Small Ramsey Numbers

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ABSTRACT: We present data which, to the best of our knowledge, includes all known nontrivial values and bounds for specific graph, hypergraph and multicolor Ramsey numbers, where the avoided graphs are complete or complete without one edge. Many results pertaining to other more studied cases are also presented. We give references to all cited bounds and values, as well as to previous similar compilations. We do not attempt complete coverage of asymptotic behavior of Ramsey numbers, but concentrate on their specific values.

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1. Scope and Notation

There is a vast literature on Ramsey type problems starting in 1930 with the original paper of Ramsey [Ram]. Graham, Rothschild and Spencer in their book [GRS] present an exciting development of Ramsey Theory. The subject has grown amazingly, in particular with regard to asymptotic bounds for various types of Ramsey numbers (see the survey paper [GrRö]), but the progress on evaluating the basic numbers themselves has been very unsatisfactory for a long time. In the last decade, however, considerable progress has been obtained in this area, mostly by employing computer algorithms. The few known exact values and several bounds for different numbers are scattered among many technical papers. This compilation is a fast source of references for the best results known for specific numbers. It is not supposed to serve as a source of definitions or theorems, but these can be easily accessed via the references gathered here.

Ramsey Theory studies conditions when a combinatorial object contains necessarily some smaller given objects. The role of Ramsey numbers is to quantify some of the general existential theorems in Ramsey Theory.

Let $G_1, G_2, \ldots, G_m$ be graphs or s-uniform hypergraphs (s is the number of vertices in each edge). $R(G_1, G_2, \ldots, G_m; s)$ denotes the m-color **Ramsey number** for s-uniform graphs/hypergraphs, avoiding G_i in color i for $1 \le i \le m$. It is defined as the least integer n such that, in any coloring with m colors of the s-subsets of a set of n elements, for some i the s-subsets of color i contain a sub-(hyper)graph isomorphic to G_i (not necessarily induced). The value of $R(G_1, G_2, \ldots, G_m; s)$ is fixed under permutations of the first m arguments.

If $s = 2$ (standard graphs) then s can be omitted. If G_i is a complete graph K_k, then we can write k instead of G_i, and if $G_i = G$ for all i we can use the abbreviation $R_m(G; s)$ or $R_m(G)$. For $s = 2$, $K_k - e$ denotes a K_k without one edge, and for $s = 3$, $K_k - t$ denotes a K_k without one triangle (hyperedge). P_i is a **path** on i vertices, C_i is a **cycle** of length i, and W_i is a **wheel** with $i-1$ spokes, i.e. a graph formed by some vertex x, connected to all vertices of some cycle C_{i-1}. $K_{n,m}$ is a complete n by m bipartite graph, in particular $K_{1,n}$ is a **star** graph. The **book** graph $B_i = K_2 + \bar{K}_i = K_1 + K_{1,i}$ has $i+2$ vertices, and can be seen as i triangular pages attached to a single edge. The **fan** graph F_n is defined by $F_n = K_1 + nK_2$. For a graph G, $n(G)$ and $e(G)$ denote the number of vertices and edges, respectively. Finally, let $\chi(G)$ be the chromatic number of G, and let nG denote n disjoint copies of G.

Section 2 contains the data for the classical two color Ramsey numbers $R(k, l)$ for complete graphs, and section 3 for the two color case when the avoided graphs are complete or have the form $K_k - e$, but not both are complete. Section 4 lists the most studied two color cases for other graphs. The multicolor and hypergraph cases are gathered in sections 5 and 6, respectively. Finally, section 7 gives pointers to cumulative data and to some previous surveys, especially those containing data not subsumed by this compilation.

2. Classical Two Color Ramsey Numbers

k \ l	3	4	5	6	7	8	9	10	11	12	13	14	15
3	6	9	14	18	23	28	36	40 43	46 51	52 59	59 69	66 78	73 88
4		18	25	35 41	49 61	56 84	69 115	80 149	96 191	128 238	133 291	141 349	153 417
5			43 49	58 87	80 143	95 216	121 316	141 442	153	181	193	221	242
6				102 165	111 298	127 495	153 780	177 1171	253	262	278	292	374
7					205 540	216 1031	 1713	 2826	322	416	511		
8						282 1870	 3583	316 6090			635		703
9							565 6588	580 12677					
10								798 23556					

Table I. Known nontrivial values and bounds for two color Ramsey numbers $R(k,l) = R(k,l;2)$.

k \ l	4	5	6	7	8	9	10	11	12	13	14	15
3	GG	GG	Kéry	Ka2 GY	GR MZ	Ka2 GR	Ex5 RK2	Ka2 RK2	Ex12 Les	Piw1 RK2	Ex8 RK2	WW Les
4	GG	Ka1 MR4	Ex9 MR5	Ex3 Mac	Ex15 Mac	RK1 Mac	Piw1 Mac	Piw1 Spe2	SLL2 Spe2	XX1 Spe2	XX1 Spe2	XX1 Spe2
5		Ex4 MR5	Ex9 HZ1	CET Spe2	Piw1 Spe2	Haa Mac	Ex12 Mac	Ex12	Ex12	Ex12	Ex12	SLLL
6			Ka1 Mac	XX1 Mac	XX1 Mac	Ex12 Mac	XX1 Mac	XX1	XX1	XX1	XX1	SLLL
7				She1 Mac	XX1 Mac	 HZ1	 Mac	XX1	XX1	XX1		
8					BR Mac	 Ea1	XX1 HZ1			XX1		XX1
9						She1 ShZ1	XX1 Ea1					
10							She1 Shi2					

References for Table I.

We split the data into the table of values and a table with corresponding references for the Table I. Known exact values appear as centered entries, lower bounds as top entries, and upper bounds as bottom entries.

The task of proving $R(3,3) \leq 6$ was the second problem in Part I of the William Lowell Putnam Mathematical Competition held in March 1953 [Bush].

The construction by Mathon [Mat] (see also sections 4.16 and 5.3.(h)), using data obtained by Shearer [She1], gives the following lower bounds for higher diagonal numbers: $R(11,11) \geq 1597$, $R(13,13) \geq 2557$, $R(14,14) \geq 2989$, $R(15,15) \geq 5485$, and $R(16,16) \geq 5605$. Similarly, $R(17,17) \geq 8917$, $R(18,18) \geq 11005$ and $R(19,19) \geq 17885$ were obtained in [LSL]. The same approach does not improve on an easy bound $R(12,12) \geq 1597 + 11 + 10$. The best known construction for this case showing $R(12,12) \geq 1637$ is given in [XX1] (for the general case see section 4.16).

All the critical graphs for the numbers $R(k,l)$ (graphs on $R(k,l)-1$ vertices without K_k and without K_l in the complement) are known for $k=3$ and $l=3$, 4, 5 [Kéry], 6 [Ka2], 7 [RK3, MZ], and there are 1, 3, 1, 7 and 191 of them, respectively. All $(3,k)$-graphs, for $k \leq 6$, were enumerated in [RK3], and all (4,4)-graphs in [MR2]. There exists a unique critical graph for $R(4,4)$ [Ka2]. There are 4 such graphs known for $R(3,8)$ [RK2], 1 for $R(3,9)$ [Ka2] and 350904 for $R(4,5)$ [MR4], but there might be more of them. In [MR5] evidence is given for the conjecture that $R(5,5)=43$ and that there exist 656 critical graphs on 42 vertices.

The claim that $R(5,5)=50$ published on the web [Stone] is in error, and despite being shown so more than once, this incorrect value is being cited by some authors. The bound $R(3,13) \geq 60$ [XZ] cited in the 1995 version of this survey was shown to be incorrect in [Piw1]. The graphs constructed by Exoo in [Ex12, Ex15], and some others, are available electronically from http://isu.indstate.edu/ge/RAMSEY.

By taking a disjoint union of two critical graphs one can easily see that $R(k,p) \geq s$ and $R(k,q) \geq t$ imply $R(k,p+q-1) \geq s+t-1$. Xu and Xie [XX1] improved this construction to yield better general lower bounds, in particular $R(k,p+q-1) \geq s+t+k-3$. For example, this gives a lower bound $R(4,13) \geq 133$ with $p=2$, $q=12$. Only some higher lower bounds implied this way are shown. Some upper bounds implied by $R(k,l) \leq R(k-1,l)+R(k,l-1)$, or by its slight improvement with strict inequality when both terms on the right hand side are even, are marked [Ea1]. There are obvious generalizations of these inequalities for graphs other than complete.

The bound $R(6,6) \leq 166$, only 1 more than the best known [Mac], is an easy consequence of theorem 1 in [Walk] (see section 4.16) and the inequality $R(4,6) \leq 41$. T. Spencer [Spe2], Mackey [Mac], and Huang and Zhang [HZ1], using the bounds for minimum and maximum number of edges in (4,5) Ramsey graphs listed in [MR3, MR5], were able to establish new upper bounds for several higher Ramsey numbers, improving all the previous longstanding results of Giraud [Gi3, Gi5, Gi6]. We have recomputed the bounds marked [HZ1] using the method from the paper [HZ1], because the bounds there relied on an overly optimistic personal communication from T. Spencer. Further refinements of this method are studied in [HZ2, ShZ1, Shi2].

For a more in depth study of triangle-free graphs in relation to the case of $R(3,k)$, for which considerable progress has been obtained in recent years, see also [AKS, Alon2, BBH1, BBH2, CPR, FL, Fra1, Fra2, Gri, Loc, KM1, RK3, RK4, She2, Stat, Yu1]. In 1995, Kim [Kim] obtained a breakthrough by proving that $R(3,k)$ has order of magnitude exactly

$\Theta(k^2/\log k)$. Good asymptotic bounds for $R(k,k)$ can be found, for example, in [Chu3, McS] (lower bound) and [Tho] (upper bound), and for many other asymptotic bounds in the general case of $R(k,l)$ consult [Spe1, GRS, GrRö, AP].

The lower bounds marked [XX1] were obtained by a method referenced in section 4.16. All other lower bounds for higher numbers listed in Table II were obtained by construction of cyclic graphs, except for the bound $R(5,17) \geq 284$ established by Exoo [Ex15] using pq-groups.

k \ l	15	16	17	18	19	20	21	22	23
3	73 WW	79 WW	92 WWY1	98 WWY1	106 WWY1	109 WWY1	122 WWY1	125 WWY1	136 WWY1
4	153 XX1		182 LSS		198 LSZL	230 SLZL	242 SLZL	282 SL	
5	242 SLLL	278 LSS	284 Ex15		338 SLZL	380 LSS		422 LSZL	434 LSZL
6	374 SLLL	434 SLLL	548 SLLL	614 SLLL	710 SLLL	878 SLLL		1070 SLLL	
7			628 XX1	722 XX1	908 SLLL		1214 SLLL		
8	703 XX1		737 XX1	871 XX1	1054 XX1	1094 SLLL	1328 SLLL		

Table II. Known nontrivial lower bounds for higher two color Ramsey numbers $R(k,l)$, with references.

Exoo [Ex15] gives also the bounds $R(3,27) \geq 158$ and $R(3,31) \geq 198$. The constructions establishing $R(3,26) \geq 150$, $R(3,29) \geq 174$, $R(3,31) \geq 198$ and $R(3,32) \geq 212$ are presented in [SLL1], [SLL3], [LSS] and [LSZL], respectively. Yu [Yu2] constructed a special class of triangle-free cyclic graphs establishing several lower bounds for $R(3,k)$, for $k \geq 61$. Only two of these bounds, $R(3,61) \geq 479$ and $R(3,103) \geq 955$, cannot be easily improved by the inequality $R(3,4k+1) \geq 6R(3,k+1)-5$ from [CCD] and data from tables I and II. Finally, $R(5,24) \geq 488$ was given in [SLL3], and $R(9,17) \geq 1411$ and $R(10,16) \geq 1189$ in [XX1].

3. Two Colors - Dropping One Edge from Complete Graph

G \ H	K_3-e	K_4-e	K_5-e	K_6-e	K_7-e	K_8-e	K_9-e	$K_{10}-e$	$K_{11}-e$
K_3-e	3	5	7	9	11	13	15	17	19
K_3	5	7	11	17	21	25	31	37 38	42 47
K_4-e	5	10	13	17	28	29 38	34	41	
K_4	7	11	19	27 36	37 52				
K_5-e	7	13	22	31 39	40 66				
K_5	9	16	30 34	43 67	112				
K_6-e	9	17	31 39	45 70	59 135				
K_6	11	21	37 55	119	205				
K_7-e	11	28	40 66	59 135	251				
K_7	13	28 34	51 88	204					

Table III. Two types of Ramsey numbers $R(G,H)$,
includes all known nontrivial values.

The exact values in Table III above involving K_3-e are trivial, since easily $R(K_3-e, K_k) = R(K_3-e, K_{k+1}-e) = 2k-1$, for all $k \geq 2$. Other bounds (not shown in Table III) can be obtained by using Table I, an obvious generalization of the inequality $R(k,l) \leq R(k-1,l) + R(k,l-1)$, and by monotonicity of Ramsey numbers, in this case $R(K_{k-1}, G) \leq R(K_k - e, G) \leq R(K_k, G)$.

For the following numbers it was established that the critical graphs are unique: $R(K_3, K_l - e)$ for $l = 3$ [Tr], 6 and 7 [Ra1], $R(K_4-e, K_4-e)$ [FRS2], $R(K_5-e, K_5-e)$ [Ra3] and $R(K_4-e, K_7-e)$ [McR]. The number of $R(K_3, K_l - e)$ critical graphs for $l = 4, 5$ and 8 is 4, 2 and 9, respectively [MPR]. Also in [MPR] a bound $R(K_3, K_{12}-e) \geq 46$ is given. Wang, Wang and Yan in [WWY2] constructed cyclic graphs showing $R(K_3, K_{13}-e) \geq 54$, $R(K_3, K_{14}-e) \geq 59$ and $R(K_3, K_{15}-e) \geq 69$. The upper bounds in [HZ2] were obtained by a reasoning generalizing the bounds for classical numbers in [HZ1].

G \ H	K_4-e	K_5-e	K_6-e	K_7-e	K_8-e	K_9-e	$K_{10}-e$	$K_{11}-e$
K_3	CH2	Clan	FRS1	GH	Ra1	Ra1	MPR MPR	WWY2 MPR
K_4-e	CH1	FRS2	McR	McR	Ea1 HZ2	Ex14	Ex14	
K_4	CH2	EHM1	Ex11 Ea1	Ex14 HZ2				
K_5-e	FRS2	CEHMS	Ex14 Ea1	Ex14 HZ2				
K_5	BH	Ex8 Ex8	Ea1 HZ2	HZ2				
K_6-e	McR	Ex14 Ea1	Ex14 HZ2	Ex14 HZ2				
K_6	McN	Ex14 Ea1	ShZ2	ShZ2				
K_7-e	McR	Ex14 HZ2	Ex14 HZ2	ShZ1				
K_7	Ea1 Ea1	Ex14 ShZ2	ShZ2					

References for Table III.

4. General Graph Numbers in Two Colors

This section includes data with respect to general graph results. We tried to include all nontrivial values and identities regarding exact results (or references to them), but only those out of general bounds and other results which, in our opinion, have a direct connection to the evaluation of specific numbers. If some small value cannot be found below, it may be covered by the cumulative data gathered in section 7, or be a special case of a general result listed in this section. Note that $B_1 = F_1 = C_3 = W_3 = K_3$, $B_2 = K_4 - e$, $P_3 = K_3 - e$, $W_4 = K_4$ and $C_4 = K_{2,2}$ imply other identities not mentioned explicitly.

4.1. Paths

$R(P_n, P_m) = n + \lfloor m/2 \rfloor - 1$ for all $n \ge m \ge 2$ [GeGy]

4.2. Cycles

$R(C_3, C_3) = 6$ [GG]
$R(C_4, C_4) = 6$ [CH1]

Result obtained independently in [Ros] and [FS1], new simple proof in [KáRos]:

$$R(C_n, C_m) = \begin{cases} 2n-1 & \text{for } 3 \le m \le n,\ m \text{ odd, } (n,m) \ne (3,3) \\ n-1+m/2 & \text{for } 4 \le m \le n,\ m \text{ and } n \text{ even, } (n,m) \ne (4,4) \\ \max\{n-1+m/2, 2m-1\} & \text{for } 4 \le m < n,\ m \text{ even and } n \text{ odd} \end{cases}$$

Unions of cycles, $R(nC_p, mC_q)$, [MS, Den]

$R(nC_3, mC_3) = 3n + 2m$ for $n \ge m \ge 1$, $n \ge 2$ [BES]

$R(nC_4, mC_4) = 2n + 4m - 1$ for $m \ge n \ge 1$, $(n,m) \ne (1,1)$ [LiWa]

4.3. Wheels

$R(W_3, W_5) = 11$ [Clan]
$R(W_3, W_n) = 2n - 1$ for all $n \ge 6$ [BE2]
All critical colorings for $R(W_3, W_n)$ for all $n \ge 3$ [RaJi]
$R(W_4, W_5) = 17$ [He3]
$R(W_5, W_5) = 15$ [HM2, He2]
$R(W_4, W_6) = 19$, $R(W_5, W_6) = 17$ and $R(W_6, W_6) = 17$,
and all critical colorings (2, 1 and 2) for these numbers [FM]

4.4. Books

$R(B_1, B_n) = 2n+3$ for all $n > 1$ [RS1]
$R(B_3, B_3) = 14$ [RS1, HM2]
$R(B_2, B_5) = 16$, $R(B_3, B_5) = 17$, $R(B_5, B_5) = 21$,
$R(B_4, B_4) = 18$, $R(B_4, B_6) = 22$, $R(B_6, B_6) = 26$ [RS1]
$254 \le R(B_{37}, B_{88}) \le 255$ [Par6]

in general $R(B_n, B_n) = 4n+2$ for $4n+1$ a prime power,
and several other general equalities and bounds for $R(B_n, B_m)$ [RS1, FRS7, Par6].

4.5. Complete bipartite graphs

$R(K_{2,3}, K_{2,3}) = 10$ [Bu4]
$R(K_{2,3}, K_{2,4}) = 12$ [ExRe]
$R(K_{2,3}, K_{1,7}) = 13$ [Par4]
$R(K_{2,3}, K_{3,3}) = 13$ and $R(K_{3,3}, K_{3,3}) = 18$ [HM3]
$R(K_{2,2}, K_{2,8}) = 15$ and $R(K_{2,2}, K_{2,11}) = 18$ [HM]
$R(K_{2,2}, K_{1,15}) = 20$ [La2]
$R(nK_{1,3}, mK_{1,3}) = 4n+m-1$ for $n \ge m \ge 1$, $n \ge 2$ [BES]
Asymptotics for $K_{2,m}$ versus K_n [CLRZ]

$R(K_{1,n}, K_{1,m}) = n+m-\varepsilon$, where $\varepsilon = 1$ if both n and m are even and $\varepsilon = 0$ otherwise [Har1]. It is also a special case of multicolor numbers for stars obtained in [BuRo1].

$R(K_{2,n}, K_{2,n}) \le 4n-2$ for all $n \ge 2$, exact values 6, 10, 14, 18, 21, 26, 30, 33, 38, 42, 46, 50, 54, 57 and 62 of $R(K_{2,n}, K_{2,n})$ for $2 \le n \le 16$, respectively.

The first open case is $65 \le R(K_{2,17}, K_{2,17}) \le 66$ [EHM2].

4.6. Triangle versus other graphs

$R(3,k) = \Theta(k^2/\log k)$ [Kim]

Explicit construction for $R(3, 4k+1) \ge 6R(3, k+1) - 5$, for all $k \ge 1$ [CCD]

Explicit triangle-free graphs with independence k on $\Omega(k^{3/2})$ vertices [Alon2, CPR]

$R(K_3, K_7 - 2P_2) = R(K_3, K_7 - 3P_2) = 18$ [SchSch2]

$R(K_3, K_3 + \overline{K}_m) = R(K_3, K_3 + \overline{C}_m) = 2m+5$ for $m \ge 212$ [Zhou1]

$R(K_3, G) = 2n(G) - 1$ for any connected G on at least 4 vertices and with at most $(17n(G)+1)/15$ edges, in particular for $G = P_i$ and $G = C_i$, for all $i \ge 4$ [BEFRS1]

$R(K_3, G) \le 2e(G) + 1$ for any graph G without isolated vertices [Sid3, GK]
$R(K_3, G) \le n(G) + e(G)$ for all G, a conjecture [Sid2]
$R(K_3, K_n)$, see section 2
$R(K_3, K_n - e)$, see section 3
$R(K_3, G)$ for all connected G up to 9 vertices, see section 7.1

Since $B_1 = F_1 = C_3 = W_3 = K_3$, other sections apply. See also [AKS, BBH1, BBH2, FL, Fra1, Fra2, Gri, Loc, KM1, RK3, RK4, She2, Spe1, Stat, Yu1].

4.7. Paths versus other graphs

Paths versus stars [Par2, BEFRS2]
Paths versus trees [FS4]
Paths versus books [RS2]
Paths versus cycles [FLPS, BEFRS2]
Paths versus K_n [Par1]
Paths versus $K_{n,m}$ [Häg]
Paths versus W_5 and W_6 [SuBa1]
Paths versus W_7 and W_8 [Bas]
Paths versus wheels [BaSu, ChZZ]
Paths and cycles versus trees [FSS1]
Sparse graphs versus paths and cycles [BEFRS2]
Graphs with long tails [Bu2, BG]

4.8. Cycles versus complete graphs

$R(C_4, K_3) = R(C_4, C_3) = 7$ [CS]
$R(C_4, K_4) = 10$ [CH2]
$R(C_4, K_5) = 14$ [Clan]
$R(C_4, K_6) = 18$ [Ex2] [RoJa1]
$R(C_4, K_7) = 22$ [RT] [JR1]
$R(C_4, K_8) = 26$ [RT]
$30 \le R(C_4, K_9)$, $34 \le R(C_4, K_{10})$ [RT]
C_4 versus K_n [CLRZ]

$R(C_5, K_3) = R(C_5, C_3) = 9$ [CS]
$R(C_5, K_4) = 13$ [He2, JR4]
$R(C_5, K_5) = 17$ [He2, JR4]
$R(C_5, K_6) = 21$ [JR5]
$R(C_5, K_7) = 25$ [Schi2]

$R(C_6, K_4) = 16$ [JR2]
$R(C_6, K_5) = 21$ [JR2, YHZ2, BJYHRZ]
$R(C_6, K_6) = 26$ [Schi1]
$R(C_7, K_5) = 25$ [YHZ2, BJYHRZ]
Cycles versus K_n [BoEr, Spe1, FS4, EFRS2, CLRZ, Sud]

$R(C_n, K_m) = (n-1)(m-1) + 1$, for $n \ge m^2 - 2$ [BoEr], for $n > 3 = m$ [FS1], for $n \ge 4 = m$ [YHZ1], for $n \ge 5 = m$ [BJYHRZ], for $n \ge 6 = m$, and for $n \ge m \ge 7$ with $n \ge m(m-2)$ [Schi1]. Since 1976, it was conjectured to be true for all $n \ge m \ge 3$, except $n = m = 3$ [FS4, EFRS2].

4.9. Cycles versus other graphs

C_4 versus stars [Par3, Par5, BEFRS5, Chen]
C_4 versus trees [EFRS4, Bu6, BEFRS5, Chen]
C_4 versus $K_{m,n}$ [HM]
C_4 versus all graphs on six vertices [JR3]
$R(C_4, B_n)$ = 7, 9, 11, 12, 13 and 16, for $2 \le n \le 7$, respectively [FRS6]
$R(C_4, B_n)$ = 17, 18, 19, 20 and 21, for $8 \le n \le 12$, respectively [Tse]
$R(C_4, W_n)$ = 10, 9, 10, 9, 11, 12, 13, 14, 16 and 17, for $4 \le n \le 13$, respectively [Tse]
$R(C_4, G) \le 2q+1$ for any isolate-free graph G with q edges [RoJa2]
$R(C_4, G) \le p+q-1$ for any connected graph G on p vertices and q edges [RoJa2]

$R(C_5, W_6) = 13$ [ChvS]
$R(C_5, K_6 - e) = 17$ [JR4]
$R(C_5, B_1) = R(C_5, B_2) = 9$ [CRSPS]
$R(C_5, B_3) = 10$, and in general $R(C_5, B_n) = 2n+3$ for $n \ge 4$ [FRS8]
C_5 versus all graphs on six vertices [JR4]
$R(C_6, K_5 - e) = 17$ [JR2]
C_6 versus all graphs on five vertices [JR2]

$R(C_n, G) \le 2q + \lfloor n/2 \rfloor - 1$, for $3 \le n \le 5$, for any isolate-free graph G with $q > 3$ edges. It is conjectured that it also holds for other n [RoJa2].

Cycles versus paths [FLPS, BEFRS2]
Cycles versus stars [La1, Clark, see Par6]
Cycles versus trees [FSS1]
Cycles versus books [FRS6, FRS8, Zhou1]
Cycles versus W_5 and W_6 [SuBB2]
Cycles versus wheels [Zhou2]
See also bipartite graphs for $K_{2,2} = C_4$

4.10. Stars versus other graphs

Stars versus C_4 [Par3, Par5, Chen]
Stars versus W_5 and W_6 [SuBa1]
Stars versus paths [Par2, BEFRS2]
Stars versus cycles [La1, Clark, see Par6]
Stars versus books [CRSPS, RS2]
Stars versus trees [Bu1, GV, ZZ]
Stars versus stripes [CL, Lor]
Stars versus $K_{2,n}$ [Par4]
Stars versus $K_{n,m}$ [Stev, Par3]
Stars versus $K_n - tK_2$ [Hua1, Hua2]
Stars versus $2K_2$ [MO]
Union of two stars [Gro2]

4.11. Books versus other graphs

$R(B_3, K_4) = 14$ [He3]
$20 \le R(B_3, K_5) \le 22$ [He2]
Books versus paths [RS2]
Books versus trees [EFRS7]
Books versus stars [CRSPS, RS2]
Books versus cycles [FRS6, FRS8, Zhou1, Tse]
Books versus wheels [Zhou3]
Books versus $K_2 + C_n$ [Zhou3]
Books and $(K_1 + tree)$ versus K_n [LR1]

4.12. Wheels versus other graphs

$R(W_5, K_5 - e) = 17$ [He2][YH]
$27 \le R(W_5, K_5) \le 29$ [He2]
W_5 and W_6 versus stars and paths [SuBa1]
W_5 and W_6 versus trees [BSNM]
W_5 and W_6 versus cycles [SuBB2]
W_7 and W_8 versus paths [Bas]
Wheels versus paths [BaSu, ChZZ]
Odd wheels versus star-like trees [SuBB]
$R(W_6, C_5) = 13$ [ChvS]
Wheels versus C_4 [Tse]
Wheels versus cycles [Zhou2]
Wheels versus books [Zhou3]
Wheels versus linear forests [SuBa2]

4.13. Trees and Forests

Trees, forests [EG, GRS, FSS1, GV, CsKo]
Trees versus C_4 [EFRS4, Bu6, Chen]
Trees versus paths [FS4]
Trees versus paths and cycles [FSS1]
Trees versus stars [Bu1, GV, ZZ]
Trees versus books [EFRS7]
Trees versus W_5 and W_6 [BSNM]
Star-like trees versus odd wheels [SuBB]
Trees versus K_n [Chv]
Trees versus $K_n + \overline{K}_m$ [RS2, FSR]
Trees versus bipartite graphs [BEFRS5, EFRS6]
Trees versus almost complete graphs [GJ2]
Trees versus small $(n(G) \le 5)$ connected G [FRS4]
Linear forests, forests [BuRo2, FS3, CsKo]
Linear forests versus wheels [SuBa2]

Forests versus K_n [Stahl]
Forests versus almost complete graphs [CGP]

4.14. Mixed special cases:

$R(C_5+e, K_5)=17$ [He5]
$R(W_5, K_5-e)=17$ [He2][YH]
$20 \le R(B_3, K_5) \le 22$ [He2]
$27 \le R(W_5, K_5) \le 29$ [He2]
$25 \le R(K_5-P_3, K_5) \le 28$ [He2]
$26 \le R(K_{2,2,2}, K_{2,2,2})$, $K_{2,2,2}$ is an octahedron [Ex8]

4.15. Mixed general cases

Unicyclic graphs [Gro1, Köh, KrRod]
$K_{2,m}$ and C_{2m} versus K_n [CLRZ]
$K_{2,n}$ versus any graph [RoJa2]
nK_3 versus mK_3, in particular $R(nK_3, nK_3) = 5n$ for $n \ge 2$ [BES]
nK_3 versus mK_4 [LorMu]
$R(nK_4, nK_4) = 7n+4$ for large n [Bu7]
Stripes [CL, Lor]
Union of two stars [Gro2]
Double stars* [GHK]
Graphs with bridge versus K_n [Li]
Fans $F_n = K_1 + nK_2$ versus K_m [LR2]
$R(F_1, F_n) = R(K_3, F_n) = 4n+1$ and bounds for $R(F_m, F_n)$ [GGS]
Multipartite complete graphs [BEFRS3, EFRS4, FRS3, Stev]
Multipartite complete graphs versus trees [EFRS8, BEFRSGJ]
Disconnected graphs versus any graph [GJ1]
Graphs with long tails [Bu2, BG]
Brooms+ [EFRS3]

4.16. Other general results

[Chv] $R(K_n, T_m)=(n-1)(m-1)+1$ for any tree T on m vertices.

[CH2] $R(G,H) \ge (\chi(G)-1)(c(H)-1)+1$, where $\chi(G)$ is the chromatic number of G, and $c(H)$ is the size of the largest connected component of H.

[Walk] $R(k,k) \le 4R(k,k-2)+2$.

[Spe1] Probabilistic asymptotic lower bounds for $R(k,l)$, also weaker bounds but with an explicit constructive approach in [AP].

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* - A double star is a union of two stars with their centers joined by an edge.

+ - A broom is a star with a path attached to its center.

[Mat] If the quadratic residues Paley graph Q_p of prime order $p = 4t + 1$ contains no K_k, then $R(k,k) \geq p + 1$ and $R(k+1, k+1) \geq 2p + 3$. More data was obtained in [She1, LSL].

[XX1] For $2 \leq p \leq q$ and $3 \leq k$, if (k,p)-graph G and (k,q)-graph H have a common induced subgraph on m vertices without K_{k-1}, then $R(k, p+q-1) > n(G) + n(H) + m$. In particular, we have $R(k, p+q-1) \geq R(k,p) + R(k,q) + k - 3$ and $R(k, p+q-1) \geq R(k,p) + R(k,q) + p - 2$.

[BE1] $R(G,G) \geq \lfloor (4n(G)-1)/3 \rfloor$ for any connected G.

[BE2] Graphs yielding $R(K_n, G) = (n-1)(n(G)-1)+1$ and related results (see also [EFRS5]).

[BES] Study of Ramsey numbers for multiple copies of graphs. See also [Bu1, LorMu].

[Zeng] $R(nK_3, nG)$ for all isolate-free graphs G on 4 vertices.

[Bu7] Study of Ramsey numbers for large disjoint unions of graphs, in particular $R(nK_k, nK_l) = n(k+l-1) + R(K_{k-1}, K_{l-1}) - 2$, for n large enough. See also [Bu8].

[Bu2] Graphs H yielding $R(G,H) = (\chi(G)-1)(n(H)-1) + s(G)$, where $s(G)$ is a chromatic surplus of G, defined as the minimum number of vertices in some color class under all vertex colorings in $\chi(G)$ colors (such H's are called G-good). This idea, initiated in [Bu2], is a basis of a number of exact results for $R(G,H)$ for large and sparse graphs H [BG, BEFRS2, BEFRS4, Bu5, FS, EFRS4, FRS3, BEFSRGJ, BF, LR4]. A survey of this area appeared in [FRS5].

[BEFS] Bounds for the difference between consecutive Ramsey numbers.

[Par3] Relations between some Ramsey graphs and block designs. See also [Par4].

[Bra3] $R(G,H) > h(G,d)\,n(H)$ for all nonbipartite G and almost every d-regular H, for some h unbounded in d.

[CSRT] $R(G,G) \leq c_d\, n(G)$ for all G, where constant c_d depends only on the maximum degree d in G. The constant was improved in [GRR1]. Tight lower and upper bounds for bipartite G [GRR2].

[ChenS] $R(G,G) \leq c_d\, n$ for all d-arrangeable graphs G on n vertices, in particular with the same constant for all planar graphs. The constant c_d was improved in [Eaton]. An extension to graphs not containing a subdivision of K_d [RöTh]. Progress towards a conjecture that the same inequality holds for all d-generate graphs G [KoRö].

[EFRS9] Study of graphs G, called *Ramsey size linear,* for which there exists a constant c_G such that for all H with no isolates $R(G,H) \leq c_G\, e(H)$. An overview and further results were given in [BSS].

[LRS] $R(G,G) < 6n$ for all n-vertex graphs G, in which no two vertices of degree at least 3 are adjacent. This improves the result $R(G,G) \leq 12n$ in [Alon1].

[Shi1] $R(Q_n, Q_n) \le 2^{(3+\sqrt{5})n/2+o(n)}$, for the n-dimensional cube Q_n with 2^n vertices. This bound can be also derived from a theorem in [KoRö].

[Gro1] Conjecture that $R(G,G) = 2n(G) - 1$ if G is unicyclic of odd girth. Further support for the conjecture was given in [Köh, KrRod].

[RoJa2] $R(K_{2,k}, G) \le kq + 1$, for $k \ge 2$, for isolate-free graphs G with $q \ge 2$ edges.

[FSS1] Discussion of the conjecture that $R(T_1, T_2) \le n(T_1) + n(T_2) - 2$ holds for all trees T_1, T_2.

[FM] $R(W_6, W_6) = 17$ and $\chi(W_6) = 4$. This gives a counterexample $G = W_6$ to the Erdös conjecture (see [GRS]) $R(G,G) \ge R(K_{\chi(G)}, K_{\chi(G)})$.

[LR3] Bounds on $R(H + \overline{K}_n, K_n)$ for general H. Also, for fixed k and m, as $n \to \infty$, $R(K_k + \overline{K}_m, K_n) \le (m + o(1))\, n^k / (\log n)^{k-1}$ [LRZ].

[-] Special cases of multicolor results listed in section 5.5.

[-] See also surveys listed in section 7.

5. Multicolor Graph Numbers

The only known value of a multicolor classical Ramsey number:

$R_3(3) = R(3,3,3) = R(3,3,3;2) = 17$	[GG]
2 critical colorings	[KS, LayMa]

5.1. Bounds for multicolor classical numbers

$51 \le R_4(3) = R(3,3,3,3) \le 62$	[Chu1] [FKR]
$162 \le R_5(3) \le 307$	[Ex10]; [FKR], 5.3.(a)
$538 \le R_6(3) \le 1838$	[FreSw], 5.3.(a)
$1682 \le R_7(3) \le 12861$	[FreSw], 5.3.(a)
$128 \le R(4,4,4) \le 236$	[HI], 5.3.(a)
$578 \le R(4,4,4,4)$	[ExRa]
$2295 \le R(4,4,4,4,4)$	[XX2]
$415 \le R(5,5,5)$	[ExRa]
$2501 \le R(5,5,5,5)$	5.3.(j)
$25922 \le R(5,5,5,5,5)$	5.3.(j)
$1070 \le R(6,6,6)$	[Mat], 5.3.(h)
$10407 \le R(6,6,6,6)$	[XX2]
$3211 \le R(7,7,7)$	[Mat], 5.3.(h)
$42117 \le R(7,7,7,7)$	[XX2]
$4930 \le R(8,8,8)$	[XX2]
$8461 \le R(9,9,9)$	[XX2]
$30 \le R(3,3,4) \le 31$	[Ka2] [PR1, PR2]
$45 \le R(3,3,5) \le 57$	[Ex2, KLR], 5.3.(a)
$60 \le R(3,3,6)$	[Rob3, Rob4]
$79 \le R(3,3,7)$	[Ex16]
$98 \le R(3,3,8)$	[ZSL]
$110 \le R(3,3,9)$	[SLZL]
$141 \le R(3,3,10)$	[XX2], 5.3.(c)
$157 \le R(3,3,11)$	[XX2], 5.3.(c)
$181 \le R(3,3,12)$	[XX2], 5.3.(c)
$205 \le R(3,3,13)$	[XX2], 5.3.(c)
$233 \le R(3,3,14)$	5.3.(c)

$55 \leq R(3,4,4) \leq 79$	[KLR], 5.3.(a)
$80 \leq R(3,4,5) \leq 160$	[Ex12], 5.3.(a)
$99 \leq R(3,4,6)$	5.3.(g)
$123 \leq R(3,5,5)$	5.3.(g)
$93 \leq R(3,3,3,4) \leq 153$	[Ex16], 5.3.(a)
$137 \leq R(3,3,3,5)$	[Rob3]
$171 \leq R(3,3,4,4)$	[Ex16]
$561 \leq R(3,3,3,11)$	[XX2]

The best published upper bound $R_4(3) \leq 64$ by Sánchez-Flores [San] improved a very old bound $R_4(3) \leq 65$ obtained by Folkman [Fo] in 1974. In [PR1] it is conjectured that $R(3,3,4) = 30$, and the results in [PR2] eliminate some cases which could give $R(3,3,4) = 31$.

Lower bounds for higher numbers can be obtained by applying various constructive inequalities from the section 5.3 below. For example, the bounds $261 \leq R(3,3,15)$, $241 \leq R(3,3,3,7)$ and $2501 \leq R(5,5,5,5)$ were not published explicitly but are implied by general constructions in 5.3.(c), [Rob3] and [Abb1]. Similarly, other lower bounds for parameters not listed here can be easily derived.

5.2. Multicolor special cases

$R_3(C_4) = 11$	[BS, see also Clap]
$R_3(C_5) = 17$	[YR1]
$R_3(C_6) = 12$	[YR2]
$R_3(C_7) = 25$	[FSS2]
$18 \leq R_4(C_4) \leq 21$	[Ex2] [Ir]
$27 \leq R_5(C_4) \leq 29$	[LaWo1]
$R(C_4, C_4, K_3) = 12$	[Schu]
$R(C_4, K_3, K_3) = 17$	[ExRe]
$13 \leq R(C_3, C_4, C_5)$	[Rao]
$R(K_{1,3}, C_4, K_4) = 16$	[KM2]
$R(K_4 - e, K_4 - e, P_3) = 11$	[Ex7]
$28 \leq R_3(K_4 - e) \leq 30$	[Ex7] [Piw2]
$R(C_4, C_4, C_4, T) = 16$ for $T = P_4$ and $T = K_{1,3}$	[ExRe]
$25 \leq R(C_3, C_3, C_4, C_4)$	[Rao]

All colorings on at least 14 vertices for the parameters (K_3, K_3, K_3), and all colorings for $(K_4 - e, K_4 - e, P_3)$ were found in [Piw2].

5.3. Multicolor results for complete graphs

(a) $R(k_1, \ldots, k_r) \le 2 - r + \sum_{i=1}^{r} R(k_1, \ldots, k_{i-1}, k_i - 1, k_{i+1}, \ldots, k_r)$, implicit in [GG]

Inequality in (a) is strict if the right hand side is even, and at least one of the terms in the summation is even. It is suspected that this upper bound is never tight for $r \ge 3$ and $k_i \ge 3$, except for $r = k_1 = k_2 = k_3 = 3$. However, the only known such case is $R_4(3) \le 62$, for which (a) gives an upper bound of 66.

(b) $R_k(3) \ge 3R_{k-1}(3) + R_{k-3}(3) - 3$ [Chu1]

(c) $R(3, k_1, \ldots, k_r) \ge 4R(k_1 - 1, k_2, \ldots, k_r) - 3$ for $k_1 \ge 5$, $r \ge 2$ and $k_i \ge 3$ [XX2]

(d) $R(3, 3, 3, k_1, \ldots, k_r) \ge 3R(3, 3, k_1, \ldots, k_r) + R(k_1, \ldots, k_r) - 3$ [Rob2]

(e) Bounds for $R_k(3)$ [AH, Fre, Chu2, ChGri, GrRö, Wan]

(f) $R(k_1, \ldots, k_r) \ge S(k_1, \ldots, k_r) + 2$, where $S(k_1, \ldots, k_r)$ is the generalized Schur number [AH, Gi1, Gi2]. In particular, the special case $k_1 = \ldots = k_r = 3$ has been widely studied [Fre, FreSw, Ex10, Rob3].

(g) $R(k_1, \ldots, k_r) \ge L(k_1, \ldots, k_r) + 1$, where $L(k_1, \ldots, k_r)$ is the maximal order of cyclic $(k_1, \ldots, k_r)$-coloring, which can be considered a special case of Schur partitions defining (symmetric) Schur numbers. Many lower bounds for Ramsey numbers were established by cyclic colorings, and thus the following recurrence can be used to derive lower bounds for higher parameters.

$$L(k_1, \ldots, k_r, k_{r+1}) \ge (2k_{r+1} - 3)L(k_1, \ldots, k_r) - k_{r+1} + 2 \text{ [Gi2]}$$

(h) $R_r(m) \ge p + 1$ and $R_r(m+1) \ge r(p+1) + 1$ if there exists a K_m-free cyclotomic r-class association scheme of order p [Mat].

(i) $R_r(m) \ge c_m (2m-3)^r$, and some slight improvements of the exponent r for small values of m [AH, Gi1, Gi2, Song2].

(j) $R_r(pq+1) > (R_r(p+1) - 1)(R_r(q+1) - 1)$ [Abb1]

(k) $R(p_1 q_1 + 1, \ldots, p_r q_r + 1) > (R(p_1 + 1, \ldots, p_r + 1) - 1)(R(q_1 + 1, \ldots, q_r + 1) - 1)$ [Song3]

(l) $R_{r+s}(m) > (R_r(m) - 1)(R_s(m) - 1)$ [Song2]

(m) $R(k_1, k_2, \ldots, k_r) > (R(k_1, \ldots, k_i) - 1)(R(k_{i+1}, \ldots, k_r) - 1)$ in [Song1], see [ExRa, XX2].

(n) $R(k_1, k_2, \ldots, k_r) > (k_1 + 1)(R(k_2 - k_1 + 1, k_3, \ldots, k_r) - 1)$ [Rob5]

(o) Constructions in [XX2] show how to increase the right hand side in (k) and (m) by a suitable choice and composition of colorings for smaller parameters. Several of the best known constructions for specific parameters can be obtained this way.

All lower bounds in (b) through (o) above are constructive. (d) generalizes (b), (k) generalizes both (j) and (m), and (m) generalizes (l). Observe that the construction (k) with $q_1 = \ldots = q_i = 1 = p_{i+1} = \ldots = p_r$ is the same as (m).

5.4. Multicolor results for cycles

- $R_3(C_4) = 11$ [BS] and $R_4(C_4) \geq 18$ [Ex2].
- $R(C_n, C_n, C_n) \leq (4 + o(1))n$, with equality for odd n [Łuc]. It was conjectured in [BoEr, Erd] that for all odd $n \geq 5$ we have $R(C_n, C_n, C_n) = 4n - 3$.
- Formulas for $R(C_n, C_m, C_k)$ and $R(C_n, C_m, C_k, C_l)$ for n sufficiently large [EFRS1].
- $R_k(C_4) \leq k^2 + k + 1$ for all $k \geq 1$, $R_k(C_4) \geq k^2 - k + 2$ for all $k - 1$ which is a prime power [Ir, Chu2, ChGra], and $R_k(C_4) \geq k^2 + 2$ for odd prime power k [LaWo1]. The latter was extended to all prime power k in [Ling, LaMu].
- Bounds for $R_k(C_m)$ [GRS]
- See also section 5.2 above.

5.5. Other general multicolor results

- General bounds for $R_k(G)$ [CH3, Par6]
- Formulas for $R_k(G)$ for G being P_3, $2K_2$ and $K_{1,3}$ for all k, and for P_4 if k is not divisible by 3 [Ir]. Wallis [Wall] showed $R_6(P_4) = 13$, which already implied $R_{3t}(P_4) = 6t + 1$, for all $t \geq 2$. Independently, the case $R_k(P_4)$ for $k \neq 3^m$ was completed by Lindström in [Lind], and later Bierbrauer proved $R_{3^m}(P_4) = 2 \cdot 3^m + 1$ for all $m \geq 1$.
- Bounds on $R_k(K_{s,t})$, in particular for $K_{2,2} = C_4$ and $K_{2,t}$ [ChGra, AFM].
- Bounds on $R_k(G)$ for unicyclic graphs G of odd girth. Some exact values for special graphs G, for $k = 3$ and $k = 4$ [KrRod].
- $tk^2 + 1 \leq R_k(K_{2,t+1}) \leq tk^2 + k + 2$, where upper bound is general, and lower bound holds when both t and k are prime powers [LaMu].
- Monotone paths and cycles [Lef].
- Formulas for $R(P_{n_1}, \cdots, P_{n_k})$, except few cases [FS2].
- Formulas for $R(S_1, \cdots, S_k)$, where S_i's are arbitrary stars [BuRo1].
- Formulas for $R(S_1, \cdots, S_k, K_n)$, where S_i's are arbitrary stars [Jac].
- Formulas for $R(S_1, \cdots, S_k, nP_2)$, where S_i's are arbitrary stars [CL2].
- Formulas for $R(S_1, \cdots, S_k, T)$, where S_i's are stars and T is a tree [ZZ].
- Formulas for $R(pP_3, qP_3, rP_3)$ and $R(pP_4, qP_4, rP_4)$ [Scob].
- Cockayne and Lorimer [CL1] found the exact formula for $R(n_1P_2, \cdots, n_kP_2)$, and later Lorimer [Lor] extended it to a more general case of $R(K_m, n_1P_2, \cdots, n_kP_2)$. Still more general cases of the latter, with multiple copies of the complete graph and forests, were studied in [Stahl, LorSe, LorSo].
- If G is connected and $R(K_k, G) = (k-1)(n(G) - 1) + 1$, in particular if G is any tree, then $R(K_{n_1}, \cdots, K_{n_k}, G) = (R(K_{n_1}, \cdots, K_{n_k}) - 1)(n(G) - 1) + 1$ [BE2]. A generali-

zation for connected $G_1, \ldots, G_n$ in place of G appeared in [Jac].

- Study of $R(S, G_1, \cdots, G_k)$ for large sparse S [EFRS1, Bu3].
- Constructive bound $R(G_1, \ldots, G_{t^{n-1}}) \geq t^n + 1$ for some families of decompositions of K_{t^n} [LaWo1, LaWo2].
- Bounds for trees $R_k(T)$ and forests $R_k(F)$ [EG, GRS, BB, GT, Bra1, Bra2, SwPr].
- See also surveys listed in section 7.

6. Hypergraph Numbers

The only known value of a classical Ramsey number for hypergraphs:

$R(4,4;3) = 13$	[MR1]
more than 200000 critical colorings	

Other hypergraph cases:

$33 \leq R(4,5;3)$	[Ex13]
$63 \leq R(5,5;3)$	[Ea1]
$56 \leq R(4,4,4;3)$	[Ex8]
$34 \leq R(5,5;4)$	[Ex11]
$R(K_4 - t, K_4 - t;3) = 7$	[Ea2]
$R(K_4 - t, K_4;3) = 8$	[Sob, Ex1, MR1]
$14 \leq R(K_4 - t, K_5;3)$	[Ex1]
$13 \leq R(K_4 - t, K_4 - t, K_4 - t;3) \leq 17$	[Ex1] [Ea1]

The computer evaluation of $R(4,4;3)$ in [MR1] consisted of an improvement of the upper bound from 15 to 13, which followed an extensive theoretical study of this number in [Gi4, Is1, Sid1]. Exoo in [Ex1] announced the bounds $R(4,5;3) \geq 30$ and $R(5,5;4) \geq 27$ without presenting the constructions. The bound of $R(4,5;3) \geq 24$ was obtained by Isbell [Is2]. Shastri in [Sha] shows a weak bound $R(5,5;4) \geq 19$ (now 34 in [Ex11]), nevertheless his lemmas and those in [Ka3, Abb2, GRS, HuSo] can be used to derive other lower bounds for higher numbers.

Several lower bound constructions for 3-uniform hypergraphs were presented in [HuSo]. Study of lower bounds on $R(p,q;4)$ can be found in [Song3] and [SYL, Song4] (the latter two papers are almost identical in contents). Most lower bounds in these papers can be easily improved by using the same techniques, but starting with better constructions for small parameters listed above.

Let $H^{(r)}(s,t)$ be the complete r-partite r-uniform hypergraph with $r-2$ parts of size 1, one part of size s, and one part of size t (for example, for $r=2$ it is the same as $K_{s,t}$). For the multicolor numbers, Lazebnik and Mubayi [LaMu] proved that

$$tk^2 - k + 1 \le R_k(H^{(r)}(2,t+1)) \le tk^2 + k + r,$$

where the lower bound holds when both t and k are prime powers. For the general case of $H^{(r)}(s,t)$, more bounds are presented in [LaMu].

Lower bounds on $R_m(k;s)$ are discussed in [DLR, AW]. In [AS], it is shown that for some values of a,b the numbers $R(m,a,b;3)$ are at least exponential in m. General lower bounds for large number of colors were given in an early paper by Hirschfeld [Hir], and some of them were later improved in [AL]. Other theoretical results on hypergraph numbers are gathered in [GrRö, GRS].

7. Cumulative Data and Surveys

7.1. Cumulative data for two colors

- [CH1] $R(G,G)$ for all graphs G without isolates on at most 4 vertices.
- [CH2] $R(G,H)$ for all graphs G and H without isolates on at most 4 vertices.
- [Clan] $R(G,H)$ for all graphs G on at most 4 vertices and H on 5 vertices, except five entries (now all solved).
- [He4] All critical colorings for $R(G,H)$, for isolate-free graphs G and H as in [Clan] above.
- [Bu4] $R(G,G)$ for all graphs G without isolates and with at most 6 edges.
- [He1] $R(G,G)$ for all graphs G without isolates and with at most 7 edges.
- [HM2] $R(G,G)$ for all graphs G on 5 vertices and with 7 or 8 edges.
- [He2] $R(G,H)$ for all graphs G and H on 5 vertices without isolates, except 7 entries (5 still open).
- [HoMe] $R(G,H)$ for $G=K_{1,3}+e$ and $G=K_4-e$ versus all connected graphs H on 6 vertices, except $R(K_4-e,K_6)$. The result $R(K_4-e,K_6)=21$ was claimed by McNamara [McN, unpublished].
- [FRS4] $R(G,T)$ for all connected graphs G on at most 5 vertices and all (except some cases) trees T.
- [FRS1] $R(K_3,G)$ for all connected graphs G on 6 vertices.
- [Jin] $R(K_3,G)$ for all connected graphs G on 7 vertices. Some errors in [Jin] were found by [SchSch1].
- [Brin] $R(K_3,G)$ for all connected graphs G on at most 8 vertices. The numbers for K_3 versus sets of graphs with fixed number of edges, on at most 8 vertices,

were presented in [KM1].

[BBH1] $R(K_3, G)$ for all connected graphs G on 9 vertices. See also [BBH2].

[JR3] $R(C_4, G)$ for all graphs G on at most 6 vertices.

[JR4] $R(C_5, G)$ for all graphs G on at most 6 vertices.

[JR2] $R(C_6, G)$ for all graphs G on at most 5 vertices.

Chvátal and Harary [CH1, CH2] formulated several simple but very useful observations how to discover values of some numbers. All five missing entries in the tables of Clancy [Clan] have been solved. Out of 7 open cases in [He2] 2 have been solved, the bounds for 2 were improved, and the status of the other 3 did not change. Section 4.14 lists 4 of them (labeled [He2]): 1 solved and 3 still open. $R(4,5) = R(G_{19}, G_{23}) = 25$ is the second solved case. The other 2 open entries are K_5 versus K_5 (see section 2) and K_5 versus $K_5 - e$ (see section 3).

7.2. Cumulative data for three colors

[YR3] $R_3(G)$ for all graphs G with at most 4 edges and no isolates.

[YR1] $R_3(G)$ for all graphs G with 5 edges and no isolates, except $K_4 - e$. The case of $R_3(K_4 - e)$ remains open (see section 5.2).

[YY] $R_3(G)$ for all graphs G with 6 edges and no isolates, except 10 cases.

[AKM] $R(F, G, H)$ for most triples of isolate-free graphs with at most 4 vertices. Some of the missing cases completed in [KM2].

7.3. Surveys

[Bu1] A general survey of results in Ramsey graph theory by S. A. Burr (1974)

[Par6] A general survey of results in Ramsey graph theory by T. D. Parsons (1978)

[Har2] Summary of progress by Frank Harary (1981)

[ChGri] A general survey of bounds and values by F. R. K. Chung and C. M. Grinstead (1983)

[JGT] Special volume of *the Journal of Graph Theory* (1983)

[Rob1] Nice textbook-type review of Ramsey graph theory for newcomers (1984)

[Bu6] What can we hope to accomplish in generalized Ramsey Theory ? (1987)

[GrRö] Survey of asymptotic problems by R. L. Graham and V. Rödl (1987)

[GRS] An excellent book by R. L. Graham, B. L. Rothschild and J. H. Spencer, second edition (1990)

[FRS5] Survey of graph goodness results, i.e. conditions for the formula $R(G, H) = (\chi(G) - 1)(n(H) - 1) + s(G)$ (1991)

[Neš] A chapter in *Handbook of Combinatorics* (1996)

[Caro] Survey of zero-sum Ramsey theory (1996)

[Chu4] Among 114 open problems and conjectures of Paul Erdös, presented and commented by F. R. K. Chung, 31 are concerned directly with Ramsey numbers. 216 references are given. (1997)

The surveys by S. A. Burr [Bu1] and T. D. Parsons [Par6] contain extensive chapters on general exact results in graph Ramsey theory. F. Harary presented the state of the theory in 1981 in [Har2], where he also gathered many references including seven to other survey papers. Two decades ago, Chung and Grinstead in their survey paper [ChGri] gave less data than in this note, but included a broad discussion of different methods used in Ramsey computations in the classical case. S. A. Burr, one of the most experienced researchers in Ramsey graph theory, formulated in [Bu6] seven conjectures on Ramsey numbers for sufficiently large and sparse graphs, and reviewed the evidence for them found in the literature. Three of them have been refuted in [Bra3].

For newer extensive presentations see [GRS, GrRö, FRS5, Neš], though these focus on asymptotic theory not on the numbers themselves. Finally, this compilation could not pretend to be complete without mentioning a special 1983 volume of the Journal of Graph Theory [JGT] dedicated entirely to Ramsey theory. Besides a number of research papers, it includes historical notes and presents to us Frank P. Ramsey (1903-1930) as a person.

8. Concluding Remarks

This compilation does not include information on numerous variations of Ramsey numbers, nor related topics, like size Ramsey numbers, zero-sum Ramsey numbers, irredundant Ramsey numbers, induced Ramsey numbers, local Ramsey numbers, connected Ramsey numbers, chromatic Ramsey numbers, avoiding sets of graphs in some colors, coloring graphs other than complete, or the so called Ramsey multiplicities. Interested reader can find such information in the surveys listed in section 7 here.

The author apologizes for any omissions or other errors in reporting results belonging to the scope of this work. Suggestions for any kind of corrections or additions will be greatly appreciated and considered for inclusion in the next year revision of this survey.

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References

We mark the papers containing results obtained with the help of computer algorithms with stars. We identify two categories of such papers: marked with * involving some use of computers, where the results are easily verifiable with some computations, and those marked with **, where cpu intensive algorithms have to be implemented to replicate or verify the results. The first category contains mostly constructions done by algorithms, while the second mostly nonexistence results or claims of complete enumerations of special kinds of graphs.

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