392	753	1336	2303	3751
K_8	15	42	123	300	657	1349	2558	4722	8200

Table IIIa. Two types of Ramsey numbers $R(G,H)$,
includes all known nontrivial values.

(a) The exact values in Table IIIa involving K_3-e are obvious, since one can easily see that $R(K_3-e, K_k) = R(K_3-e, K_{k+1}-e) = 2k-1$ for all $k \geq 2$.

(b) More bounds (beyond those shown in Tables IIIa/b) can be easily obtained using Table I, an obvious generalization of the inequality $R(k,l) \leq R(k-1,l) + R(k,l-1)$, and by monotonicity of Ramsey numbers, in this case $R(K_{k-1}, G) \leq R(K_k - e, G) \leq R(K_k, G)$.

(c) If the quadratic residues Paley graph Q_p of prime order $p = 4t+1$ contains no $K_k - e$, then $R(K_{k+1}-e, K_{k+1}-e) \geq 2p+1$. In particular, $R(K_{14}-e, K_{14}-e) \geq 2987$ [LiShen]. This was generalized to $K_k - F$ for some small graphs F instead of an edge e $(=K_2)$ [WaLi]. See also item 2.3.i.

G \ H	K_4-e	K_5-e	K_6-e	K_7-e	K_8-e	K_9-e	$K_{10}-e$	$K_{11}-e$
K_3	CH2	Clan	FRS1	GH	Ra1	Ra1	MPR GoR2	WWY2 GoR2
K_4-e	CH1	FRS2	McR	McR	Ea1 HZ2	Ex14	Ex14	
K_4	CH2	EHM1	Boza6 Boza5	Ex14 HZ2	BZ2	BZ2	BZ2	Ea1
K_5-e	FRS2	CE+	Ex14 Ea1	Ex14 HZ2				
K_5	BoH	Ex6 Ex8	Ea1 HZ2	HZ2	BZ2	BZ2	BZ2	BZ2
K_6-e	McR	Ex14 Ea1	Ex14 HZ2	Ex14 HZ2				
K_6	McN/ ShWR	Ex14 BZ1	BZ2	ShZ2	BZ2	BZ2	BZ2	BZ2
K_7-e	McR	Ex14 HZ2	Ex14 HZ2	ShZ1				
K_7	Ea1 BoPo	Ex14 Ea1	Ea1	BZ2	BZ2	BZ2	BZ2	BZ2
K_8	BZ1	BZ1	BZ2	BZ2	BZ2	BZ2	BZ2	BZ2

References for Table IIIa;
CE+ abbreviates CEHMS, for some details on BZ1 and BZ2 see item 3.1.d below.

k	11	12	13	14	15	16
lower bound	42 WWY2	47 Ea1	55 GoR2	59 Ea1	69 WWY2	73 Ea1
upper bound	45 GoR2	53 GoR2	62 GoR2	71 GoR2	80 GoR2	91 GoR2

Table IIIb. Lower and upper bounds for $R(K_3, K_k - e)$ for $11 \le k \le 16$;
lower bounds for $k = 12, 14, 16$ are the same as for $R(K_3, K_{k-1})$.

(d) This item follows personal communication from Boza [Boza5]. The upper bounds marked [BZ1] were obtained until 2012, while ones marked [BZ2] are from 2013. They are implied by [Boza6], the previous work [Boza1, Boza3, BoPo], the method of [HZ2], and the bounds given in [GoR2]. The enumeration of all (K_6, K_4-e)-graphs [ShWR] is used in [BoPo].

(e) All $(K_3, K_k - e)$-graphs were enumerated for $k \le 6$ [Ra1] and $k = 7$ [Fid2, GoR2]. Full sets of $(K_l, K_k - e)$-graphs were posted for the parameters $(K_3, K_k - e)$ for $k \le 7$, $(K_4, K_k - e)$ for $k \le 5$, and $(K_5, K_k - e)$ for $k \le 4$ at [Fid2], and other full and restricted families at [BrCGM, Fuj1].

(f) The number of $(K_3, K_l - e)$-critical graphs for $l = 4, 5$ and 8 is 4, 2 and 9, respectively [MPR]. There are 7 critical graphs for $R(K_3, K_9 - e)$, and at least 40 such graphs for $R(K_3, K_{10} - e)$ [GoR2].

(g) The critical graphs are unique for: $R(K_3, K_l - e)$ for $l = 3$ [Tr], 6 and 7 [Ra1], $R(K_4 - e, K_4 - e)$ [FRS2], $R(K_5 - e, K_5 - e)$ [Ra3] and $R(K_4 - e, K_7 - e)$ [McR].

(h) All of the critical graphs for the cases $R(K_4 - e, K_4)$ [EHM1], $R(K_4 - e, K_5)$ and $R(K_5 - e, K_4)$ [DzFi1] are known, and there are 5, 13 and 6 of them, respectively. The unpublished value of $R(K_4 - e, K_6)$ [McN] was confirmed in [ShWR], where in addition all 24976 critical graphs were found.

(i) It is known that $R(K_4, K_{12} - e) \geq 128$ [Shao] by using one color of the (4,4,4;127)-coloring defined in [HiIr].

(j) $R(K_k - e, K_k - e) \leq 4R(K_{k-2}, K_k - e) - 2$ [LiShen].
For a similar inequality for complete graphs see 2.3.b.

(k) Study of the cases $R(K_m, K_n - K_{1,s})$ and $R(K_m - e, K_n - K_{1,s})$, with several exact values for special parameters [ChaMR].

(l) The upper bounds from [ShZ1, ShZ2] are subsumed by a later article [Shi2].

(m) The upper bounds in [HZ2] were obtained by a reasoning generalizing the bounds for classical numbers in [HZ1]. Several other results from section 2.3 apply, though checking in which situation they do may require looking inside the proofs whether they still hold for $K_n - e$.

3.2. Triangle versus other graphs

(a) $R(3, k) = \Theta(k^2/\log k)$ [Kim].
For more comments on asymptotics see section 2.3 and the item 3.2.o/p below.

(b) Explicit construction for $R(3, 3k+1) \geq 4R(3, k+1) - 3$, for all $k \geq 2$ [CleDa],
explicit construction for $R(3, 4k+1) \geq 6R(3, k+1) - 5$, for all $k \geq 1$ [ChCD].

(c) Explicit triangle-free graphs with independence k on $\Omega(k^{3/2})$ vertices [Alon2, CPR].

(d) $R(K_3, K_7 - 2P_2) = R(K_3, K_7 - 3P_2) = 18$ [SchSch2].

(e) $R(K_3, K_3 + \overline{K}_m) = R(K_3, K_3 + C_m) = 2m + 5$ for $m \geq 212$ [Zhou1].

(f) $R(K_3, K_2 + T_n) = 2n + 3$ for n-vertex trees T_n, for $n \geq 4$ [SonGQ].

(g) $R(K_3, G) = 2n(G) - 1$ for any connected G on at least 4 vertices and with at most $(17n(G)+1)/15$ edges, in particular for $G = P_i$ and $G = C_i$, for all $i \geq 4$ [BEFRS1].

(h) $R(K_3, Q_n) = 2^{n+1} - 1$ for large n [FizGMSS], where Q_n is the n-dimensional hypercube. For the general case of $R(K_m, Q_n)$ see item 5.15.n.

(i) Relations between $R(3, k)$ and graphs with large $\chi(G)$ [Für],
further detailed study of the relation between $R(3, k)$ and the chromatic gap [GySeT].

(j) $R(K_3, G) \leq 2e(G)+1$ for any graph G without isolated vertices [Sid3, GoK].

(k) $R(K_3, G) \leq n(G)+e(G)$ for all G, a conjecture [Sid2].

(l) $R(K_3, G)$ for all connected G up to 9 vertices [BBH1, BBH2].

(m) $R(K_3, G)$ for all graphs G on 10 vertices [BrGS], except 10 cases (three of which, including $G = K_{10} - e$, were solved [GoR2]). See also section 8.1.

(n) Formulas for $R(nK_3, mG)$ for all G of order 4 without isolates [Zeng].

(o) For every positive constant c, Δ, and n large enough, there exists graph G with $\Delta(G) \leq \Delta$ for which $R(K_3, G) > cn$ [Bra3].

(p) $R(K_3, K_{k,k}) = \Theta(k^2/\log k)$ [LinLi2].

(q) For $R(K_3, K_n)$ see section 2, and for $R(K_3, K_n - e)$ see section 3.1.

(r) Since $B_1 = F_1 = C_3 = W_3 = K_3$, other sections apply. See also [Boh, AjKS, BBH1, BBH2, FrLo, Fra1, Fra2, Für, Gri, GySeT, Loc, KlaM1, LiZa1, RK2, RK3, RK4, She1, She3, Spe2, Stat, Yu1].

3.3. Complete bipartite graphs

Note: This subsection gathers information on Ramsey numbers where specific bipartite graphs are avoided in edge colorings of K_n (as everywhere in this survey), in contrast to the often studied bipartite Ramsey numbers, which are not covered in this survey, where the edges of complete bipartite graphs $K_{n,m}$ are colored.

3.3.1. Numbers

The following Tables IVa and IVb gather information mostly from the surveys by Lortz and Mengersen [LoM3, LoM4]. All cases involving $K_{1,2} = P_3$ are solved by a formula for $R(P_3, G)$, which holds for all isolate-free graphs G, derived in [CH2]. All star versus star numbers are given below in the item 3.3.2.a and in section 5.5.

p,q / m,n	1, 2	1, 3	1, 4	1, 5	1, 6	2, 2	2, 3	2, 4	2, 5	3, 3	3, 4
2, 2	4 CH2	6 CH2	7 Par3	8 Par3	9 FRS4	6 CH1					
2, 3	5 CH2	7 FRS4	9 Stev	10 FRS4	11 FRS4	8 HaMe4	10 Bu4				
2, 4	6 CH2	8 HaMe3	9 Stev	11 HaMe4	13 LoM4	9 HaMe4	12 ExRe	14 EHM2			
2, 5	7 CH2	9 HaMe3	11 Stev	13 Stev	14 LoM4	11 HaMe4	13 LoM3	16 LoM1	18 EHM2		
2, 6	8 CH2	10 HaMe3	11 Stev	14 Stev	15* Shao	12 HaMe4	14 LoM3	17 LoM3	20 LoM1		
3, 3	7 CH2	8 HaMe3	11 LoM4	12 LoM4	13 LoM4	11 Lortz	13 HaMe3	16 LoM4	18 LoM4	18 HaMe3	
3, 4	7 CH2	9 HaMe3	11 LoM4	13 LoM4	14 LoM4	11 Lortz	14 LoM4	17 Sh1+	≤21 LoM4	≤25 LoM2	≤30 LoM2
3, 5	9 CH2	10 HaMe3	13 Sh1+	15 Sh1+		14 HaMe4	17* Shao		≥21 Sh2+	≤28 LoM2	≤33 LoM2

Table IVa. Ramsey numbers $R(K_{m,n}, K_{p,q})$;
unpublished results are marked with a *, and Sh1+, Sh2+ abbreviate ShaXBP, ShaoWX.

m / n	2	3	4	5	6	7	8	9	10	11
6	12 HaMe4	14 LoM3	17 LoM3	20 LoM1	21 EHM2					
7	14 HaMe4	17 LoM3	19 LoM3	21 LoM3	24 LoM1	26 EMH2				
8	15 HaMe4	18 LoM3	20 LoM3	22*-23 LoM3	24-25 LoM3	28 LoM1	30 EMH2			
9	16 HaMe4	19 LoM3	22 LoM3	25* Shao	27* Shao	29* Shao	32 LoM1	33 EHM2		
10	17 HaMe4	21 LoM3	24 LoM3	27 LoM3	27-29 LoM3	28-31 LoM3	32-33 LoM3	36 LoM1	38 EHM2	
11	18 HaMe4						≤35 LoM3	36-37 LoM3	40 LoM1	42 EHM2

Table IVb. Known Ramsey numbers $R(K_{2,n}, K_{2,m})$ for $6 \le n \le 11$, $2 \le m \le 11$;
unpublished results improving over [LoM3] are marked with a *.

(a) The next few easily computed values of $R(K_{1,n}, K_{2,2})$, extending data in the first row of Table IVa, are 13, 14, 21 and 22 for n equal to 9, 10, 16 and 17, respectively. See function $f(n)$ in 3.3.2.c of the next subsection below.

(b) Formula for $R(K_{1,n}, K_{k_1,k_2,\ldots,k_t,m})$ for m large enough, in particular for $t=1, k_1=2$ with $n \le 5, m \ge 3$ and $n=6, m \ge 11$, for example $R(K_{1,5}, K_{2,7}) = 15$ [Stev].

(c) The values and bounds for higher cases of $R(K_{2,2}, K_{2,n})$ are 20, 22, 22, 24, 25, 26, 27/28, 28/29, 30 and 32 for $12 \le n \le 21$, respectively. All of them were given in [HaMe4], except those for n = 14, 15 and 18, which were obtained in [Dyb]. More exact values for prime powers $\lceil \sqrt{n} \rceil$ and $\lceil \sqrt{n} \rceil + 1$ can be found in [HaMe4].

(d) The known values of $R(K_{2,2}, K_{3,n})$ are 15, 16, 17, 20 and 22 for $6 \le n \le 10$ [Lortz], and $R(K_{2,2}, K_{3,12}) = 24$ [Shao]. See Tables IVa and IVb for the smaller cases, and [HaMe4] for upper bounds and values for some prime powers $\lceil \sqrt{n} \rceil$.

(e) $R(K_{2,n}, K_{2,n})$ is equal to 46, 50, 54, 57 and 62 for $12 \le n \le 16$, respectively.
The first open diagonal case is $65 \le R(K_{2,17}, K_{2,17}) \le 66$ [EHM2].
The status of all higher cases for $n < 30$ is listed in [LoM1].

(f) $R(K_{1,4}, K_{4,4}) = R(K_{1,5}, K_{4,4}) = 13$ [ShaXPB]
$R(K_{1,4}, K_{1,2,3}) = R(K_{1,4}, K_{2,2,2}) = 11$ [GuSL]
$R(K_{1,7}, K_{2,3}) = 13$ [Par4, Par6]
$R(K_{1,15}, K_{2,2}) = 20$ [La2]
$R(K_{2,2}, K_{4,4}) = 14$ [HaMe4]
$R(K_{2,2}, K_{4,5}) = 15$ [Shao]
$R(K_{2,2}, K_{4,6}) = 16$ [Shao]
$R(K_{2,2}, K_{5,5}) = R(K_{2,3}, K_{3,5}) = 17$ [Shao]

(g) A number of general upper and lower bounds for $R(K_{s,t}, K_{s,t})$, in particular for small fixed s, and for some slightly off-diagonal cases were obtained in [LoM2]. They can be used to derive the upper bounds for the cases listed in (h) and (i) below.

(h) Several lower bounds of the form $R(K_{s,t}, K_{s,t}) \ge m$ from distance colorings, a slightly more general concept than circular graphs, were presented in [HaKr2] for the following triples (s, t, m): (3,6,38), (3,7,42), (3,8,43), (3,9,54), (4,5,42), (4,6,43), (4,7,54), (5,5,54).

(i) $30 \le R(K_{3,5}, K_{3,5}) \le 38$ [HaKr2][LoM2]
$30 \le R(K_{4,4}, K_{4,4}) \le 62$ [HaKr2][LoM2]

3.3.2. General results

(a) $R(K_{1,n}, K_{1,m}) = n + m - \varepsilon$, where $\varepsilon = 1$ if both n and m are even and $\varepsilon = 0$ otherwise [Har1]. It is also a special case of multicolor numbers for stars obtained in [BuRo1].

(b) $R(K_{1,3}, K_{m,n}) = m + n + 2$ for $m, n \ge 1$ [HaMe3].

(c) $R(K_{1,n}, K_{2,2}) = f(n) \le n + \sqrt{n} + 1$, with $f(q^2) = q^2 + q + 1$ and $f(q^2+1) = q^2 + q + 2$ for every q which is a prime power [Par3]. Furthermore, $f(n) \ge n + \sqrt{n} - 6n^{11/40}$ [BEFRS4]. For more bounds on $f(n)$ see [Par5, Chen, ChenJ, MoCa, WuSZR].

(d) $R(K_{1,n+1}, K_{2,2}) \le R(K_{1,n}, K_{2,2}) + 2$ [Chen].

(e) $R(K_{2,\lambda+1}, K_{1,v-k+1})$ is either $v+1$ or $v+2$ if there exists a (v, k, λ)-difference set. This and other related results are presented in [Par4, Par5]. See also [GoCM, GuLi].

(f) Formulas and bounds on $R(K_{2,2}, K_{2,n})$, and bounds on $R(K_{2,2}, K_{m,n})$. In particular, we have $R(K_{2,2}, K_{2,k}) = n + k\sqrt{n} + c$, for $k = 2, 3, 4$, some prime powers $\lceil \sqrt{n} \rceil$ and $\lceil \sqrt{n} \rceil + 1$, and some $-1 \le c \le 3$ [HaMe4]. An improvement of the latter for some special cases of n was obtained in [Dyb].

(g) $R(K_{2,n}, K_{2,n}) \le 4n - 2$ for all $n \ge 2$, and the equality holds if and only if there exists a strongly regular $(4n-3, 2n-2, n-2, n-1)$-graph [EHM2].

(h) Conjecture that $4n - 3 \le R(K_{2,n}, K_{2,n}) \le 4n - 2$ for all $n \ge 2$. Many special cases are solved and several others are discussed in [LoM1].

(i) $R(K_{2,n-1}, K_{2,n}) \le 4n - 4$ for all $n \ge 3$, with the equality if there exists a symmetric Hadamard matrix of order $4n - 4$. There are only 4 cases in which the equality is still open for $3 \le n \le 58$, namely 30, 40, 44 and 48 [LoM1].

(j) $R(K_{2,n-s}, K_{2,n}) \le 4n - 2s - 3$ for $s \ge 2$ and $n \ge s + 2$, with the equality in many cases involving Hadamard matrices or strongly regular graphs. Asymptotics of $R(K_{2,n}, K_{2,m})$ for $m \gg n$ [LoM3].

(k) Some algebraic lower and upper bounds on $R(K_{s,n}, K_{t,m})$ for various combinations of n, m and $1 \le t, s \le 3$ [BaiLi, BaLX]. A general lower bound $R(K_{m,n}) \ge 2^m(n - n^{0.525})$ for large n [Dong].

(l) Upper bounds for $R(K_{2,2}, K_{m,n})$ for $m, n \ge 2$, with several cases identified for which the equality holds. Special focus on the cases for $m = 2$ [HaMe4].

(m) Bounds for the numbers of the form $R(K_{k,n}, K_{k,m})$, specially for fixed k and close to the diagonal cases. Asymptotics of $R(K_{3,n}, K_{3,m})$ for $m \gg n$ [LoM2].

(n) $R(nK_{1,3}, mK_{1,3}) = 4n + m - 1$ for $n \ge m \ge 1$, $n \ge 2$ [BES].

(o) Asymptotics for $K_{2,m}$ versus K_n [CaLRZ]. Upper bound asymptotics for $K_{k,m}$ versus K_n [LiZa1] and for some bipartite graphs K_n [JiSa].

(p) Special two-color cases apply in the study of asymptotics for multicolor Ramsey numbers for complete bipartite graphs [ChGra1].

4. Two Colors: Numbers Involving Cycles

4.1. Cycles, cycles versus paths and stars

Note: The paper *Ramsey Numbers Involving Cycles* [Ra4] is based on the revision #12 of this survey. It collects and comments on the results involving cycles versus any graphs, in two or more colors. It contains some more details than this survey, but only until 2009.

Cycles

(a) $R(C_3, C_3) = 6$ [GG, Bush],
$R(C_4, C_4) = 6$ [CH1].

(b) $R(C_3, C_n) = 2n-1$ for $n \geq 4$, $R(C_4, C_n) = n+1$ for $n \geq 6$,
$R(C_5, C_n) = 2n-1$ for $n \geq 5$, and $R(C_6, C_6) = 8$ [ChaS].

(c) Result obtained independently in [Ros1] and [FS1], a new simpler proof in [KáRos]:

$$R(C_m, C_n) = \begin{cases} 2n-1 & \text{for } 3 \leq m \leq n,\ m \text{ odd, } (m,n) \neq (3,3), \\ n-1+m/2 & \text{for } 4 \leq m \leq n,\ m \text{ and } n \text{ even, } (m,n) \neq (4,4), \\ \max\{n-1+m/2, 2m-1\} & \text{for } 4 \leq m < n,\ m \text{ even and } n \text{ odd.} \end{cases}$$

(d) Characterization of all graphs critical for $R(C_4, C_n)$ [WuSR].

(e) $R(mC_3, nC_3) = 3n + 2m$ for $n \geq m \geq 1$, $n \geq 2$ [BES].

(f) $R(mC_4, nC_4) = 2n + 4m - 1$ for $m \geq n \geq 1$, $(n,m) \neq (1,1)$ [LiWa1].

(g) Formulas for $R(mC_4, nC_5)$ [LiWa2].

(h) Formulas and bounds for $R(nC_m, nC_m)$ [Den2, Biel1].

(i) Study of $R(S_1, S_2)$, where S_1 and S_2 are sets of cycles [Hans].

(j) Unions of cycles, formulas and bounds for various cases including diagonal, different lengths, different multiplicities [MiSa, Den2], powers of cycles [AllBS], disjoint cycles versus K_n [Fuj2], and their relation to 2-local Ramsey numbers [Biel1].

Cycles versus paths

Result obtained by Faudree, Lawrence, Parsons and Schelp in 1974 [FLPS]:

$$R(C_m, P_n) = \begin{cases} 2n-1 & \text{for } 3 \leq m \leq n,\ m \text{ odd,} \\ n-1+m/2 & \text{for } 4 \leq m \leq n,\ m \text{ even,} \\ \max\{m-1+\lfloor n/2 \rfloor, 2n-1\} & \text{for } 2 \leq n \leq m,\ m \text{ odd,} \\ m-1+\lfloor n/2 \rfloor & \text{for } 2 \leq n \leq m,\ m \text{ even.} \end{cases}$$

For all n and m it holds that $R(P_m, P_n) \leq R(C_m, P_n) \leq R(C_m, C_n)$. Each of the two inequalities can become an equality, and, as derived in [FLPS], all four possible combinations of $<$ and $=$ hold for an infinite number of pairs (m,n). For example, if both m and n are even, and at least one of them is greater than 4, then $R(P_m, P_n) = R(C_m, P_n) = R(C_m, C_n)$.

For related generalizations see [BEFRS2].

Cycles versus stars

Only partial results for C_m versus stars are known. Lawrence [La1] settled the cases for odd m and for long cycles (see also [Clark, Par6]). The case for short even cycles is open, and it is related in particular to bipartite graphs. Partial results for $C_4 = K_{2,2}$ are pointed to in subsections 3.3.1 and 3.3.2. The most known exact result in [La1] is:

$$R(C_m, K_{1,n}) = \begin{cases} 2n+1 & \text{for odd } m \le 2n+1, \\ m & \text{for } m \ge 2n. \end{cases}$$

4.2. Cycles versus complete graphs

Since 1976, it was conjectured that $R(C_n, K_m) = (n-1)(m-1) + 1$ for all $n \ge m \ge 3$, except $n = m = 3$ [FS4, EFRS2]. Various parts of this conjecture were proved as follows: for $n \ge m^2 - 2$ [BoEr], for $n > 3 = m$ [ChaS], for $n \ge 4 = m$ [YHZ1], for $n \ge 5 = m$ [BolJY+], for $n \ge 6 = m$ [Schi1], for $n \ge m \ge 7$ with $n \ge m(m-2)$ [Schi1], for $n \ge 7 = m$ [ChenCZ1], and for $n \ge 4m+2$, $m \ge 3$ [Nik]. Open conjectured cases are marked in Table V by "conj."

	C_3	C_4	C_5	C_6	C_7	C_8	C_9	...	C_n for $n \ge m$
K_3	6 GG-Bush	7 ChaS	9 ...	11	13	15	17	... ...	$2n-1$ ChaS
K_4	9 GG	10 CH2	13 He4/JR4	16 JR2	19 YHZ1	22 ...	25	... ...	$3n-2$ YHZ1
K_5	14 GG	14 Clan	17 He2/JR4	21 JR2	25 YHZ2	29 BolJY+	33 ...	... ...	$4n-3$ BolJY+
K_6	18 Kéry	18 Ex2-RoJa1	21 JR5	26 Schi1	31 ...	36	41	... ...	$5n-4$ Schi1
K_7	23 Ka2-GrY	22 RT-JR1	25 Schi2	31 CheCZN	37 CheCZN	43 JaBa/Ch+	49 Ch+	... ...	$6n-5$ Ch+
K_8	28 GR-MZ	26 RT	29-33 JaAl2	36 ChenCX	43 ChenCZ1	50 JaAl1/ZZ3	57 BatJA	... ...	$7n-6$ conj.
K_9	36 Ka2-GR	30 RT-LivLR					65 conj.	... ...	$8n-7$ conj.
K_{10}	40-42 Ex5-GoR1	36 LivLR						...	$9n-8$ conj.
K_{11}	47-50 Ex20-GoR1	39-44 LivLR						...	$10n-9$ conj.

Table V. Known Ramsey numbers $R(C_n, K_m)$;
Ch+ abbreviates ChenCZ1, for comments on joint credits see 4.2.b.

(a) The first column in Table V gives data from the first row in Table I.

(b) Joint credit [He2/JR4] in Table V refers to two cases in which Hendry [He2] announced the values without presenting the proofs, which later were given in [JR4]. The special cases of $R(C_6, K_5) = 21$ [JR2] and $R(C_7, K_5) = 25$ were solved independently in [YHZ2] and [BolJY+]. The double pointer [JaBa/ChenCZ1] refers to two independent papers, similarly as [JaAl1/ZZ3], except that in the latter case [ZZ3] refers to an unpublished manuscript. For joint credits marked in Table V with "-", the first reference is for the lower bound and the second for the upper bound.

(c) Erdős et al. [EFRS2] asked what is the minimum value of $R(C_n, K_m)$ for fixed m, and they suggested that it might be possible that $R(C_n, K_m)$ first decreases monotonically, then attains a unique minimum, then increases monotonically with n.

(d) There exist constants $c_1, c_2 > 0$ such that $c_1(m^{3/2}/\log m) \le R(C_4, K_m) \le c_2(m/\log m)^2$. The lower bound, recently obtained by Bohman and Keevash ([BohK1], see also 4.2.h below) improved over an almost 40 years old bound $c(m/\log m)^{3/2}$ by Spencer [Spe2], using the probabilistic method. The upper bound was reported in a paper by Caro, Li, Rousseau and Zhang [CaLRZ], who in turn give the credit to an unpublished work by Szemerédi from 1980.

(e) Erdős, in 1981, in the Ramsey problems section of the paper [Erd3] formulated a challenge by asking for a proof of $R(C_4, K_m) < m^{2-\varepsilon}$, for some $\varepsilon > 0$. To date, no such proof is known.

(f) Let $C_{\le m}$ be the set of cycles of length at most m, and let the *girth* $g(G)$ be the length of the shortest cycle in graph G. Probabilistic lower bound asymptotics for $R(C_{\le m}, K_k)$ [Spe2] currently is the same as for $R(C_m, K_k)$, for fixed m. However, there are clear differences already for girth 4 and 5 and small k: Backelin [Back1, Back2] found that $R(C_{\le 4}, K_k)$ = 6, 8, 11, 15, 18 for k = 3, 4, 5, 6, 7, and that $R(C_{\le 5}, K_k)$ = 5, 8, 10, 13, 15, also for k = 3, 4, 5, 6, 7, respectively.

(g) Erdős et al. [EFRS2] proved various facts about $R(C_{\le m}, K_k)$, and in particular that it is equal to $2n-1$ for $m \ge 2n-1$, and to $2n$ for $n < m < 2n-1$. The upper asymptotics for $R(C_{\le m}, K_k)$ is implied in the study of independence number in graphs with odd girth m [Den1].

(h) The best known lower bound asymptotics $R(C_n, K_m) = \Omega(m^{(n-1)/(n-2)}/\log m)$, for fixed n and large m, was obtained by Bohman and Keevash [BohK1]. Note that for $n = 4$ it gives the lower bound in 4.2.d above. See also [Spe2, FS4, AlRö] for previous results.

(i) Upper bound asymptotics [BoEr, FS4, EFRS2, CaLRZ, Sud1, LiZa2, AlRö, DoLL2].

4.3. Cycles versus wheels

Note: In this survey the wheel graph $W_n = K_1 + C_{n-1}$ has n vertices, while some authors use the definition $W_n = K_1 + C_n$ with $n+1$ vertices. For the cases involving $W_3 = C_3$ versus C_m see sections 3.2 and 4.2.

	C_3	C_4	C_5	C_6	C_7	C_8	C_m	for
W_4	9 GG	10 CH2	13 He4	16 JR2	19 YHZ1	22 ...	$3m-2$ ...	$m \geq 4$ YHZ1
W_5	11 Clan	9 Clan	9 He2	11 JR2	13 SuBB2	15 ...	$2m-1$ ...	$m \geq 5$ SuBB2
W_6	11 BE3	10 JR3	13 ChvS	16 SuBB2	19 ...	22 ...	$3m-2$ ...	$m \geq 4$ SuBB2
W_7	13 BE3	9 Tse1	13 LuLL	11 LuLL	13 LuLL		$2m-1$ ...	$m \geq 10$ Ch1
W_8	15 BE3	11 Tse1	15 LuLL	16 LuLL	19 Ch2	22 ...	$3m-2$ ...	$m \geq 6$ Ch2
W_9	17 BE3	12 Tse1	17 LuLL	13 LuLL	17 LuLL		$2m-1$	$m \geq 13$ Ch1
W_{10}	19 BE3	13 Tse1					$3m-2$	$m \geq 9$ Ch2
W_n for	... $2n-1$ $n \geq 6$ BE3		$2n-1$ $n \geq 19$ Zhou2		$2n-1$ $n \geq 29$ Zhou2		large wheels	cycles

Table VI. Ramsey numbers $R(W_n, C_m)$ for $n \leq 10$, $m \leq 8$;
Ch1 abbreviates ChenCMN, Ch2 abbreviates ChenCNZ.

(a) $R(C_3, W_n) = 2n-1$ for $n \geq 6$ [BE3]. All critical graphs have been enumerated. The critical graphs are unique for $n = 3, 5$, and for no other n [RaJi].

(b) $R(C_4, W_n) = 14, 16, 17$ for $n = 11, 12, 13$, respectively [Tse1], $R(C_4, W_n) = 18, 19, 20, 21$ for $n = 14, 15, 16, 17$, respectively [DyDz2], and several higher values and bounds, including 9 cases of n between 18 and 44 [WuSR, WuSZR].

(c) $R(C_4, W_n) \leq n + \lceil (n-1)/3 \rceil$ for $n \geq 7$ [SuBUB], which was improved to $R(C_4, W_n) \leq n + \sqrt{n-2} + 1$ for $n \geq 11$ [DyDz2].

(d) $R(C_4, W_{q^2+1}) = q^2 + q + 1$ for prime power $q \geq 4$ [DyDz2], exact values of $R(C_4, W_{q^2+2})$ and $R(C_4, W_{q^2-i})$ for special q and small i [WuSZR].

(e) $R(W_n, C_m) = 2n-1$ for odd m with $n \geq 5m-6$ [Zhou2].

(f) $R(W_n, C_m) = 3m-2$ for even $n \geq 4$ with $m \geq n-1$, $m \neq 3$, was conjectured by Surahmat et al. [SuBT1, SuBT2, Sur]. Parts of this conjecture were proved in [SuBT1, ZhaCC1, Shi5], and the proof was completed in [ChenCNZ].

(g) Conjecture that $R(W_n, C_m) = 2m - 1$ for odd $n \geq 3$ and all $m \geq 5$ with $m > n$ [Sur]. It was proved for $2m \geq 5n - 7$ [SuBT1], and further for $2m \geq 3n - 1$ [ChenCMN]. See also [Shi5].

(h) Observe apparently four distinct situations with respect to parity of m and n.

(i) Cycles are Ramsey unsaturated for some wheels [AliSur], see also comments on [BaLS] in subsection 5.16.

(j) Study of cycles versus generalized wheels $W_{k,n}$ [Sur, SuBTB, Shi5].

4.4. Cycles versus books

	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_m	for
B_2	7 RS1	7 F*al*6	9 C*al*	11 F*al*8	13 ...	15	17	19	21	$2m-1$ ...	$m \geq 4$ F*al*8
B_3	9 RS1	9 F*al*6	10 F*al*8	11 JR2	13 Shi5	15 F*al*8	17 ...	19	21	$2m-1$ ...	$m \geq 6$ F*al*8
B_4	11 RS1	11 F*al*6	11 F*al*8	12 S*al*1	13 S*al*1	15 Shi5	17 Shi5	19 F*al*8	21 ...	$2m-1$ ...	$m \geq 7$ F*al*8
B_5	13 RS1	12 F*al*6	13 F*al*8	14 S*al*1	15 S*al*1	15 S*al*2	17 S*al*2	19 Shi5	21 Shi5	$2m-1$ ...	$m \geq 8$ F*al*8
B_6	15 RS1	13 F*al*6	15 F*al*8	16 S*al*2	17 S*al*2	18 S*al*2	18 S*al*2		21 Shi5	$2m-1$ ...	$m \geq 11$ Shi5
B_7	17 RS1	16 F*al*6	17 F*al*8	16 S*al*2	19 S*al*2	20 S*al*2	21 S*al*2			$2m-1$	$m \geq 13$ Shi5
B_8	19 RS1	17 Tse1	19 F*al*8	17 S*al*2	19 S*al*2	22 S*al*2	≥ 23 S*al*2			$2m-1$	$m \geq 14$ Shi5
B_9	21 RS1	18 Tse1	21 F*al*8	18 S*al*2			≥ 25 S*al*2	≥ 26 S*al*2		$2m-1$	$m \geq 16$ Shi5
B_{10}	23 RS1	19 Tse1	23 F*al*8	19 S*al*2				≥ 28 S*al*2		$2m-1$	$m \geq 17$ Shi5
B_{11}	25 RS1	20 Tse1	25 F*al*8							$2m-1$	$m \geq 19$ Shi5
B_n for	... $2n+3$ $n \geq 2$ RS1	$\approx n$ some (c)	... $2n+3$ $n \geq 4$ F*al*8		$2n+3$ $n \geq 15$ F*al*8		$2n+3$ $n \geq 23$ F*al*8		$2n+3$ $n \geq 31$ F*al*8	large books	cycles

Table VII. Ramsey numbers $R(B_n, C_m)$ for $n, m \leq 11$;
et al. abbreviations: F*al*/FRS, C*al*/CRSPS, S*al*1/ShaXBP, S*al*2/ShaXB.

(a) For the cases of $B_1 = K_3$ versus C_m see section 4.2. The exact values for the cases (3,7), (4,8), (4,9), (5,10), (5,11) were obtained independently in [S*al*1, S*al*2]/[ShaXBP, ShaXB] using computer algorithms.

(b) $R(C_4, B_{12}) = 21$ [Tse1], $R(C_4, B_{13}) = 22$, $R(C_4, B_{14}) = 24$ [Tse2].
$R(C_4, B_8) = 17$ [Tse2] (it was reported incorrectly in [FRS7] to be 16).

(c) $q^2 + q + 2 \le R(C_4, B_{q^2 - q + 1}) \le q^2 + q + 4$ for prime power q [FRS7]. B_n is a subgraph of B_{n+1}, hence likely $R(C_4, B_n) = n + O(\sqrt{n})$ (compare to $R(C_4, K_{2,n})$ in section 3.3).

(d) $R(B_n, C_m) = 2n + 3$ for odd $m \ge 5$ with $n \ge 4m - 13$ [FRS9].

(e) $R(B_n, C_m) = 2m - 1$ for $n \ge 1$, $m \ge 2n + 2$ [FRS9]. The range of m was extended to $m \ge 2n - 1 \ge 7$ in [ShaXB], and to $m > (6n + 7)/4$ in [Shi5].

(f) $R(B_n, C_n) \ge 3n - 2$ and $R(B_{n-1}, C_n) \ge 3n - 4$ for $n \ge 3$ [ShaXB].

(g) More theorems on $R(B_n, C_m)$ in [FRS7, FRS9, NiRo4, Zhou1].

(h) Cycles versus some generalized books [Shi5].

4.5. Cycles versus other graphs

(a) C_4 versus stars [Par3, Par4, Par5, BEFRS4, Chen, ChenJ, GoMC, MoCa, WuSZR]. For several exact results see $K_{2,2}$ in Tables IVa and IVb, and for general results see items 3.3.1.a, 3.3.2.c and 3.3.2.d.

(b) C_4 versus unions of stars [HaABS, Has]

(c) C_4 versus trees [EFRS4, Bu7, BEFRS4, Chen]

(d) C_4 versus all graphs on six vertices [JR3]

(e) C_4 versus various types of complete bipartite graphs, see section 3.3

(f) $R(C_4, G) \le 2q + 1$ for any isolate-free graph G with q edges [RoJa2]

(g) $R(C_4, G) \le p + q - 1$ for any connected graph G on p vertices and q edges [RoJa2]

(h) $R(C_5, K_6 - e) = 17$ [JR4]

(i) $R(C_5, K_4 - e) = 9$ [CRSPS]

(j) C_5 versus all graphs on six vertices [JR4]

(k) $R(C_6, K_5 - e) = 17$ [JR2]

(l) C_6 versus all graphs on five vertices [JR2]

(m) $R(C_{2m+1}, G) = 2n - 1$ for sufficiently large sparse graphs G on n vertices, in particular $R(C_{2m+1}, T_n) = 2n - 1$ for all $n > 1512m + 756$, for n-vertex trees T_n [BEFRS2].

(n) $R(C_n, G) \le 2q + \lfloor n/2 \rfloor - 1$, for $3 \le n \le 5$, for any isolate-free graph G with $q > 3$ edges. It is conjectured that it also holds for other n [RoJa2].

(o) Cycles versus trees [BEFRS2, FSS1]

(p) Monotone paths and cycles [Lef]

(q) Cycles versus $K_{n,m}$ and multipartite complete graphs [BoEr]

(r) Cycles versus generalized books and wheels [Shi5, Sur, SuBTB], and versus other special graphs of the form $K_n + G$ with small $n \le 3$ and sparse G [Shi5].

5. General Graph Numbers in Two Colors

This section includes data with respect to general graph results. We tried to include all nontrivial values and identities regarding exact results, or references to them, but only those out of general bounds and other results which, in our opinion, may have a direct connection to the evaluation of specific numbers. If some small value cannot be found below, it may be covered by the cumulative data gathered in section 8, or be a special case of a general result listed in this section. Note that $P_2 = K_2$, $B_1 = F_1 = C_3 = W_3 = K_3$, $B_2 = K_4 - e$, $P_3 = K_3 - e$, $W_4 = K_4$ and $C_4 = K_{2,2}$ imply other identities not mentioned explicitly.

5.1. Paths

$R(P_m, P_n) = n + \lfloor m/2 \rfloor - 1$ for all $n \geq m \geq 2$ [GeGy]

Stripes mP_2 [CocL1, CocL2, Lor]

Disjoint unions of paths (also called linear forests) [BuRo2, FS2]

5.2. Wheels

Note: In this survey the wheel graph $W_n = K_1 + C_{n-1}$ has n vertices,
while some authors use the definition $W_n = K_1 + C_n$ with $n+1$ vertices.

m \ n	3	4	5	6	7
3	6	9 GG	11 Clan	11 BE3	13 BE3
4		18 GG	17 He3	19 FM	
5			15 He2	17 FM	
6				17 FM	

Table VIII. Ramsey numbers $R(W_m, W_n)$ for $m \leq n \leq 7$.

(a) $R(W_3, W_n) = 2n-1$ for all $n \geq 6$ [BE3],
All critical colorings for $R(W_3, W_n)$ for all $n \geq 3$ [RaJi].

(b) The value $R(W_5, W_5) = 15$ was given in the Hendry's table [He2] without a proof. Later the proof was published in [HaMe2].

(c) All critical colorings (2, 1 and 2) for $R(W_n, W_6)$, for $n = 4, 5, 6$ [FM].

(d) $R(W_6, W_6) = 17$, $R(4,4) = 18$ and $\chi(W_6) = 4$ give a counterexample $G = W_6$ to the Erdős conjecture (Erd2, see also [GRS]) that $R(G, G) \geq R(K_{\chi(G)}, K_{\chi(G)})$.

5.3. Books

n / m	1	2	3	4	5	6	7
1	6	7 CH2	9 Clan	11 RS1	13 RS1	15 RS1	17 RS1
2		10 CH1	11 Clan	13 Rou	16 RS1	17 Rou	18 BlLR
3			14 RS1	15 Sh1+	17 RS1		
4				18 RS1	≤ 20 RS1	22 RS1	
5					21 RS1		
6						26 RS1	

Table IX. Ramsey numbers $R(B_m, B_n)$ for $m, n \leq 7$;
Sh1+ abbreviates ShaXBP.

(a) $254 \leq R(B_{37}, B_{88}) \leq 255$ [Par6].

(b) Unpublished result $R(B_2, B_6) = 17$ [Rou] was confirmed in [BlLR].

(c) There are 4 Ramsey-critical graphs for $R(B_2, B_3)$, a unique graph for $R(B_3, B_4)$ [ShaXBP], 3 for $R(B_2, B_6)$ and 65 for $R(B_2, B_7)$ [BlLR].

(d) $R(B_1, B_n) = 2n + 3$ for all $n > 1$ [RS1].

(e) $R(B_n, B_m) = 2n + 3$ for all $n \geq cm$ for some $c < 10^6$ [NiRo2, NiRo3].

(f) $R(B_n, B_n) = (4 + o(1))n$ [RS1, NiRS].

(g) In general, $R(B_n, B_n) = 4n + 2$ for $4n + 1$ a prime power. Several other specific values (like $R(B_{62}, B_{65}) = 256$) and general equalities and bounds for $R(B_n, B_m)$ can be found in [RS1, FRS8, Par6, NiRS, LiRZ2].

5.4. Trees and forests

In this subsection T_n and F_n denote n-vertex tree and forest, respectively.

(a) $R(T_n, T_n) \leq 4n + 1$ [EG].

(b) $R(T_n, T_n) \geq \lfloor (4n - 1)/3 \rfloor$ [BE2], see also section 5.15.

(c) Conjecture that $R(T_n, T_n)$ is at most $2n - 2$ for even n and $2n - 3$ for odd n [BE2]. Note that this is the same as asking if $R(T_n, T_n) \leq R(K_{1,n-1}, K_{1,n-1})$. Zhao proved that $R(T_n, T_n) \leq 2n - 2$ and thus confirmed the conjecture for even n. Independently, Ajtai et al. [AjKSS] announced a full proof for large n. This recent progress subsumes some of the results pointed to in items (d)-(l) below.

(d) For general discussion of related problems see [Bu7, FSS1, ChGra2], in particular of the conjecture that $R(T_m, T_n) \le n + m - 2$ holds for all trees [FSS1].

(e) If $\Delta(T_m) = m - 2$ and $\Delta(T_n) = n - 2$ then the exact values of $R(T_m, T_n)$ are known, and they are between $n + m - 5$ and $n + m - 3$ depending on n and m. In particular, for $n = 2k + 1$ we have $R(T_{2k+1}, T_{2k+1}) = 2n - 5$ [GuoV].

(f) Examples of families T_m and T_n (including P_n) for which $R(T_m, T_n) = n + m - c$, $c = 3, 4, 5$ [SunZ], extending the results in [GuoV].

(g) View tree T as a bipartite graph with parts t_1 and t_2, $t_2 \ge t_1$. Define $b(T) = \max\{2t_1 + t_2 - 1, 2t_2 - 1\}$. Then the bound $R(T, T) \ge b(T)$ holds always, $R(T, T) = b(T)$ holds for many classes of trees [EFRS3, GeGy], and asymptotically [HaŁT], but cases for nonequality have been found [GHK].

(h) Comments in [BaLS] about some conjectures on Ramsey saturation of non-star trees, which would imply that $R(T_n, T_n) \le 2n - 2$ holds for sufficiently large n.

(i) Formulas for $R(T_m, T_n)$ for some subcases of when T_m and T_n satisfy $\Delta(T_m) = m - 3$ and $\Delta(T_n) \le n - 3$ [SunWW].

(j) $R(T_m, K_{1,n}) \le m + n - 1$, with equality for $(m - 1) \mid (n - 1)$ [Bu1].

(k) $R(T_m, K_{1,n}) = m + n - 1$ for sufficiently large n for almost all trees T_m [Bu1]. Many cases were identified for which $R(T_m, K_{1,n}) = m + n - 2$ [Coc, ZZ1], see also [Bu1].

(l) $R(T_m, K_{1,n}) \le m + n$ if T_n is not a star and $(m - 1) \nmid (n - 1)$,
some classes of trees and stars for which the equality holds [GuoV].

(m) $R(F_n, F_n) > n + \log_2 n - O(\log\log n)$ [BE2], forests are tight for this bound [CsKo].

(n) Forests, linear forests (unions of paths) [BuRo2, FS3, CsKo].

(o) Paths versus trees [FSS1], see also other parts of this survey involving special graphs, in particular sections 5.5, 5.6, 5.10, 5.12 and 5.15.

5.5. Stars, stars versus other graphs

$R(K_{1,n}, K_{1,m}) = n + m - \varepsilon$, where $\varepsilon = 1$ for even n and m, and $\varepsilon = 0$ otherwise [Har1]. This is also a special case of multicolor numbers for stars 6.6.e obtained in [BuRo1].

$R(K_{1,n}, K_m) = n(m - 1) + 1$ by Chvátal's theorem [Chv].

Stars versus C_4 [Par3, Par4, Par5, BEFRS4, Chen, ChenJ, GoMC, MoCa, WuSZR]
Stars versus $K_{2,n}$ [Par4, GoMC]
Stars versus $K_{n,m}$ [Stev, Par3, Par4]
See also section 3.3

$R(K_{1,4}, B_4) = 11$ [RS2]
$R(K_{1,4}, K_{1,2,3}) = R(K_{1,4}, K_{2,2,2}) = 11$ [GuSL]

Stars versus paths [Par2, BEFRS2]
Stars versus cycles [La1, Clark], see also [Par6] and section 4.1
Stars versus $2K_2$ [MeO]
Stars versus stripes mP_2 [CocL1, CocL2, Lor]
Stars versus W_5 and W_6 [SuBa1]
$nK_{1,m}$ versus W_5 [BaHA]
Stars versus W_9 [Zhang2, ZhaCZ1]
Stars versus wheels [HaBA1, ChenZZ2, Kor]
Stars versus books [CRSPS, RS2]
Stars versus trees [Bu1, Cheng, Coc, GuoV, SunZ, ZZ1]
Stars versus $K_n - tK_2$ [Hua1, Hua2]
Union of two stars [Gros2]
Unions of stars versus C_4 and W_5 [HaABS, Has]
Unions of stars versus wheels [BaHA, HaBA2, SuBAU1]

5.6. Paths versus other graphs

Note: for cycles versus P_n see section 4.1.

P_3 versus all isolate-free graphs [CH2]
Paths versus stars [Par2, BEFRS2]
Paths versus trees [FS4, FSS1, SunZ]
Paths versus books [RS2]
Paths versus K_n [Par1]
Paths versus $2K_n$ [SuAM, SuAAM]
Paths versus $K_{n,m}$ [Häg]
Paths versus some balanced complete multipartite graphs [Pokr]
Paths versus W_5 and W_6 [SuBa1]
Paths versus W_7 and W_8 [Bas]
Paths versus wheels [BaSu, ChenZZ1, SaBr3, Zhang1]
$R(P_n, mW_4) = 2n + m - 2$ [Sudar]
Paths versus beaded wheels [AliBT2]
Paths versus powers of paths [Pokr, AllBS]
Paths versus fans [SaBr2]
Paths versus $K_1 + P_m$ [SaBr1, SaBr4]
Paths and cycles versus trees [FSS1]
Powers of paths [AllBS]
Unions of paths [BuRo2]
Paths and unions of paths versus Jahangir graphs [AliBas, AliBT1, AliSur]
Paths and unions of paths versus $K_{2m} - mK_2$ [AliBB]
Sparse graphs versus paths and cycles [BEFRS2]
Graphs with long tails [Bu2, BG]
Monotone paths and cycles [Lef]

5.7. Fans, fans versus other graphs

$R(F_1, F_n) = R(K_3, F_n) = 4n+1$ for $n \geq 2$, and bounds for $R(F_m, F_n)$ [LiR2, GGS]

$R(F_2, F_n) = 4n+1$ for $n \geq 2$ and $R(F_m, F_n) \leq 4n+2m$ for $n \geq m \geq 2$ [LinLi1]

$R(K_4, F_n) = 6n+1$ for $n \geq 3$ [SuBB3]

Fans versus paths, formulas for a number of cases including $R(P_6, F_n)$ [SaBr2].
Missing case $R(P_6, F_4) = 12$ solved in [Shao].

Fans versus cycles [Shi5]
Fans versus K_n [LiR2]
Lower bounds on $R(F_2, K_n)$ from cyclic graphs for $n \leq 9$ [Shao]

5.8. Wheels versus other graphs

Notes: In this survey the wheel graph $W_n = K_1 + C_{n-1}$ has n vertices,
while some authors use the definition $W_n = K_1 + C_n$ with $n+1$ vertices.
For cycles versus W_n see section 4.3.

$R(W_5, K_5 - e) = 17$ [He2][YH]
$R(W_5, K_5) = 27$ [He2][RST]
$R(W_5, K_6) \geq 33$, $R(W_5, K_7) \geq 43$ [Shao, ShaoWX]
W_5 and W_6 versus stars and paths [SuBa1]
W_5 versus $nK_{1,m}$ [BaHA]
W_5 versus unions of stars [Has]
W_5 and W_6 versus trees [BaSNM]
W_7 and W_8 versus paths [Bas]

W_7 versus trees T_n with $\Delta(T_n) \geq n-3$, other special trees T,
and for $n \leq 8$ [ChenZZ3, ChenZZ5, ChenZZ6]

W_7 and W_8 versus trees [ChenZZ4, ChenZZ5]
W_9 versus stars [Zhang2, ZhaCZ1, ZhaCC2]
W_9 versus trees of high degree [ZhaCZ2]

Wheels versus stars [HaBA1, ChenZZ2, Kor]
Wheels W_n, for even n, versus star-like trees [SuBB1]
Wheels versus paths [BaSu, ChenZZ1, SaBr3, Zhang1]
Wheels versus books [Zhou3]
Wheels versus unions of stars [BaHA, HaBA2, SuBAU1]
Wheels versus linear forests (disjoint unions of paths) [SuBa2]
Generalized wheels versus cycles [Shi5]
Upper asymptotics for $R(W_n, K_m)$ [Song5, SonBL]
Upper asymptotics for generalized wheels versus K_n [Song9]

5.9. Books versus other graphs

Note: for cycles versus B_n see section 4.4.

$R(B_3, K_4) = 14$ [He3]
$R(B_3, K_5) = 20$ [He2][BaRT]
$R(B_4, K_{1,4}) = 11$ [RS2]

Cyclic lower bounds for $R(B_m, K_n)$ for $m \leq 7$, $n \leq 9$
and for $R(B_3, K_n - e)$ for $n \leq 7$ [Shao, ShaoWX]

Books versus paths [RS2]
Books versus stars [CRSPS, RS2]
Books versus trees [EFRS7]
Books versus K_n [LiR1, Sud2]
Books versus wheels [Zhou3]
Books versus $K_2 + C_n$ [Zhou3]
Books and $(K_1 + tree)$ versus K_n [LiR1]
Generalized books $K_3 + qK_1$ versus cycles [Shi5]
Generalized books $K_r + qK_1$ versus K_n [NiRo1, NiRo4]

5.10. Trees and forests versus other graphs

In this subsection T_n and F_n denote n-vertex tree and forest, respectively.

$R(T_n, K_m) = (n-1)(m-1)+1$ [Chv]

$R(T_n, C_{2m+1}) = 2n-1$ for all $n > 1512m + 756$ [BEFRS2]

$R(T_n, B_m) = 2n-1$ for all $n \geq 3m-3$ [EFRS7]

$R(F_{nk}, K_m) = (n-1)(m-2)+nk$ for all forests F_{nk} consisting of k trees with
n vertices each, also exact formula for all other cases of forests versus K_m [Stahl]

Exact results for almost all small $(n(G) \leq 5)$ connected graphs G versus all trees [FRS4]

Trees versus stars [Bu1, Cheng, Coc, GuoV, ZZ1]
Trees versus paths [FS4, FSS1]
Trees versus C_4 [EFRS4, Bu7, BEFRSS5, Chen]
Trees versus cycles [FSS1, EFRS6]
Trees versus books [EFRS7]
Trees versus W_5 and W_6 [BaSNM]
Trees versus W_7 and W_8 [ChenZZ4, ChenZZ5]

Trees T_n with $\Delta(T_n) \geq n-3$, other special trees T,
and for $n \leq 8$ versus W_7 [ChenZZ3, ChenZZ5, ChenZZ6]
Trees T_n with $\Delta(T_n) \geq n-4$ versus W_9 [ZhaCZ2]

Star-like trees versus odd wheels [SuBB1, ChenZZ3]
Trees versus $K_n + \overline{K}_m$ [RS2, FSR]
Trees versus bipartite graphs [BEFRS4, EFRS6]

Trees versus almost complete graphs [GoJa2]
Trees versus multipartite complete graphs [EFRS8, BEFRSGJ]

Linear forests versus $3K_3$ and $2K_4$ [SuBAU2]
Linear forests versus $2K_m$ [SuAAM]
Linear forests versus wheels [SuBa2]
Forests versus almost complete graphs [ChGP]
Forests versus complete graphs [BE1, Stahl, BaHA]

Study of graphs G for which all or almost all trees are G-good [BF, BEFRSGJ], see also section 5.15 and 5.16, item [Bu2], for the definition and more pointers.

See also various parts of this survey for special trees, and section 5.4.

5.11. Cases for $n(G)$, $n(H) \leq 5$

Clancy [Clan], in 1977, presented a table of $R(G,H)$ for all isolate-free graphs G with $n(G) = 5$ and H with $n(H) \leq 4$, except 5 entries. All five of the open entries have been solved as follows:

$R(B_3, K_4) = 14$	[He3]
$R(K_5, K_4 - e) = 16$	[BoH]
$R(W_5, K_4) = 17$	[He2]
$R(K_5 - e, K_4) = 19$	[EHM1]
$R(K_5, K_4) = R(4,5) = 25$	[MR4]

An interesting case in [Clan] is

$$R(K_4, K_5 - P_3) = R(K_4, K_4 + e) = R(4,4) = 18.$$

Hendry [He2], in 1989, presented a table of $R(G,H)$ for all graphs G and H on 5 vertices without isolates, except 7 entries. Five of the open entries have been solved:

$R(K_5, K_4 + e) = R(4,5) = 25$	[Ka1][MR4]
$R(K_5, K_5 - P_3) = 25$	[Ka1][Boza2, CalSR]
$R(K_5, B_3) = 20$	[He2][BaRT]
$R(K_5, W_5) = 27$	[He2][RST]
$R(W_5, K_5 - e) = 17$	[He2][YH]

The still open cases for K_5 versus $K_5 - e$ and K_5 are:

$30 \leq R(K_5, K_5 - e) \leq 34$	[Ex6][Ex8]
$43 \leq R(K_5, K_5) \leq 49$	[Ex4][MR5]

All critical colorings for the case $R(C_5 + e, K_5) = 17$ were found by Hendry [He5].

5.12. Mixed cases

$12 \le R(Q_3, Q_3)$, where Q_3 is the 8-vertex 3-dimensional cube graph,
$19 \le R(P, P)$, where P is the 10-vertex Petersen graph,
$30 \le R(K_{2,2,2}, K_{2,2,2})$, where $K_{2,2,2}$ is the octahedron [HaKr2].

Unicyclic graphs [Gros1, Köh, KrRod]
$K_{2,m}$ and C_{2m} versus K_n [CaLRZ]
$K_{2,n}$ versus any graph [RoJa2]
Union of two stars [Gros2]
Double stars* [GHK, BahS]
Brooms+ [EFRS3]
Graphs with bridge versus K_n [Li1]
Multipartite complete graphs [BFRS, FRS3, Stev]
Multipartite complete graphs versus trees [EFRS8, BEFRSGJ]
Multipartite complete graphs versus sparse graphs [EFRS4]
Graphs with long tails [Bu2, BG]

5.13. Multiple copies of graphs, disconnected graphs

(a) $2K_2$ versus all isolate-free graphs [CH2]

(b) nK_2 versus mK_2, in particular $R(nK_2, nK_2) = 3n - 1$ for $n \ge 1$ [CocL1, CocL2, Lor]

(c) nK_3 versus mK_3, in particular $R(nK_3, nK_3) = 5n$ for $n \ge 2$ [BES], see also section 4.1

(d) nK_3 versus mK_4 [LorMu]

(e) $nK_{1,m}$ versus W_5 [BaHA]

(f) $R(nK_4, nK_4) = 7n + 4$ for large n [Bu8]

(g) Stripes mP_2 [CocL1, CocL2, Lor]

(h) $R(G, H)$ for all disconnected isolate-free graphs H on at most 6 vertices versus all G on at most 5 vertices, except 3 cases [LoM5]. Missing cases were completed in [KroMe].

(i) $R(F, G \cup H) \le \max\{ R(F, G) + n(H), R(F, G) \}$ [Par6]

(j) $R(mG, nH) \le (m-1)n(G) + (n-1)n(H) + R(G, H)$ [BES],
Formulas for $R(nK_3, mG)$ for all isolate-free graphs G on 4 vertices [Zeng],
Variety of results for numbers $R(nG, mH)$ [Bu1, BES, HaBA2, SuBAU1].

(k) Disjoint unions of paths (linear forests) [BuRo2, FS2]
Linear forests versus $3K_3 \cup 2K_4$ [SuBAU2]

(l) Forests versus K_n [Stahl, BaHA] and W_n [BaHA]. Generalizations to forests versus other graphs G in terms of $\chi(G)$ and the chromatic surplus of G [Biel4], and for linear forests versus $2K_n$ [SuAM].

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* double star is a union of two stars with their centers joined by an edge

+ broom is a star with a path attached to its center

(m) Disconnected graphs versus other graphs [BE1, GoJa1]

(n) See section 4.1 for cases involving unions of cycles

(o) See also [Bu9, BE1, LorMu, MiSa, Den2, Biel1, Biel2]

5.14. General results for special graphs

(a) $R(K_m^p, K_n^q) = R(K_m, K_n)$ for $m, n \geq 3$, $m + n \geq 8$, $p \leq m/(n-1)$ and $q \leq n/(m-1)$, where K_s^t is a K_s with additional vertex connected to it by t edges [BEFS]. Some applications can be found in [BlLR].

(b) $R(K_{2,k}, G) \leq kq + 1$, for $k \geq 2$, for isolate-free graphs G with $q \geq 2$ edges [RoJa2].

(c) $R(W_6, W_6) = 17$ and $\chi(W_6) = 4$ [FM]. This gives a counterexample $G = W_6$ to the Erdős conjecture (see [GRS]) $R(G, G) \geq R(K_{\chi(G)}, K_{\chi(G)})$, since $R(4,4) = 18$.

(d) $R(G + K_1, H) \leq R(K_{1, R(G,H)}, H)$ [BE1].

(e) $R(\overline{K}_2 + G, \overline{K}_2 + G) \leq 4R(G, \overline{K}_2 + G) - 2$ [LiShen].

(f) Study of $R(G + K_1, nH + K_1)$ [LinLD].

(g) $R(K_{p+1}, B_q^r) = p(q + r - 1) + 1$ for generalized books $B_q^r = K_r + qK_1$, for sufficiently large q [NiRo1].

(h) Study of the cases $R(K_m, K_n - K_{1,s})$ and $R(K_m - e, K_n - K_{1,s})$, with several exact values for special parameters [ChaMR].

(i) Study of $R(T + K_1, K_n)$ for trees T [LiR1]. Asymptotic upper bounds for $R(T + K_2, K_n)$ [Song7], see also [SonGQ].

(j) Bounds on $R(H + \overline{K}_n, K_n)$ for general H [LiR3]. Also, for fixed k and m, as $n \to \infty$, $R(K_k + \overline{K}_m, K_n) \leq (m + o(1))\, n^k / (\log n)^{k-1}$ [LiRZ1].

(k) Asymptotics of $R(H + \overline{K}_n, K_n)$. In particular, the order of magnitude of $R(K_{m,n}, K_n)$ is $n^{m+1}/(\log n)^m$ [LiTZ]. Upper asymptotics for $R(K_s + K_{m,n}, K_k)$ [Song9].

(l) Study of the largest k such that if the star $K_{1,k}$ is removed from K_r, $r = R(G, H)$, any edge 2-coloring of the remaining part still contains monochromatic G or H, as for K_r, for various special G and H [HoIs].

(m) Let G'' be a graph obtained from G by deleting two vertices with adjacent edges. Then $R(G, H) \leq A + B + 2 + 2\sqrt{(A^2 + AB + B^2)/3}$, where $A = R(G'', H)$ and $B = R(G, H'')$ [LiRZ2].

5.15. General results for sparse graphs

(a) $R(K_n, T_m) = (n-1)(m-1) + 1$ for any tree T_m on m vertices [Chv].

(b) Graphs yielding $R(K_n, G) = (n-1)(n(G) - 1) + 1$, called Ramsey n-good [BE3], and related results [EFRS5]. An extensive survey and further study of n-goodness appeared in [NiRo4].

(c) $R(C_{2m+1}, G) = 2n - 1$ for sufficiently large sparse graphs G on n vertices, little more complicated formulas for P_{2m+1} instead of C_{2m+1} [BEFRS2].

(d) $R(G,G) \le c_d n(G)$ for all G, where constant c_d depends only on the maximum degree d in G [CRST]. The constant was improved in [GRR1, FoxSu1]. Tight lower and upper bounds for bipartite G [GRR2, Con2]. Further improvements of the constant c_d in general were obtained in [ConFS4], and for graphs with bounded bandwidth in [AllBS].

(e) Study of L-sets, which are sets of pairs of graphs whose Ramsey numbers are linear in the number of vertices. Conjecture that Ramsey numbers grow linearly for d-degenerate graphs (graph is d-degenerate if all its subgraphs have minimum degree at most d) [BE1]. Progress towards this conjecture was obtained by several authors, including [KoRö1, KoRö2, KoSu, FoxSu1, FoxSu2].

(f) $R(G,G) \le c_d n$ for all d-arrangeable graphs G on n vertices, in particular with the same constant for all planar graphs [ChenS]. The constant c_d was improved in [Eaton]. An extension to graphs not containing a subdivision of K_d [RöTh].

(g) Conjecture that $R(G,G) \le 12n(G)$ for all planar G, for sufficiently large n [AllBS].

(h) Ramsey numbers grow linearly for degenerate graphs versus some sparser graphs, arrangeable graphs, crowns, graphs with bounded maximum degree, planar graphs, and graphs without any topological minor of a fixed clique [Shi3].

(i) Discussion of various old and new classes of Ramsey linear graphs [NeOs].

(j) Study of graphs G, called *Ramsey size linear,* for which there exists a constant c_G such that for all H with no isolates $R(G,H) \le c_G e(H)$ [EFRS9]. An overview and further results were given in [BaSS].

(k) $R(G,G) < 6n$ for all n-vertex graphs G, in which no two vertices of degree at least 3 are adjacent [LiRS]. This improves the result $R(G,G) \le 12n$ in [Alon1]. In an early paper by Burr and Erdős [BE1] it was proved that if any two points of degree at least 3 are at distance at least 3 then $R(G,G) \le 18n$.

(l) $R(G_{a,b}, G_{a,b}) = (3/2 + o(1))ab$, where $G_{a,b}$ is the rectangular $a \times b$ grid graph. Other similar results follow for bipartite planar graphs with bounded degree and grids of higher dimension [MoSST].

(m) $R(Q_n, Q_n) \le 2^{(3+\sqrt{5})n/2+o(n)}$, for the n-dimensional hypercube Q_n with 2^n vertices [Shi1]. This bound can also be derived from a theorem in [KoRö1]. An improvement was obtained in [Shi4], and a further one to $R(Q_n, Q_n) \le 2^{2n+5}n$ in [FoxSu1]. A lower bound construction for $12 \le R(Q_3, Q_3)$ was presented in [HaKr2].

(n) $R(K_m, Q_n) = (m-1)(2^n - 1) + 1$ for every fixed m and sufficiently large n [FizGMSS].

(o) Conjecture that $R(G,G) = 2n(G) - 1$ if G is unicyclic of odd girth [Gros1]. Further support for the conjecture was given in [Köh, KrRod].

(p) See also earlier subsections 5.* for various specific sparse graphs.

5.16. General results

[CH2] $R(G,H) \geq (\chi(G)-1)(c(H)-1)+1$, where $\chi(G)$ is the chromatic number of G, and $c(H)$ is the size of the largest connected component of H.

[CH3] $R(G,G) > (s\,2^{e(G)-1})^{1/n(G)}$, where s is the number of automorphisms of G. Hence $R(K_{n,n},K_{n,n}) > 2^n$, see also item 6.7.i.

[BE2] $R(G,G) \geq \lfloor (4n(G)-1)/3 \rfloor$ for any connected G, and $R(G,G) \geq 2n-1$ for any connected nonbipartite G. These bounds can be achieved for all $n \geq 4$.

[Bu2] Graphs H yielding $R(G,H) = (\chi(G)-1)(n(H)-1)+s(G)$, where $s(G)$ is a chromatic surplus of G, defined as the minimum number of vertices in some color class under all vertex colorings in $\chi(G)$ colors (such H's are called G-good). This idea, initiated in [Bu2], is a basis of a number of exact results for $R(G,H)$ for large and sparse graphs H [BG, BEFRS2, BEFRS3, Bu5, FS, EFRS4, FRS3, BEFSRGJ, BF, LiR4, Biel2, SuBAU3, Song6, AllBS]. Surveys of this area appeared in [FRS5, NiRo4].

[BaLS] Graph G is Ramsey saturated if $R(G+e,G+e) > R(G,G)$ for every edge e in $\overline{G}$. This paper contains several theorems involving cycles, cycles with chords and trees on Ramsey saturated and unsaturated graphs, and also seven conjectures including one stating that almost all graphs are Ramsey unsaturated. Some classes of graphs were proved to be Ramsey unsaturated [Ho]. Special cases involving cycles and Jahangir graphs were studied in [AliSur].

[Für] Relations between $R(3,k)$ and graphs with large $\chi(G)$. Further detailed study of the relation between $R(3,k)$ and the chromatic gap [GySeT].

[Bra3] $R(G,H) > h(G,d)\,n(H)$ for all nonbipartite G and almost every d-regular H, for some h unbounded in d.

[DoLL1] Lower asymptotics of $R(G,H)$ depending on the average degree of G and the size of H. This continues the study initiated in [EFRS5], later much enhanced for both lower and upper bounds in [Sud3].

[LiZa1] Lower bound asymptotics of $R(G,H)$ for large dense H.

[Erd4] A conjecture posed by Erdős in 1983 that there exists a constant c such that $R(G,G) \leq 2^{c\sqrt{e(G)}}$ for all isolate-free graphs G. Discussion of this conjecture and partial results, proof for bipartite graphs and progress in other cases are included in [AlKS]. In 2011, Sudakov [Sud4] completed the proof of this conjecture. An extension of the latter to some off-diagonal cases is presented in [MaOm], and an improvement of the constant for bipartite graphs is given in [JoPe]. For the multicolor case see item 6.7.i.

[Kriv] Lower bound on $R(G,K_n)$ depending on the density of subgraphs of G. This construction for $G = K_m$ produces a bound similar to the best known probabilistic lower bound by Spencer [Spe2]. Further lower and upper bounds on $R(G,K_n)$ in terms of n and $e(G)$ can be found in [Sud3].

[Con3] Upper bounds on $R(G, K_n)$ for dense graphs G.

[BE1] Relations between the cases of G or $G + K_1$ versus H or $H + K_1$.

[HaKr1] Study of cyclic graphs yielding lower bounds for Ramsey numbers. Exact formulas for paths and cycles, and values for small complete graphs and for graphs with up to five vertices.

[Par3] Relations between some Ramsey graphs and block designs. See also [Par4].

[Li2] Relations between the Shannon capacity of noisy communication channels and graph Ramsey numbers. See also section 6 in [Ros2], and [XuR3].

[Bu6] Given integer m and graphs G and H, determining whether $R(G, H) \leq m$ holds is NP−hard. Further complexity results related to Ramsey theory were presented in [Bu10].

[Scha] Ramsey arrowing is Π_2^p−complete, a rare natural example of a problem higher than NP in the polynomial hierarchy of computational complexity theory.

[-] Special cases of multicolor results listed in section 6.

[-] See also surveys listed in section 8.

6. Multicolor Ramsey Numbers

The only known value of a multicolor classical Ramsey number:

$R_3(3) = R(3,3,3) = R(3,3,3;2) = 17$	[GG]
2 critical colorings (on 16 vertices)	[KaSt, LayMa]
2 colorings on 15 vertices	[Hein]
115 colorings on 14 vertices	[PR1]

6.1. Bounds for classical numbers

General upper bound, implicit in [GG]:

$$R(k_1, \ldots, k_r) \le 2 - r + \sum_{i=1}^{r} R(k_1, \ldots, k_{i-1}, k_i - 1, k_{i+1}, \ldots, k_r) \qquad \text{(a)}$$

Inequality in (a) is strict if the right hand side is even and at least one of the terms in the summation is even. It is suspected that this upper bound is never tight for $r \ge 3$ and $k_i \ge 3$, except for $r = k_1 = k_2 = k_3 = 3$. However, only two cases are known to improve over (a), namely $R_4(3) \le 62$ [FKR] and $R(3,3,4) \le 31$ [PR1, PR2], for which (a) produces the bounds of 66 and 34, respectively.

Diagonal Cases

r \ m	3	4	5	6	7	8	9
3	17 GG	128 HiIr	417 Ex16	1070 Mat	3214 XuR1	6079 XSR2	13761 XXER
4	51 Chu1	634 XXER	3049 Xu	15202 XXER	62017 XXER		
5	162 Ex10	3416 XXER	26912 Xu				
6	538 FreSw						
7	1682 FreSw						

Table X. Known nontrivial lower bounds for diagonal multicolor Ramsey numbers $R_r(m)$, with references.

The best published bounds corresponding to the entries in Table X marked as personal communications [Ex16] and [Xu] are $415 \le R_3(5)$, $2721 \le R_4(5)$ and $26082 \le R_5(5)$ [XXER].

The most studied and intriguing open case is

$$\text{[Chu1]} \quad 51 \le R_4(3) = R(3,3,3,3) \le 62 \quad \text{[FKR]}$$

The construction for $51 \le R_4(3)$ as described in [Chu1] is correct, but be warned of a typo found by Christopher Frederick in 2003 (there is a triangle (31,7,28) in color 1 in the displayed matrix). The inequality 6.1.a implies $R_4(3) \le 66$, Folkman [Fol] in 1974 improved this bound to 65, and Sánchez-Flores [San] in 1995 proved $R_4(3) \le 64$.

The upper bounds in $162 \le R_5(3) \le 307$, $538 \le R_6(3) \le 1838$, $1682 \le R_7(3) \le 12861$, $128 \le R_3(4) \le 236$ and $634 \le R_4(4) \le 6474$ are implied by 6.1.a (we repeat lower bounds from Table X just to see easily the ranges). All the latter and other upper bounds obtainable from known smaller bounds and 6.1.a can be computed with the help of a LISP program written by Kerber and Rowat [KerRo].

Off-Diagonal Cases

Three colors:

k \ m	4	5	6	7	8	9	10	11	12	13	14
3	30 Ka2	45 Ex2	60 Rob3	85 Ex18	103 Ex18	129 Ex18	147 Ex18	162 Ex18	185 6.2.f	212 LSS2	233 6.2.f
4	55 KLR	89 Ex17	117 Ex17	145 Ex17	193 6.2.f	229 6.2.f					
5	89 Ex17	139 Ex17	181 Ex17	237 6.2.f							

Table XI. Known nontrivial lower bounds for 3-color Ramsey numbers of the form $R(3,k,m)$, with references.

In addition, the bounds $303 \le R(3,6,6)$, $609 \le R(3,7,7)$ and $1689 \le R(3,9,9)$ were derived in [XXER] (used there for building other lower bounds for some diagonal cases).

The other most studied, and perhaps the only open case of a classical multicolor Ramsey number, for which we can anticipate exact evaluation in the not-too-distance future is

$$\text{[Ka2]} \quad 30 \le R(3,3,4) \le 31 \quad \text{[PR1, PR2]}$$

In [PR1] it is conjectured that $R(3,3,4) = 30$, and the results in [PR2] eliminate some cases which could give $R(3,3,4) = 31$. The upper bounds in $45 \le R(3,3,5) \le 57$, $55 \le R(3,4,4) \le 79$, and $89 \le R(3,4,5) \le 160$ are implied by 6.1.a. We repeat lower bounds from Table XI to show explicitly the current ranges.

Four colors:

$97 \le R(3,3,3,4) \le 153$	[Ex17], 6.1.a
$171 \le R(3,3,4,4) \le 462$	[Ex15, XXER], 6.1.a
$381 \le R(3,4,4,4) \le 1619$	6.2.j, 6.1.a
$162 \le R(3,3,3,5)$	[XXER]
$565 \le R(3,3,3,11)$	6.2.f
$681 \le R(3,4,5,5)$	[XXER]

Lower bounds for higher numbers can be obtained by using general constructive results from section 6.2 below. For example, the bounds $261 \le R(3,3,15)$ and $247 \le R(3,3,3,7)$ were not published explicitly but are implied by 6.2.f and 6.2.g, respectively.

6.2. General results for complete graphs

(a) $R(k_1, \dots, k_r) \le 2 - r + \sum_{i=1}^{r} R(k_1, \dots, k_{i-1}, k_i - 1, k_{i+1}, \dots, k_r)$ [GG].

(b) $R_r(3) \ge 3R_{r-1}(3) + R_{r-3}(3) - 3$ [Chu1].

(c) $R_r(m) \ge c_m (2m-3)^r$, and some slight improvements of this bound for small values of m were described in [AbbH, Gi1, Gi2, Song2]. For $m = 3$, the best known lower bound is $R_r(3) \ge (3.199...)^r$ [XXER].

(d) $R_r(3) \le r!(e - e^{-1} + 3)/2 \approx 2.67r!$ [Wan], which improves the classical $3r!$ [GRS].

(e) The limit $L = \lim_{r \to \infty} R_r(3)^{1/r}$ exists, though it can be infinite [ChGri].
It is known that $3.199 < L$, as implied by (c) above. For more related results, mostly on the asymptotics of $R_r(3)$, see [AbbH, Fre, Chu2, GRS, GrRö].

(f) $R(3, k, l) \ge 4R(k, l-1) - 3$ for $k \ge 3$, $l \ge 5$, and in general for $r \ge 2$ and $k_i \ge 2$ it holds
$R(3, k_1, \dots, k_r) \ge 4R(k_1 - 1, k_2, \dots, k_r) - 3$ for $k_1 \ge 5$, and
$R(k_1, 2k_2 - 1, k_3, \dots, k_r) \ge 4R(k_1 - 1, k_2, \dots, k_r) - 3$ for $k_1 \ge 5$ [XX2, XXER].

(g) $R(3, 3, 3, k_1, \dots, k_r) \ge 3R(3, 3, k_1, \dots, k_r) + R(k_1, \dots, k_r) - 3$ [Rob2].

(h) For $r+1$ colors, avoiding K_3 in the first r colors and avoiding K_m in the last color, $R(3, \dots, 3, m) \le r!\, m^{r+1}$ [Sár].

(i) $R(k_1, \dots, k_r) \ge S(k_1, \dots, k_r) + 2$, where $S(k_1, \dots, k_r)$ is the generalized Schur number [AbbH, Gi1, Gi2]. In particular, the special case $k_1 = \dots = k_r = 3$ has been widely studied [Fre, FreSw, Ex10, Rob3].

(j) $R(k_1, \dots, k_r) \ge L(k_1, \dots, k_r) + 1$, where $L(k_1, \dots, k_r)$ is the maximal order of any cyclic $(k_1, \dots, k_r)$–coloring, which can be considered a special case of Schur partitions defining (symmetric) Schur numbers. Many lower bounds for Ramsey numbers were established by cyclic colorings. The following recurrence can be used to derive lower bounds for higher parameters. For $k_i \geq 3$ [Gi2],

$$L(k_1, \ldots, k_r, k_{r+1}) \geq (2k_{r+1} - 3)L(k_1, \ldots, k_r) - k_{r+1} + 2.$$

(k) $R_r(m) \geq p+1$ and $R_r(m+1) \geq r(p+1)+1$ if there exists a K_m–free cyclotomic r–class association scheme of order p [Mat].

(l) If the quadratic residues Paley graph Q_p of prime order $p = 4t+1$ contains no K_k, then $R(s, k+1, k+1) \geq 4ps - 6p + 3$ [XXER].

(m) $R_r(pq+1) > (R_r(p+1)-1)(R_r(q+1)-1)$ [Abb1]

(n) $R_r(pq+1) > R_r(p+1)(R_r(q+1)-1)$ for $p \geq q$ [XXER]

(o) $R(p_1q_1+1, \ldots, p_rq_r+1) > (R(p_1+1, \ldots, p_r+1)-1)(R(q_1+1, \ldots, q_r+1)-1)$ [Song3]

(p) $R_{r+s}(m) > (R_r(m)-1)(R_s(m)-1)$ [Song2]

(q) $R(k_1, k_2, \ldots, k_r) > (R(k_1, \ldots, k_i) - 1)(R(k_{i+1}, \ldots, k_r) - 1)$ in [Song1], see [XXER].

(r) $R(k_1, k_2, \ldots, k_r) > (k_1 + 1)(R(k_2 - k_1 + 1, k_3, \ldots, k_r) - 1)$ [Rob4]

(s) Further lower bound constructions, though with more complicated assumptions, were presented in [XX2, XXER].

(t) Grolmusz [Grol1] generalized the classical constructive lower bound by Frankl and Wilson [FraWi] (item 2.3.6) to more colors and to hypergraphs [Grol3] (item 7.4.k).

(u) Exact asymptotics of a very special but important case is known, namely $R(3,3,n) = \Theta(n^3 \text{poly–}\log n)$ [AlRö]. For general upper bounds and more asymptotics see in particular [Chu4, ChGra2, ChGri, GRS, GrRö].

All lower bounds in (b) through (t) above are constructive. Item (g) generalizes (b), (o) generalizes both (m) and (q), and (q) generalizes (p). (n) is stronger than (m). Finally, we note that the construction in (o) with $q_1 = \ldots = q_i = 1 = p_{i+1} = \ldots = p_r$ is the same as (q).

6.3. Cycles

Note: The paper *Ramsey Numbers Involving Cycles* [Ra4] is based on the revision #12 of this survey. It collects and comments on the results involving cycles versus any graphs, in two or more colors. It contains some more details than this survey, but only until 2009.

6.3.1. Three colors

(a) One long cycle.

The first larger paper in this area by Erdős, Faudree, Rousseau and Schelp [EFRS1] appeared in 1976. It gives several formulas and bounds for $R(C_m, C_n, C_k)$ and $R(C_m, C_n, C_k, C_l)$ for large m. For three colors [EFRS1] includes:

$R(C_m, C_{2p+1}, C_{2q+1}) = 4m - 3$ for $p \geq 2$, $q \geq 1$,
$R(C_m, C_{2p}, C_{2q+1}) = 2(m+p) - 3$ and
$R(C_m, C_{2p}, C_{2q}) = m + p + q - 2$ for p, $q \geq 1$ and large m.

m n k	$R(C_m, C_n, C_k)$	references	general results
3 3 3	**17**	GG	page 36
3 3 4	17	ExRe	
3 3 5	21	Sun1+/Tse3	$5k-4$ for $k \geq 5$, $m=n=3$ [Sun1+]
3 3 6	26	Sun1+	
3 3 7	31	Sun1+	
3 4 4	12	Schu	
3 4 5	13	Sun1+/Rao/Tse3	
3 4 6	13	Sun1+/Tse3	
3 4 7	15	Sun1+/Tse3	
3 5 5	≥17	Tse3	
3 5 6	21	Sun1+	
3 5 7	25	Sun1+	
3 6 6			
3 6 7	21	Sun1+	
3 7 7			
4 4 4	**11**	BiaS	
4 4 5	12	Sun2+/Tse3	
4 4 6	12	Sun2+/Tse3	$k+2$ for $k \geq 11$, $m=n=4$ [Sun2+]
4 4 7	12	Sun2+/Tse3	values for $k = 8, 9, 10$ are 12, 13, 13 [Sun2+]
4 5 5	13	Tse3	
4 5 6	13	Sun1+	
4 5 7	15	Sun1+	
4 6 6	11	Tse3	
4 6 7	13	Sun1+/Tse3	
4 7 7			
5 5 5	**17**	YR1	
5 5 6	21	Sun1+	
5 5 7	25	Sun1+	
5 6 6			
5 6 7	21	Sun1+	
5 7 7			
6 6 6	**12**	YR2	$R_3(C_{2q}) \geq 4q$ for $q \geq 2$ [DzNS]
6 6 7	15	Sun1+	see 6.3.1.a for larger parameters
6 7 7			see 6.3.1.a for larger parameters
7 7 7	**25**	FSS2	$R_3(C_{2q+1}) = 8q+1$ for large q [KoSS1, KoSS2]
8 8 8	**16**	Sun/SunY	$R_3(C_{2q}) = 4q$ for large q [BenSk]

Table XII. Ramsey numbers $R(C_m, C_n, C_k)$ for $m, n, k \leq 7$ and $m = n = k = 8$;
Sun1+ abbreviates SunYWLX, Sun2+ abbreviates SunYLZ2,
the work in [SunYWLX] and [SunYLZ2] is independent from [Tse3].

(b) Triple even cycles.

$R_3(C_{2m}) \geq 4m$ for all $m \geq 2$ [DzNS], see also 6.3.2.d/e/f. It was proven that $R(C_n, C_n, C_n) = (2+o(1))n$ for even n [FiŁu1, GyRSS], which was improved to exactly $2n$, for large n, by Benevides and Skokan [BenSk]. In 2005, Dzido [Dzi1] conjectured that $R_3(C_{2m}) = 4m$ for all $m \geq 3$. The first open case is for $R_3(C_{10})$, known to be at least 20. A more general result holds for slightly off-diagonal cases [FiŁu1]:

$$R(C_{2\lfloor\alpha_1 n\rfloor}, C_{2\lfloor\alpha_2 n\rfloor}, C_{2\lfloor\alpha_3 n\rfloor}) = (\alpha_1 + \alpha_2 + \alpha_3 + \max\{\alpha_1, \alpha_2, \alpha_3\} + o(1))\,n, \quad \textit{for all} \quad \alpha_1, \alpha_2, \alpha_3 > 0.$$

The conjectured equality $R_3(C_{2m}) = 4m$, whenever true, implies $R_3(P_{2m+1}) = 4m+1$ [DyDR] (see also section 6.4).

(c) Triple odd cycles.

Bondy and Erdős conjectured that $R(C_n, C_n, C_n) \le 4n-3$ for all $n \ge 4$ (see for example [Erd2]). If true, then for all odd $n \ge 5$ we have $R(C_n, C_n, C_n) = 4n-3$. The first open case is for $R_3(C_9)$, known to be at least 33. Erdős [Erd3] and other authors credit this conjecture to Bondy and Erdős, often pointing to a 1973 paper [BoEr]. Interestingly, however, the conjecture is not mentioned in this paper.

Łuczak proved that $R(C_n, C_n, C_n) \le (4+o(1))\,n$, with equality for odd n [Łuc]. The result $R_3(C_{2m+1}) = 8m+1$ for all sufficiently large m, or equivalently $R(C_n, C_n, C_n) = 4n-3$ for large odd n, was announced with an outline of the proof by Kohayakawa, Simonovits and Skokan [KoSS1], followed by the full proof in [KoSS2].

(d) $R(C_3, C_3, C_k) = 5k-4$ for $k \ge 5$ [SunYWLX], and $R(C_4, C_4, C_k) = k+2$ for $k \ge 11$ [SunYLZ2]. All exceptions to these formulas for small k are listed in Table XII.

(e) Asymptotics for triples of cycles of mixed parity similar in form to (b) [FiŁu2].

(f) Almost all of the off-diagonal cases in Table XII required the use of computers.

6.3.2. More colors

k \ m	3	4	5	6	7	8
3	17	11	17	12	25	16
4	51 62	18	33 158	18 20	49	20
5	162 307	27 29	65	26	97	28
6	538 1838	32 43	129		193	

Table XIII. Known values and bounds for $R_k(C_m)$ for small k, m;

(a) For the entries in the row $k = 3$ and in the column $m = 3$ in Table XIII, more details and all corresponding references are in sections 6.3.1 and 6.1, respectively. The lower bounds for $m = 5, 7$ are implied by 6.3.2.k, $R_k(C_m) \le 158$ follows from 6.3.2.j, and references to other cases with $k, m \ge 4$ can be found below in this section.

$R_4(C_4) = 18$	[Ex2] [SunYLZ1]
$18 \le R_4(C_6) \le 20$	[SunYJLS][ZhaSW]
$27 \le R_5(C_4) \le 29$	[LaWo1]
$R_5(C_6) = 26$	[SunYJLS] [SunYW]
$24 \le R(C_3, C_4, C_4, C_4) \le 27$	[DyDz1] [XuR2]
$30 \le R(C_3, C_3, C_4, C_4) \le 36$	[DyDz1] [XuR2]
$49 \le R(C_3, C_3, C_3, C_4)$	6.7.e
$18 \le R(C_4, C_6, C_6, C_6) \le 20$	[ZhaSW]
$18 \le R(C_4, C_4, C_6, C_6) \le 20$	[ZhaSW]
$R(C_4, C_4, C_4, C_6) = 19$	[ZhaSW]

(b) $R_k(C_4) \le k^2 + k + 1$ for all $k \ge 1$, $R_k(C_4) \ge k^2 - k + 2$ for all $k - 1$ which is a prime power [Ir, Chu2, ChGra1], and $R_k(C_4) \ge k^2 + 2$ for odd prime power k [LaWo1]. The latter was extended to any prime power k in [Ling, LaMu].

(c) Formulas for $R(C_m, C_n, C_k, C_l)$ for large m [EFRS1].

Bounds in (d) through (i) below cover different situations and each is best in some respect.

(d) $R_k(C_{2m}) \ge (k+1)m$ for odd k and $m \ge 2$, and
$R_k(C_{2m}) \ge (k+1)m - 1$ for even k and $m \ge 2$ [DzNS].

(e) $R_k(C_{2m}) \ge 2(k-1)(m-1) + 2$ [SunYXL].

(f) $R_k(C_{2m}) \ge k^2 + 2m - k$ for $2m \ge k+1$ and prime power k [SunYJLS].

(g) $R_k(C_{2m}) = \Theta(k^{m/(m-1)})$ for fixed $m = 2, 3$ and 5 [LiLih].

(h) $R_k(C_{2m}) \le 201km$ for $k \le 10^m/201m$ [EG].

(i) $R_k(C_{2m}) \le 2km + o(m)$ for all fixed $k \ge 2$ [ŁucSS].

(j) $R_k(C_5) < \sqrt{18^k k!}/10$ [Li4].

(k) $2^k m < R_k(C_{2m+1}) \le (k+2)!(2m+1)$ [BoEr].
Better upper bound $R_k(C_{2m+1}) < 2(k+2)!m$ was obtained in [EG].
Much better upper bound $R_k(C_{2m+1}) \le (c^k k!)^{1/m}$, for some positive constant c,
if all Ramsey-critical colorings for C_{2m+1} are not far from regular, was obtained in [Li4].

(l) Conjecture that $R_k(C_{2m+1}) = 2^k m + 1$ for all $m \ge 2$ was credited by several authors to Bondy and Erdős [BoEr], though only lower bound, not the conjecture, is in this paper.

(m) $R(C_n, C_{l_1}, \ldots, C_{l_k}) = 2^k(n-1) + 1$ for all l_i's odd with $l_i > 2^i$, and sufficiently large n, and support for the conjecture that $R_k(C_n) = 2^{k-1}(n-1) + 1$ for large odd n [AllBS].

(n) $R_k(C_{2m+1}) \le k2^k(2m+1) + o(m)$ for all fixed $k \ge 4$ [ŁucSS].

(o) Asymptotic bounds for $R_k(C_n)$ [Bu1, GRS, ChGra2, Li4, LiLih, ŁucSS].

(p) Survey of multicolor cycle cases [Li3].

6.3.3. Cycles versus other graphs

(a) Some cases involving C_4:

$20 \le R(C_4, C_4, K_4) \le 22$	[DyDz1] [XSR1]
$27 \le R(C_3, C_4, K_4) \le 32$	[DyDz1] [XSR1]
$52 \le R(C_4, K_4, K_4) \le 72$	[XSR1]
$34 \le R(C_4, C_4, C_4, K_4) \le 50$	[DyDz1] [XSR1]
$43 \le R(C_3, C_4, C_4, K_4) \le 76$	[DyDz1] [XSR1]
$87 \le R(C_4, C_4, K_4, K_4) \le 179$	[XSR1]
$R(K_{1,3}, C_4, K_4) = 16$	[KlaM2]
$R(C_4, C_4, K_4 - e) = 16$	[DyDz1]
$R(C_4, C_4, C_4, T) = 16$ for $T = P_4$ and $T = K_{1,3}$	[ExRe]

(b) Study of $R(C_n, K_{t_1}, \ldots, K_{t_k})$ and $R(C_n, K_{t_1, s_1}, \ldots, K_{t_k, s_k})$ for large n [EFRS1].

(c) $R(C_n, K_{t_1}, \ldots, K_{t_k}) = (n-1)(r-1)$ for $n \ge 4r+2$, where $r = R(K_{t_1}, \ldots, K_{t_k})$ [OmRa2].

(d) Study of asymptotics for $R(C_m, \ldots, C_m, K_n)$, in particular for any fixed number of colors $k \ge 4$ we have $R(C_4, C_4, \ldots, C_4, K_n) = \Theta(n^2/\log^2 n)$ [AlRö].

(e) Study of asymptotics for $R(C_{2m}, C_{2m}, K_n)$ for fixed m [AlRö, ShiuLL], in particular $R(C_4, C_4, K_n) = \Theta(n^2 \text{poly--}\log n)$ [AlRö].

(f) Monotone paths and cycles [Lef].

(g) For combinations of C_3 and K_n see sections 2.2, 3.2, 4.2, 6.1 and 6.2.

6.4. Paths, paths versus other graphs

In 2007, Gyárfás, Ruszinkó, Sárközy and Szemerédi [GyRSS] established that for all n large enough we have

$$R(P_n, P_n, P_n) = 2n - 2 + (n \bmod 2).$$

Faudree and Schelp [FS2] conjectured that the latter holds for all $n \ge 1$. It is true for $n \le 9$ (see (c) below), and the first open case is that for P_{10}. The conjectured equality $R(C_{2m}, C_{2m}, C_{2m}) = 4m$ (see 6.3.1.a), whenever true, implies the above for three paths P_{2m+1} [DyDR].

6.4.1. Three color path and path-cycle cases

(a) $R(P_m, P_n, P_k) = m + \lfloor n/2 \rfloor + \lfloor k/2 \rfloor - 2$ for $m \ge 6(n+k)^2$ [FS2],
the equality holds asymptotically for $m \ge n \ge k$ with an extra term $o(m)$ [FiŁu1],
extensions of the range of m, n, k for which (a) holds were obtained in [Biel3].

(b) $R(P_3, P_m, P_n) = m + \lfloor n/2 \rfloor - 1$ for $m \ge n$ and $(m, n) \ne (3,3), (4,3)$ [MaORS2].

(c) $R_3(P_3) = 5$ [Ea1], $R_3(P_4) = 6$ [Ir],
$R(P_m, P_n, P_k) = 5$ for other m–n–k combinations with $3 \le m, n, k \le 4$ [ArKM],
$R_3(P_5) = 9$ [YR1], $R_3(P_6) = 10$ [YR1], and $R_3(P_7) = 13$ [YY],
$R_3(P_8) = 14$, $R_3(P_9) = 17$ [DyDR].

(d) $R(P_4, P_4, P_{2n}) = 2n + 2$ for $n \ge 2$,
$R(P_5, P_5, P_5) = R(P_5, P_5, P_6) = 9$,
$R(P_5, P_5, P_n) = n + 2$ for $n \ge 7$,
$R(P_5, P_6, P_n) = R(P_4, P_6, P_n) = n + 3$ for $n \ge 6$,
$R(P_6, P_6, P_{2n}) = R(P_4, P_8, P_{2n}) = 2n + 4$ for $n \ge 14$ [OmRa1].

(e) $R(P_m, P_n, C_k) = 2n + 2\lfloor m/2 \rfloor - 3$ for large n and odd $m \ge 3$ [DzFi2],
improvements on the range of m, n, k [Biel3, Fid1].

(f) $R(P_3, P_3, C_m) = 5, 6, 6$, for $m = 3, 4$ [ArKM], 5,
$R(P_3, P_3, C_m) = m$ for $m \ge 6$ [Dzi2].
$R(P_3, P_4, C_m) = 7$ for $m = 3, 4$ [ArKM] and 5,
$R(P_3, P_4, C_m) = m + 1$ for $m \ge 6$ [Dzi2].
$R(P_4, P_4, C_m) = 9, 7, 9$ for $m = 3, 4$ [ArKM] and 5 [Dzi2],
$R(P_4, P_4, C_m) = m + 2$ for $m \ge 6$ [DzKP].

(g) $R(P_3, P_5, C_m) = 9, 7, 9, 7, 9$ for $m = 3, 4, 5, 6, 7$ [Dzi2, DzFi2],
$R(P_3, P_5, C_m) = m + 1$ for $m \ge 8$ [DzKP].
A table of $R(P_3, P_k, C_m)$ for all $3 \le k \le 8$ and $3 \le m \le 9$ [DzFi2].

(h) $R(P_4, P_5, C_m) = 11, 7, 11, 11$, and $m + 2$ for $m = 3, 4, 5, 7$ and $m \ge 23$,
$R(P_4, P_6, C_m) = 13, 8, 13, 13$, and $m + 3$ for $m = 3, 4, 5, 7$ and $m \ge 18$ [ShaXSP].

(i) $R(P_3, P_n, C_4) = n + 1$ for $n \ge 6$ [DzFi2],
$R(P_3, P_n, C_6) = n + 2$ for $n \ge 6$,
$R(P_3, P_n, C_8) = n + 3$ for $n \ge 7$ [Fid1],
$R(P_3, P_n, C_k) = 2n - 1$, and
$R(P_4, P_n, C_k) = 2n + 1$ for odd $k \ge 3$ and $n \ge k$ [DzFi2].

(j) $R(P_3, P_6, C_m) = m + 2$ for $m \ge 23$,
$R(P_6, P_6, C_m) = R(P_4, P_8, C_m) = m + 4$ for $m \ge 27$,
$R(P_6, P_7, C_m) = m + 4$ for $m \ge 57$,
$R(P_4, P_n, C_4) = R(P_5, P_n, C_4) = n + 2$ for $n \ge 5$ [OmRa1].

(k) $R(P_3, C_3, C_3) = 11$ [BE3], $R(P_3, C_4, C_4) = 8$ [ArKM], $R(P_3, C_6, C_6) = 9$ [Dzi2],
$R(P_3, C_m, C_m) = R(C_m, C_m) = 2m - 1$ for odd $m \ge 5$ [DzKP] (for $m = 5, 7$ [Dzi2]),

(l) $R(P_3, C_n, C_m) = R(C_n, C_m)$ for $n \ge 7$ and odd m, $5 \le m \le n$, and
some values and bounds on $R(P_3, C_n, C_m)$ in other cases [Fid1].

(m) $R(P_3, C_3, C_4) = 8$ [ArKM], $R(P_3, C_3, C_5) = 9$, $R(P_3, C_3, C_6) = 11$,
$R(P_3, C_3, C_7) = 13$, $R(P_3, C_4, C_5) = 8$, $R(P_3, C_4, C_6) = 8$,

$R(P_3, C_4, C_7) = 8$, $R(P_3, C_5, C_6) = 11$, $R(P_3, C_5, C_7) = 13$ and $R(P_3, C_6, C_7) = 11$ [Dzi2].

(n) Formulas for $R(pP_3, qP_3, rP_3)$ and $R(pP_4, qP_4, rP_4)$ [Scob].

(o) $R(P_3, K_4-e, K_4-e) = 11$ [Ex7]. All colorings which can form any color neighborhood for the open case $R_3(K_4-e)$ (see section 6.5) were found in [Piw2].

6.4.2. More colors

(a) $R_k(P_3) = k+1+(k \bmod 2)$, $R_k(2P_2) = k+3$ for all $k \geq 1$ [Ir].

(b) $R_k(P_4) = 2k + c_k$ for all k and some $0 \leq c_k \leq 2$. If k is not divisible by 3 then $c_k = 3 - k \bmod 3$ [Ir]. Wallis [Wall] showed $R_6(P_4) = 13$, which already implied $R_{3t}(P_4) = 6t+1$, for all $t \geq 2$. Independently, the case $R_k(P_4)$ for $k \neq 3^m$ was completed by Lindström in [Lind], and later Bierbrauer proved $R_{3^m}(P_4) = 2(3^m)+1$ for all $m > 1$. $R_3(P_4) = 6$ [Ir].

(c) Formula for $R(P_{n_1}, \ldots, P_{n_k})$ for large n_1 [FS2], and some extensions [Biel3]. Conjectures about $R(P_{n_1}, \ldots, P_{n_k})$ when all or all but one of n_i's are even [OmRa1].

(d) Formulas for $R(P_{n_1}, \ldots, P_{n_k}, C_m)$ for some cases, for large m [OmRa1].

(e) Formula for $R(n_1P_2, \ldots, n_kP_2)$, in particular $R(nP_2, nP_2, nP_2) = 4n-2$ [CocL1]. Note how close the latter is to $R(C_{2n}, C_{2n}, C_{2n}) = 4n$, see an earlier item 6.3.1b.

(f) Cockayne and Lorimer [CocL1] found the exact formula for $R(n_1P_2, \ldots, n_kP_2)$, and later Lorimer [Lor] extended it to a more general case of $R(K_m, n_1P_2, \ldots, n_kP_2)$. More general cases of the latter, with multiple copies of the complete graph, paths, stars and forests, were studied in [Stahl, LorSe, LorSo, GyRSS]. A special 3-color case $R(P_3, mP_2, nP_2) = 2m+n-1$ for $m \geq n \geq 3$ is given in [MaORS2].

(g) Multicolor cases for one large path or cycle involving small paths, cycles, complete and complete bipartite graphs [EFRS1].

(h) See sections 6.5 and 8.2, especially [ArKM, BoDD], for a number of cases for triples of small graphs.

6.5. Special cases

$R_3(K_3+e) = R_3(K_3)$ [$=17$]	[YR3, ArKM], where $K_3+e = K_4 - P_3$
$R(K_3+e, K_3+e, K_4-e) = 17$	[ShWR]
If $R_4(K_3) = 51$ then $R_4(K_3+e) = 52$, and if $R_4(K_3) > 51$ then $R_4(K_3+e) = R_4(K_3)$	[ShWR]

$28 \le R_3(K_4 - e) \le 30$	[Ex7] [Piw2]
$R(P_3, K_4 - e, K_4 - e) = 11$	[Ex7], all colorings [Piw2]
$472 \le R_3(K_6 - e)$	[HeDL]
$1102 \le R_3(K_7 - e)$	[HeDL]
$21 \le R(K_3, K_4 - e, K_4 - e) \le 27$	[ShWR]
$33 \le R(K_4, K_4 - e, K_4 - e) \le 60$	[ShWR]
$55 \le R(K_4, K_4, K_4 - e) \le 113$	[Ea1][BoDD]
$R(C_4, P_4, K_4 - e) = 11$	[ArKM]
$R(C_4, P_4, K_4) = 14$	[BoDD]
$R(C_4, C_4, K_4 - e) = 16$	[DyDz1]
$R(C_4, K_3, K_4 - e) = 17$	[BoDD]
$R(C_4, K_4 - e, K_4 - e) = 19$	[BoDD]
$28 \le R(C_4, K_4, K_4 - e) \le 36$	[BoDD]
$30 \le R(K_3, K_4, K_4 - e) \le 41$	[Ea1][BoDD]
$33 \le R(K_4, K_4 - e, K_4 - e) \le 59$	[Ea1][BoDD]

See also section 8.2 for pointers to cumulative data for three colors.

6.6. General results for special graphs

(a) Formulas for $R_k(G)$, where G is one of the graphs P_3, $2K_2$ and $K_{1,3}$, for all k, and for P_4 if k is not divisible by 3 [Ir]. For some details see section 6.4.2b.

(b) $tk^2 + 1 \le R_k(K_{2,t+1}) \le tk^2 + k + 2$, where the upper bound is general, and the lower bound holds when both t and k are prime powers [ChGra1, LaMu].

(c) $(m-1)\lfloor (k+1)/2 \rfloor < R_k(T_m) \le 2km + 1$ for any tree T_m with m edges [EG], see also [GRS]. The lower bound can be improved for special large k [EG, GRS]. The upper bound was improved to $R_k(T_m) < (m-1)(k + \sqrt{k(k-1)}) + 2$ in [GyTu].

(d) $k(\sqrt{m} - 1)/2 < R_k(F_m) < 4km$ for any forest F_m with m edges [EG], see [GRS]. See also pointers in items (p) and (r) below.

(e) $R(S_1, \ldots, S_k) = n + \varepsilon$, where S_i's are arbitrary stars, $n = n(S_1) + \ldots + n(S_k) - 2k$, and we set $\varepsilon = 1$ if n is even and some $n(S_i)$ is odd, and $\varepsilon = 2$ otherwise [BuRo1]. See also [GauST, Par6].

(f) Formula for $R(S_1, \ldots, S_k, K_n)$, where S_i's are arbitrary stars [Jac]. It was generalized to a formula for $R(S_1, \ldots, S_k, K_{k_1}, \ldots, K_{k_r})$ expressed in terms of $R(k_1, \ldots, k_r)$ and star orders [BoCGR]. A much shorter proof of the latter was presented in [OmRa2].

(g) Formula for $R(S_1, \ldots, S_k, nK_2)$, where S_i's are arbitrary stars [CocL2], and a formula for $R(n_1K_2, \ldots, n_kK_2)$ [CocL1]. See also cases involving P_2 in section 6.4.2.

(h) Formula for $R(S_1, \ldots, S_k, T)$, where S_i's are stars and T is a tree [ZZ1].

(i) Formulas for $R(S_1, \ldots, S_k)$, where each S_i's is a star or $m_i K_2$ [ZZ2, EG], formula for the case $R(S, mK_2, nK_2)$ [GySá2].

(j) Bounds on $R_k(G)$ for unicyclic graphs G of odd girth.
Some exact values for special graphs G, for $k = 3$ and $k = 4$ [KrRod].

(k) For prime $p = 3q + 1$, if the cubic residues Paley graph Q_p contains no $K_k - e$, then $R_3(K_{k+1} - e) > 3p$ [HeDL]. The cases $k = 5, 6$ give two bounds listed in section 6.5.

(l) $R_k(K_{3,3}) = (1 + o(1))\, k^3$ [AlRóS].

(m) Bounds on $R_k(K_{s,t})$, in particular for $K_{2,2} = C_4$ and $K_{2,t}$ [ChGra1, AxFM]. Asymptotics of $R_k(K_{s,t})$ for fixed k and s [DoLi, LiTZ]. Upper bounds on $R_k(K_{s,t})$ [SunLi].

(n) Exact asymptotics $R(K_{t,s}, K_{t,s}, K_{t,s}, K_m) = \Theta(m^t / \log^t m)$, for any fixed $t > 1$ and large $s \geq (t-1)! + 1$ [AlRö].

(o) Bounds on $R_k(G)$ for trees, forests, stars and cycles [Bu1].

(p) Bounds for trees $R_k(T)$ and forests $R_k(F)$ [EG, GRS, BierB, GyTu, Bra1, Bra2, SwPr].

(q) $R_3(G_{a,b}) = (2 + o(1))ab$, where $G_{a,b}$ is the rectangular $a \times b$ grid graph. Lower and upper bounds on $R_3(G)$ for graphs G with small bandwidth and bounded $\Delta(G)$ [MoSST].

(r) Study of the case $R(K_m, n_1 P_2, \ldots, n_k P_2)$ [Lor]. Other similar results include $R(P_3, mK_2, nK_2) = 2m + n - 1$ for $m \geq n \geq 3$ [MaORS2] and $R(S_n, nK_2, nK_2) = 3n - 1$ [GySá2]. More general cases, with multiple copies of the complete graph, stars and forests, were investigated in [Stahl, LorSe, LorSo, GyRSS]. See also section 6.4.2.

(s) See section 8.2, especially [ArKM, BoDD], for a number of cases for other small graphs, similar to those listed in sections 6.3 and 6.4.

6.7. General results

(a) Szemerédi's Regularity Lemma [Szem] states that the vertices of every large graph can be partitioned into similar size parts so that the edges between these parts behave almost randomly. This lemma has been used extensively in various forms to prove the upper bounds, including those studied in [BenSk, GyRSS, GySS1, HaŁP1+, HaŁP2+, KoSS1, KoSS2].

(b) $R(m_1 G_1, \ldots, m_k G_k) \leq R(G_1, \ldots, G_k) + \sum_{i=1}^{k} n(G_i)(m_i - 1)$, exercise 8.3.28 in [West].

(c) If G is connected and $R(K_k, G) = (k-1)(n(G) - 1) + 1$, in particular if G is any n-vertex tree, then $R(K_{k_1}, \ldots, K_{k_r}, G) = (R(k_1, \ldots, k_r) - 1)(n - 1) + 1$ [BE3]. A generalization for connected $G_1, \ldots, G_n$ in place of G appeared in [Jac].

(d) If F, G, H are connected graphs then $R(F, G, H) \geq (R(F, G) - 1)(\chi(H) - 1) + \min\{R(F, G), s(H)\}$, where $s(G)$ is the chromatic surplus of G (see item [Bu2] in section 5.16). This leads to several formulas and bounds for F and G being stars and/or trees when $H = K_n$ [ShiuLL].

(e) $R(K_{k_1}, \dots, K_{k_r}, G_1, \dots, G_s) \geq (R(k_1, \dots, k_r) - 1)(R(G_1, \dots, G_s) - 1) + 1$ for arbitrary graphs $G_1, \dots, G_s$ [Bev]. This generalizes 6.2.q.

(f) Constructive bound $R(G_1, \dots, G_{t^{n-1}}) \geq t^n + 1$ for decompositions of K_{t^n} [LaWo1, LaWo2].

(g) $R(G_1, \dots, G_k) \leq 32 \Delta k^{\Delta} n$, where $n \geq n(G_i)$ and $\Delta \geq \Delta(G_i)$ for all $1 \leq i \leq k$,
$R(G_1, \dots, G_k) \leq k^{2k\Delta q} n$, where $q \geq \chi(G_i)$ for all $1 \leq i \leq k$ [FoxSu1].

(h) $R_k(G) \leq k^{6e(G)^{2/3} k}$ for all isolate-free graphs G and $k \geq 3$ [JoPe].
For the original two-color conjecture, now a theorem, see item [Erd4] in section 5.16.

(i) $R_k(G) > (sk^{e(G)-1})^{1/n(G)}$, where s is the number of automorphisms of G [CH3]. Other general bounds for $R_k(G)$ [CH3, Par6].

(j) Study of $R(G_1, \dots, G_k, G)$ for large sparse G [EFRS1, Bu3].

(k) Study of asymptotics for $R(C_n, \dots, C_n, K_m)$ [AlRö]. See also sections 6.3.3.d/e.

(l) Relations between the Shannon capacity of noisy communication channels and graph Ramsey numbers. A lower bound construction for $R_k(m)$ implying that supremum of the Shannon capacity over all graphs with bounded independence cannot be achieved by any finite graph power [XuR3]. For some other links between Shannon capacity and Ramsey numbers see section 6 in [Ros2], and [Li2].

(m) See surveys listed in section 8.

7. Hypergraph Numbers

7.1. Values and bounds for numbers

The only known value of a classical Ramsey number for hypergraphs:

$R(4,4;3) = 13$,
more than 200000 critical colorings [MR1]

The computer evaluation of $R(4,4;3)$ in 1991 consisted of an improvement of the upper bound from 15 to 13. This result followed an extensive theoretical study of this number by several authors [Gi4, Isb1, Sid1].

(a)	$33 \le R(4,5;3)$	[Ex13]
	$58 \le R(4,6;3)$	[Ex18]
	$82 \le R(5,5;3)$	[Ex18]
	$56 \le R(4,4,4;3)$	[Ex8]
	$34 \le R(5,5;4)$	[Ex11]
(b)	$R(K_4 - t, K_4 - t;3) = 7$	[Ea2]
	$R(K_4 - t, K_4;3) = 8$	[Sob, Ex1, MR1]
	$14 \le R(K_4 - t, K_5;3)$	[Ex1]
	$13 \le R(K_4 - t, K_4 - t, K_4 - t;3) \le 16$	[Ex1] [Ea3]

(c) The first bound on $R(4,5;3) \ge 24$ was obtained by Isbell [Isb2]. Shastri [Shas] gave a weak bound $R(5,5;4) \ge 19$ (now 34 in [Ex11]), nevertheless his lemmas, the stepping-up lemmas by Erdős and Hajnal (see [GRS, GrRö], also 7.4.a below), and others in [Ka3, Abb2, GRS, GrRö, HuSo, SonYL] can be used to derive better lower bounds for higher numbers.

(d) Several lower bound constructions for 3-uniform hypergraphs were presented in [HuSo]. Study of lower bounds on $R(p,q;4)$ can be found in [Song3] and [SonYL, Song4] (the latter two papers are almost the same in contents). Most of the concrete lower bounds in these papers can be easily improved by using the same techniques, but starting with better constructions for small parameters as listed above.

(e) $R(p,q;4) \ge 2R(p-1,q;4) - 1$ for $p, q > 4$, and
$R(p,q;4) \ge (p-1)R(p-1,q;4) - p + 2$ for $p \ge 5$, $q \ge 7$ [SonYL].
Lower bound asymptotics for $R(p,q;4)$ [SonLi].

(f) $R(K_{1,1,c}, K_{1,1,c};3) = c + 2$ for $2 \le c \le 4$, and
a conjecture that this equality also holds for all $c \ge 5$ [MiPal].

7.2. Cycles and paths

Definitions. $P_n^{r,s}$ is called an s-*path* in an r-uniform hypergraph H, if it consists of n hyperedges $\{e_1, ..., e_n\}$ in $E(H)$, such that $|e_i \cap e_{i+1}| = s$ for all $1 \le i < n$, and all other vertices in e_j's are distinct [Peng]. An s-*cycle* $C_n^{r,s}$ is defined analogously. Several authors use the terms of *loose* paths and *loose* cycles, which are 1-path and 1-cycles, and *tight* paths and *tight* cycles, the latter most often for 3-uniform hypergraphs when they are 2-paths and 2-cycles, respectively. A 3-uniform *Berge* cycle is formed by n distinct vertices, such that all consecutive pairs of vertices are in an edge of the cycle, and all of the cycle edges are distinct. Berge cycles are not determined uniquely.

In the following items (b) to (i), when $r = 3$ or r is implied by the context, we write C_n and P_n for the r-uniform loose cycles and paths, $C_n^{r,1}$ and $P_n^{r,1}$, respectively. In other cases special comments are added.

(a) Tetrahedron is formed by four triples on the set of four points. The Ramsey number of tetrahedron is $R(4,4;3) = 13$ [MR1].

(b) For loose cycles and paths, $R(C_3, C_3; 3) = 7$, $R(C_4, C_4; 3) = 9$, and for the r-uniform case we have in general $R(P_3, P_3; r) = R(P_3, C_3; r) = R(C_3, C_3; r) + 1 = 3r - 1$ and $R(P_4, P_4; r) = R(P_4, C_4; r) = R(C_4, C_4; r) + 1 = 4r - 2$, for $r \geq 3$. These results and discussion of several related cases were presented in [GyRa].

(c) $R(P_m, P_n; 3) = R(C_m, C_n; 3) + 1 = R(P_m, C_n; 3) = 2m + \lfloor (n+1)/2 \rfloor$, for all $m \geq n$, and $R(C_m, P_n; 3) = 2m + \lfloor (n-1)/2 \rfloor$, for $m > n$ [MaORS1, OmSh].

(d) For loose cycles, $R(C_{2n}, C_{2n}; 3) > 5n - 2$ and $R(C_{2n+1}, C_{2n+1}; 3) > 5n + 1$, and asymptotically these lower bounds are tight [HaŁP1+]. Generalizations to r-uniform hypergraphs and graphs other than cycles appeared in [GySS1].

(e) For tight cycles, $R(C_{3n}, C_{3n}; 3) \approx 4n$ and $R(C_{3n+i}, C_{3n+i}; 3) \approx 6n$ for $i = 1$ or 2, and for tight paths $R(P_n, P_n; 3) \approx 4n/3$ [HaŁP2+]. Some related results are discussed in [PoRRS].

(f) Exact values for Ramsey numbers involving s-paths for even r and $s = r/2$, in particular for $P_n^{r,s}$ versus $P_3^{r,s}$ and $P_4^{r,s}$, when this value is $(n+1)s + 1$ [Peng].

(g) For 3-uniform Berge cycles and two colors, $R(C_n, C_n; 3) = n$ for $n \geq 5$ [GyLSS].

(h) For loose cycles, $R(C_3, C_3, C_3; 3) = 8$, and in general for $k \geq 4$ colors Gyárfás and Raeisi established the bounds $k + 5 \leq R_k(C_3; 3) \leq 3k$ [GyRa].

(i) For 3-uniform Berge cycles, $R_3(C_n; 3) = (1 + o(1))5n/4$ [GySá1].

(j) Lower and upper asymptotic bounds for $R(C_3^{3,1}, K_m; 3)$ and $R(C_3^{r,1}, K_m; r)$ [KosMV2].

(k) Gyárfás, Sárközy and Szemerédi proved that, for sufficiently large n, every 2-coloring of the edges of the complete 4-uniform hypergraph K_n contains a monochromatic 3-tight Berge cycle C_n [GySS2]. Special multicolor cases for r-uniform hypergraphs were studied in [GyLSS].

(l) Study of $R(G, nH; r)$ and $R(mG, nH; r)$ for loose/tight path, cycles and stars, including several exact results for large m or n [OmRa3].

7.3. General results for 3-uniform hypergraphs

(a) $2^{cn^2} < R(n, n; 3) < 2^{2^n}$ is credited to Erdős, Hajnal and Rado (see [ChGra2] p. 30).

(b) For some a, b the numbers $R(m, a, b; 3)$ are at least exponential in m [AbbS].

(c) Improved lower and upper asymptotics for $R(s, n; 3)$ for fixed s and large n, proof of related Erdős and Hajnal conjecture on the growth of $R(4, n; 3)$, and the lower bound $2^{n^{c \ln n}} < R(n, n, n; 3)$ [ConFS2].

(d) $R(G, G; 3) \leq cn(H)$ for some constant c depending only on the maximum degree of a 3-uniform hypergraph H [CooFKO1, NaORS]. Similar results were proved for r-uniform hypergraphs in [KüCFO, Ishi, CooFKO2, ConFS1], see also item 7.4.g.

(e) Asymptotic lower bounds for $R(K_{a,b,c}, K_{a,b,c}; 3)$, where $K_{a,b,c}$ is formed by all abc triples on sets of orders a, b, c [MiPal].

(f) If G is a 3-uniform H-free hypergraph, then G contains a complete or empty tripartite subgraph with parts of order $(\log n(H))^{c+1/2}$, where $c > 0$ depends only on H. Furthermore, for $k \geq 4$ no analogue of it can hold for k-uniform hypergraphs [ConFS5].

(g) Asymptotic or exact values of $R_k(H; 3)$ when H is a bow $\{abc, ade\}$, kite $\{abc, abd\}$, tight path $P_3^{3,2} = \{abc, bcd, cde\}$, or windmill $\{abc, bde, cef, bce\}$, and, among others, a special case $R_6(kite; 3) = 8$ [AxGLM].

(h) $R_k(K_3) \leq R_{4k}(K_4 - t; 3) \leq R_{4k}(K_3) + 1$ [AxGLM].

(i) Upper bounds on $R_k(H; 3)$ for complete multipartite 3-uniform hypergraphs H, a 4-color case, and some other general and special cases [ConFS1, ConFS2, ConFS3]. $R_k(H; 3)$ ranges from $\sqrt{6k}(1 + o(1))$ to double exponential in k [AxGLM].

7.4. General results

(a) If $R(n, n; r) > m$ then $R(2n + r - 4, 2n + r - 4; r + 1) > 2^m$, for $n > r \geq 3$ (see [GRS] p. 106). This is the so-called stepping-up lemma, usually credited to Erdős and Hajnal. An improvement of the stepping-up lemma implying better lower bounds for a few types of hypergraph Ramsey numbers were obtained by Conlon, Fox and Sudakov [ConFS6].

(b) Lower bounds on $R_k(n; r)$ are discussed in [AbbW, DLR].

(c) General lower bounds for large number of colors were given in an early paper by Hirschfeld [Hir], and some of them were later improved in [AbbL].

(d) Lower and upper asymptotics of $R(s, n; k)$ for fixed s [ConFS2].

(e) Exact and asymptotic results generalizing 7.2.d to r-uniform case for cycles, and 2- and 3-color cases for all r-uniform diamond matchings [GySS1].

(f) Study of $R(G, nH; r)$ and $R(mG, nH; r)$ for loose/tight path and cycles (possibly with some additions), stars, r-partite hypergraphs, including several exact results for large m or n [OmRa3].

(g) $R(H, H; r) \leq cn(H)^{1+\varepsilon}$, for some constant $c = c(\Delta, r, \varepsilon)$ depending only on the maximum degree of H, r and $\varepsilon > 0$ [KoRö3]. The proofs of the linear bound $cn(H)$ were obtained independently in [KüCFO] and [Ishi], the latter including the multicolor case, and then without regularity lemma in [ConFS1]. More discussion of lower and upper bounds for various cases can be found in [ConFS1, ConFS2, ConFS3, CooFKO2].

(h) Let T_r be an r-uniform hypergraph with r edges containing a fixed $(r-1)$-vertex set S and the $(r+1)$-st edge intersecting all former edges in one vertex outside S. Then $R(T_r, K_t; r) = O(t^r/\log t)$ [KosMV1].

(i) Let $H^r(s,t)$ be the complete r-partite r-uniform hypergraph with $r-2$ parts of size 1, one part of size s, and one part of size t (for example, for $r=2$ it is the same as $K_{s,t}$). For the multicolor numbers, Lazebnik and Mubayi [LaMu] proved that

$$tk^2 - k + 1 \le R_k(H^r(2,t+1); r) \le tk^2 + k + r,$$

where the lower bound holds when both t and k are prime powers. For the general case of $H^r(s,t)$, more bounds are presented in [LaMu].

(j) $R_k(H; r)$ is polynomial in k when a fixed r-uniform H is r-partite, otherwise it is at least exponential in k [AxGLM].

(k) Grolmusz [Grol1] generalized the classical constructive lower bound by Frankl and Wilson [FraWi] (item 2.3.6) to more colors and to hypergraphs [Grol3].

(l) Lower and upper asymptotics, and other theoretical results on hypergraph numbers, are gathered in [GrRö, GRS, ConFS1, ConFS2, ConFS3, Song8].

8. Cumulative Data and Surveys

8.1. Cumulative data for two colors

[CH1] $R(G,G)$ for all graphs G without isolates on at most 4 vertices.

[CH2] $R(G,H)$ for all graphs G and H without isolates on at most 4 vertices.

[Clan] $R(G,H)$ for all graphs G on at most 4 vertices and H on 5 vertices, except five entries (now all solved, see section 5.11). All critical colorings for the isolate-free graphs G and H studied in [Clan] were found in [He4].

[Bu4] $R(G,G)$ for all graphs G without isolates and with at most 6 edges.

[He1] $R(G,G)$ for all graphs G without isolates and with at most 7 edges.

[HaMe2] $R(G,G)$ for all graphs G on 5 vertices and with 7 or 8 edges.

[He2] $R(G,H)$ for all graphs G and H on 5 vertices without isolates, except 7 entries (2 still open, see 5.11 and the paragraph at the end of this section).

[LoM5] $R(G,H)$ for all disconnected isolate-free graphs H on at most 6 vertices versus all G on at most 5 vertices, except 3 cases. Missing cases were completed in [KroMe].

[HoMe] $R(G,H)$ for $G = K_{1,3}+e$ and $G = K_4 - e$ versus all connected graphs H on 6 vertices, except $R(K_4 - e, K_6)$. The result $R(K_4 - e, K_6) = 21$ was claimed by McNamara [McN, unpublished], now confirmed in [ShWR].

[Boza4] $R(G,H)$ for some graphs G with 4 vertices versus all graphs H with 7 vertices.

[FRS4] $R(G,T)$ for all connected graphs G with $n(G) \leq 5$, and almost all trees T.

[FRS1] $R(K_3, G)$ for all connected graphs G on 6 vertices.

[Jin] $R(K_3, G)$ for all connected graphs G on 7 vertices.
Some errors in [Jin] were found [SchSch1].

[Zeng] Formulas for $R(nK_3, mG)$ for all G of order 4 without isolates.

[Brin] $R(K_3, G)$ for all connected graphs G on at most 8 vertices. The numbers for K_3 versus sets of graphs with fixed number of edges, on at most 8 vertices, were presented in [KlaM1].

[BBH1] $R(K_3, G)$ for all connected graphs G on 9 vertices. See also [BBH2].

[BrGS] $R(K_3, G)$ for all graphs G on 10 vertices, except 10 cases (three of which, including $G = K_{10} - e$, were solved [GoR2]).

[JR3] $R(C_4, G)$ for all graphs G on at most 6 vertices.

[JR4] $R(C_5, G)$ for all graphs G on at most 6 vertices.

[JR2] $R(C_6, G)$ for all graphs G on at most 5 vertices.

[LoM3] $R(K_{2,n}, K_{2,m})$ for all $2 \leq n, m \leq 10$ except 8 cases, for which lower and upper bounds are given. Further data for other complete bipartite graphs are gathered in section 3.3 and [LoMe4].

[HaKr1] All best lower bounds up to 102 from cyclic graphs. Formulas for best cyclic lower bounds for paths and cycles, and values for small complete graphs and for graphs with up to five vertices.

Chvátal and Harary [CH1, CH2] formulated several simple but very useful observations on how to discover values of some numbers. All five missing entries in the tables of Clancy [Clan] have been solved (section 5.11). Out of 7 open cases in [He2] 5 have been solved, including $R(4,5) = R(G_{19}, G_{23}) = 25$ and other cases listed in section 5.11. The 2 cases still open are for K_5 versus K_5 (section 2.1) and K_5 versus $K_5 - e$ (section 3.1). Many extremal and other Ramsey graphs for various parameters are available at [BrCGM, McK, Ex18, Fid2, Fuj1], see section 8.3 below.

8.2. Cumulative data for three colors

[YR3] $R_3(G)$ for all graphs G with at most 4 edges and no isolates.

[YR1] $R_3(G)$ for all graphs G with 5 edges and no isolates, except $K_4 - e$.
The case of $R_3(K_4 - e)$ remains open (see section 6.5).

[YY] $R_3(G)$ for all graphs G with 6 edges and no isolates, except 10 cases.

[ArKM] $R(F,G,H)$ for many triples of isolate-free graphs with at most 4 vertices.
Some of the missing cases completed in [KlaM2].

[BoDD] Extension of [ArKM] to most triples of graphs with at most 4 vertices.

[DzFi2] $R(P_3, P_k, C_m)$ for all $3 \le k \le 8$ and $3 \le m \le 9$.

8.3. Electronic Resources

(a) W. Gasarch [Gas] maintains a website gathering over 60 pointers to literature on applications of Ramsey theory in computer science, http://www.cs.umd.edu/~gasarch/ramsey/ramsey.html.

(b) Many of the Ramsey graph constructions found by G. Exoo [Ex1-Ex20] are posted at http://ginger.indstate.edu/ge/RAMSEY.

(c) G. Brinkmann, K. Coolsaet, J. Goedgebeur and H. Mélot, *House of Graphs: A database of interesting graphs* [BrCGM], http://hog.grinvin.org.

(d) B.D. McKay, presents some graphs related to classical Ramsey numbers [McK], http://cs.anu.edu.au/people/bdm/data/ramsey.html.

(e) R. Fidytek, presents some Ramsey graphs of type $(K_n, K_m - e)$ [Fid2], see also 3.1.f, http://fidytek.inf.ug.edu.pl/ramsey.

(f) H. Fujita, some Ramsey graphs [Fuj1], http://opal.inf.kyushu-u.ac.jp/~fujita/ramsey.html.

(g) Ramsey@Home [RaHo] is a distributed computing project at the University of Wisconsin-Oshkosh designed to find new lower bounds for various Ramsey numbers. Join and help! http://www.ramseyathome.com/ramsey.

8.4. Surveys

(1974) A general survey of results in Ramsey graph theory by S.A. Burr [Bu1]

(1978) A general survey of results in Ramsey graph theory by T.D. Parsons [Par6]

(1980) Survey of results and new problems on multiplicities and Ramsey multiplicities by S.A. Burr and V. Rosta [BuRo3]

(1981) Summary of progress by Frank Harary [Har2]

(1983) A survey of bounds and values by F.R.K. Chung and C.M. Grinstead [ChGri]

(1983) Special volume of the *Journal of Graph Theory* [JGT]

(1984) A review of Ramsey graph theory for newcomers by F.S. Roberts [Rob1]

(1987) What can we hope to accomplish in generalized Ramsey Theory? [Bu7]

(1987) Survey of asymptotic problems by R.L. Graham and V. Rödl [GrRö]

(1990) *Ramsey Theory* by R.L. Graham, B.L. Rothschild and J.H. Spencer [GRS]

(1991) Survey by R.J. Faudree, C.C. Rousseau and R.H. Schelp of graph goodness results, i.e. conditions for the formula $R(G, H) = (\chi(G) - 1)(n(H) - 1) + s(G)$ [FRS5]

(1996) A chapter in *Handbook of Combinatorics* by J. Nešetřil [Neš]

(1996) Survey of zero-sum Ramsey theory by Y. Caro [Caro]

(1997) Among 114 open problems and conjectures of Paul Erdős, presented and commented by F.R.K. Chung, 31 are concerned directly with Ramsey numbers [Chu4]. 216 references are given. An extended version of this work was prepared jointly with R.L. Graham [ChGra2] in 1998.

(2001) An extensive chapter on Ramsey theory in a widely used student textbook and researcher's guide of graph theory by D. West [West]

(2002) *Ramsey Theory and Paul Erdős* by R.L. Graham and J. Nešetřil [GrNe]

(2003) Special issue of *Combinatorics, Probability and Computing* [CoPC]

(2004) Dynamic survey of Ramsey theory applications by V. Rosta [Ros2]. A website maintained by W. Gasarch [Gas] gathers over 60 pointers to literature on applications of Ramsey theory in computer science.

(2009) History, results and people of Ramsey theory. The mathematical coloring book, mathematics of coloring and the colorful life of its creators by A. Soifer [Soi1].

(2011) *Ramsey Theory. Yesterday, Today and Tomorrow,* a special volume in the series *Progress in Mathematics* [Soi2]. A survey of Ramsey numbers involving cycles by the author is included in this volume [Ra4].

(2013) *Problems in Graph Theory from Memphis,* "a summary of problems and results coming out of the 20 year collaboration between Paul Erdős and the authors", by R.J. Faudree, C.C. Rousseau and R.H. Schelp [FRS6].

The surveys by S.A. Burr [Bu1] and T.D. Parsons [Par6] contain extensive chapters on general exact results in graph Ramsey theory. F. Harary presented the state of the theory in 1981 in [Har2], where he also gathered many references including seven to other early surveys of this area. More than two decades ago, Chung and Grinstead in their survey paper [ChGri] gave less data than in this work, but included a broad discussion of different methods used in Ramsey computations in the classical case. S. A. Burr, one of the most experienced researchers in Ramsey graph theory, formulated in [Bu7] seven conjectures on Ramsey numbers for sufficiently large and sparse graphs, and reviewed the evidence for them found in the literature. Three of them have been refuted in [Bra3].

For newer extensive presentations see [GRS, GrRö, FRS5, Neš, Chu4, ChGra2], though these focus on asymptotic theory not on the numbers themselves. A very welcome addition is the 2004 compilation of applications of Ramsey theory by V. Rosta [Ros2]. This survey could not be complete without recommending special volumes of the *Journal of Graph Theory* [JGT, 1983] and *Combinatorics, Probability and Computing* [CoPC, 2003], which, besides a number of research papers, include historical notes and present to us Frank P. Ramsey (1903-1930) as a person. Finally, read a colorful book by A. Soifer [Soi1, 2009] on history and results in Ramsey theory, followed by a collection of essays and technical papers based on presentations from the 2009 Ramsey theory workshop at DIMACS [Soi2, 2011].

The historical perspective and, in particular, the timeline of progress on prior best bounds, can be obtained by checking all the previous versions of this survey since 1994 at http://www.cs.rit.edu/~spr/ElJC/eline.html.

9. Concluding Remarks

This compilation does not include information on numerous variations of Ramsey numbers, nor related topics, like size Ramsey numbers, zero-sum Ramsey numbers, irredundant Ramsey numbers, induced Ramsey numbers, planar Ramsey numbers, bipartite Ramsey numbers, on-line Ramsey numbers, mixed Ramsey numbers, local Ramsey numbers, rainbow Ramsey numbers, connected Ramsey numbers, chromatic Ramsey numbers, avoiding sets of graphs in some colors, coloring graphs other than complete, or the so called Ramsey multiplicities. Interested readers can find such information in some of the surveys listed in section 8 here.

Readers may be interested in knowing that the US patent 6965854 B2 issued on November 15, 2005 claims a method of using Ramsey numbers in "Methods, Systems and Computer Program Products for Screening Simulated Traffic for Randomness." Check the original document at http://www.uspto.gov/patft if you wish to find out whether your usage of Ramsey numbers is covered by this patent.

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The author apologizes for any omissions or other errors in reporting results belonging to the scope of this work. Suggestions for any kind of corrections or additions will be greatly appreciated and considered for inclusion in the next revision of this survey.

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Out of 661 references gathered below, most appeared in about 100 different periodicals, among which most articles were published in: *Discrete Mathematics* 68, *Journal of Combinatorial Theory* (old, Series A and B) 55, *Journal of Graph Theory* 50, *Electronic Journal of Combinatorics* 32, *Ars Combinatoria* 28, *Journal of Combinatorial Mathematics and Combinatorial Computing* 27, *European Journal of Combinatorics* 20, *Utilitas Mathematica* 18, *Combinatorica* 15, *Graphs and Combinatorics* 14, *Australasian Journal of Combinatorics* 14, *Discrete Applied Mathematics* 12, *Congressus Numerantium* 12, and *Combinatorics, Probability and Computing* 11. The results of 143 references depend on computer algorithms.

The references are ordered alphabetically by the last name of the first author, and where multiple papers have the same first author they are ordered by the last name of the second author, etc. We preferred that all work by the same author be in consecutive positions. Unfortunately, this causes that some of the abbreviations are not in alphabetical order. For example, [BaRT] is earlier on the list than [BaLS]. We also wish to explain a possible confusion with respect to the order of parts and spelling of Chinese names. We put them without any abbreviations, often with the last name written first as is customary in original. Sometimes this is different from the citations in other sources. One can obtain all variations of writing any specific name by consulting the authors database of *Mathematical Reviews* at http://www.ams.org/mathscinet/search, or *zbMATH* (formerly *Zentralblatt für Mathematik*) at http://www.zbmath.org/authors.

Papers containing results obtained with the help of computer algorithms have been marked with stars. We identify two such categories of papers: those marked with * involving some use of computers where the results are easily verifiable with some computations, and those marked with ** where cpu intensive algorithms have to be implemented to replicate or verify the results. The first category contains mostly constructions done by algorithms, while the second mostly nonexistence results or claims of complete enumerations of special classes of graphs.

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[Ea2] Unique 2-(6,3,2) design gives lower bound 7, upper bound is easy.

[Ea3] Every edge (3,3,3;2)-coloring of K_{15} has 35 edges in each color [Hein], and since the number of triangles in K_{16} is not divisible by 3, hence no required triangle-coloring of K_{16} exists.

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