

Small Ramsey Numbers

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ABSTRACT: We present data which, to the best of our knowledge, includes all known nontrivial values and bounds for specific graph, hypergraph and multicolor Ramsey numbers, where the avoided graphs are complete or complete without one edge. Many results pertaining to other more studied cases are also presented. We give references to all cited bounds and values, as well as to previous similar compilations. We do not attempt complete coverage of asymptotic behavior of Ramsey numbers, but concentrate on their specific values.

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1. Scope and Notation

There is a vast literature on Ramsey type problems starting in 1930 with the original paper of Ramsey [Ram]. Graham, Rothschild and Spencer in their book [GRS] present an exciting development of Ramsey Theory. The subject has grown amazingly, in particular with regard to asymptotic bounds for various types of Ramsey numbers (see the survey paper [GrRö]), but the progress on evaluating the basic numbers themselves has been very unsatisfactory for a long time. In the last decade, however, considerable progress has been obtained in this area, mostly by employing computer algorithms. The few known exact values and several bounds for different numbers are scattered among many technical papers. This compilation is a fast source of references for the best results known for specific numbers. It is not supposed to serve as a source of definitions or theorems, but these can be easily accessed via the references gathered here.

Ramsey Theory studies conditions when a combinatorial object contains necessarily some smaller given objects. The role of Ramsey numbers is to quantify some of the general existential theorems in Ramsey Theory.

Let G_1, G_2, \dots, G_m be graphs or s -uniform hypergraphs (s is the number of vertices in each edge). $R(G_1, G_2, \dots, G_m; s)$ denotes the m -color **Ramsey number** for s -uniform graphs/hypergraphs, avoiding G_i in color i for $1 \leq i \leq m$. It is defined as the least integer n such that, in any coloring with m colors of the s -subsets of a set of n elements, for some i the s -subsets of color i contain a sub-(hyper)graph isomorphic to G_i (not necessarily induced). The value of $R(G_1, G_2, \dots, G_m; s)$ is fixed under permutations of the first m arguments.

If $s=2$ (standard graphs) then s can be omitted. If G_i is a complete graph K_k , then we can write k instead of G_i , and if $G_i = G$ for all i we can use the abbreviation $R_m(G)$ (or $R_m(G; s)$). For $s=2$, $K_k - e$ denotes a K_k without one edge, and for $s=3$, $K_k - t$ denotes a K_k without one triangle (hyperedge). P_i is a **path** on i vertices, C_i is a **cycle** of length i , and W_i is a **wheel** with $i-1$ spokes, i.e. a graph formed by some vertex x , connected to all vertices of some cycle C_{i-1} . $K_{n,m}$ is a complete n by m bipartite graph, in particular $K_{1,n}$ is a **star** graph. The **book** graph $B_i = K_2 + \bar{K}_i = K_1 + K_{1,i}$ has $i+2$ vertices, and can be seen as i triangular pages attached to a single edge. The **fan** graph F_n is defined by $F_n = K_1 + nK_2$. For a graph G , $n(G)$ and $e(G)$ denote the number of vertices and edges, respectively. Finally, let $\chi(G)$ be the chromatic number of G , and let nG denote n disjoint copies of G .

Section 2 contains the data for the classical two color Ramsey numbers $R(k, l)$ for complete graphs, and section 3 for the two color case when the avoided graphs are complete or have the form $K_k - e$, but not both are complete. Section 4 lists the most studied two color cases for other graphs. The multicolor and hypergraph cases are gathered in sections 5 and 6, respectively. If some new bound has been not yet published, we also give a reference to the best published previous result, if any. Finally, section 7 gives pointers to cumulative data and to some previous surveys, especially those containing data not subsumed by this compilation.

2. Classical Two Color Ramsey Numbers

l	3	4	5	6	7	8	9	10	11	12	13	14	15
k													
3	6	9	14	18	23	28	36	40 43	46 51	52 59	59 69	66 78	73 88
4		18	25	35 41	49 61	55 84	69 115	80 149	96 191	128 238	131 291	136 349	145 417
5			43 49	58 87	80 143	95 216	121 316	141 442	153	181	193	221	242
6				102 165	109 298	122 495	153 780	167 1171	203	230	242	284	374
7					205 540	1031	1713	2826		312			
8						282 1870	3583	6090					
9							565 6588	12677					
10								798 23581					

Table I. Known nontrivial values and bounds for two color Ramsey numbers $R(k, l) = R(k, l ; 2)$.

l	4	5	6	7	8	9	10	11	12	13	14	15
k												
3	GG	GG	Kéry	Ka2 GY	GR MZ	Ka2 GR	Ex5 RK2	Ka2 RK2	Ex12 Les	Piw2 RK2	Ex8 RK2	WW Les
4	GG	Ka1 MR4	Ex9 MR5	Ex3 Mac	Ex12 Mac	RK1 Mac	Piw2 Mac	Piw2 Spe	SLL2 Spe	Ea3 Spe	Ea3 Spe	Ea3 Spe
5		Ex4 MR5	Ex9 HZ1	CET Spe	Piw2 Spe	Haa Mac	Ex12 Mac	Ex12	Ex12	Ex12	Ex12	SLL3
6			Ka1 Mac	Ex12 Mac	Ex12 Mac	Ex12 Mac	Ex12 Mac	Ea3	SLL3	SLZ2	SLL3	SLL3
7				She1 Mac	Mac	HZ1	Mac		SLL4			
8					BR Mac	Ea1	HZ1					
9						She1 ShZ1	Ea1					
10							She1 ShZ1					

References for Table I.

We split the data into the table of values and a table with corresponding references for the Table I. Known exact values appear as centered entries, lower bounds as top entries, and upper bounds as bottom entries.

The task of proving $R(3,3) \leq 6$ was the second problem in Part I of the William Lowell Putnam Mathematical Competition held in March 1953 [Bush].

The construction by Mathon [Mat], using data obtained by Shearer [She1], gives the following lower bounds for higher diagonal numbers: $R(11,11) \geq 1597$, $R(13,13) \geq 2557$, $R(14,14) \geq 2989$, $R(15,15) \geq 5485$, and $R(16,16) \geq 5605$. The same approach does not improve on a trivial bound $R(12,12) \geq 1597 + 11$.

All the critical graphs for the numbers $R(k,l)$ (graphs on $R(k,l) - 1$ vertices without K_k and without K_l in the complement) are known for $k = 3$ and $l = 3, 4, 5$ [Kéry], 6 [Ka2], 7 [RK3, MZ], and there are 1, 3, 1, 7 and 191 of them, respectively. All $(3,k)$ -graphs, for $k \leq 6$, were enumerated in [RK3], and all $(4,4)$ -graphs in [MR2]. There exists a unique critical graph for $R(4,4)$ [Ka2]. There are 4 such graphs known for $R(3,8)$ [RK2], 1 for $R(3,9)$ [Ka2] and 350904 for $R(4,5)$ [MR4], but there might be more of them. In [MR5] evidence is given for the conjecture that $R(5,5) = 43$ and that there exist 656 critical graphs on 42 vertices.

Most of the lower bounds for $R(4,n)$ presented by Bannani in [Ba], and two other results $R(3,13) \geq 58$ [Ka2] and $R(5,8) \geq 94$ [RK1], were improved in [Piw1, Piw2] by Piwakowski. The bound $R(3,13) \geq 60$ [XZ] cited in the 1995 version of this survey was shown to be incorrect in [Piw2]. The previously best published upper bound for $(k,l) = (5,6)$ of 94 can be found in [Wa2]. The graphs constructed by Exoo in [Ex12], and some others, are available electronically from <http://isu.indstate.edu/ge/RAMSEY>.

By taking a disjoint union of two critical graphs one can easily see that $R(k,p) \geq s$ and $R(k,q) \geq t$ imply $R(k,p+q-1) \geq s+t-1$. For example, this gives trivially a lower bound $R(4,15) \geq 145$ with $p = 4$, $q = 12$. Higher lower bounds implied this way are not shown. Some upper bounds implied by $R(k,l) \leq R(k-1,l) + R(k,l-1)$, or by its slight improvement with strict inequality when both terms on the right hand side are even, are marked [Ea1]. There are obvious generalizations of these inequalities for graphs other than complete.

The bound $R(6,6) \leq 166$ is an immediate consequence of theorem 1 in [Wa1] and $R(4,6) \leq 41$, in this case the best published bound of 169 is due to Giraud [Gi4]. T. Spencer [Spe], Mackey [Mac], and Huang and Zhang [HZ1], using the bounds for minimum and maximum number of edges in $(4,5)$ Ramsey graphs listed in [MR3, MR5], were able to establish new upper bounds for several higher Ramsey numbers, improving all the previous longstanding results of Giraud [Gi2, Gi4, Gi5]. We have recomputed the bounds marked [HZ1] using the method from the paper [HZ1], because the bounds there relied on an overly optimistic personal communication from Spencer. Further refinements of this method are studied in [HZ2, HY].

For a more in depth study of triangle-free graphs in relation to the case of $R(3,k)$, for which considerable progress has been obtained in recent years, see also [AKS, Alon2, BBH1, BBH2, CPR, FL, Fra1, Fra2, Gri, Loc, KM1, RK3, RK4, She2, Stat, Yu1]. In 1995, Kim [Kim] obtained a breakthrough by proving that $R(3,k)$ has order of magnitude exactly $\Theta(k^2/\log k)$. Good asymptotic bounds for $R(k,k)$ can be found, for example, in [Chu3, McS] (lower bound) and [Tho] (upper bound), and for many other asymptotic bounds in the general case of $R(k,l)$ consult [GRS, GrRö, AP].

All the lower bounds for higher numbers listed in Table II were obtained by construction of cyclic graphs.

k	l	15	16	17	18	19	20	21	22	23
3		73 WW	79 WW	92 WWY1	98 WWY1	106 WWY1	109 WWY1	122 WWY1	125 WWY1	136 WWY1
4		145 Ea3		164 SL	182 SL	198 LSZL	230 SLZL	242 SLZL	282 SL	
5		242 SLL3		282 SLZL		338 SLZL		374 SLZ1	422 LSZL	434 LSZL
6		374 SLL3	434 SLL3	548 SLL3	614 SLL3	710 SLL3	878 SLL3			
7				578 LSZL	618 LSZL	758 SLL3				
8		618 LSZL		678 LSZL	740 SLL3	860 SLL3	948 LSZL			

Table II. Known nontrivial lower bounds for higher two color Ramsey numbers $R(k, l)$, with references.

$R(3, 26) \geq 150$, $R(3, 29) \geq 174$ and $R(3, 32) \geq 212$ are established in [SLL1], [SLL4] and [LSZL], respectively. Yu [Yu2] constructed a special class of triangle-free cyclic graphs establishing several lower bounds for $R(3, k)$, for $k \geq 61$. Only two of these bounds, $R(3, 61) \geq 479$ and $R(3, 103) \geq 955$, cannot be easily improved by the inequality $R(3, 4k + 1) \geq 6R(3, k + 1) - 5$ from [CCD] and data from tables I and II. Finally, $R(5, 24) \geq 488$ and $R(9, 21) \geq 1278$ are given in [SLL4], and $R(10, 16) \geq 1052$ in [LSZL].

3. Two Colors - Dropping One Edge from Complete Graph

$G \quad H$	K_3-e	K_4-e	K_5-e	K_6-e	K_7-e	K_8-e	K_9-e	$K_{10}-e$	$K_{11}-e$
K_3-e	3	5	7	9	11	13	15	17	19
K_3	5	7	11	17	21	25	31	37 38	42 47
K_4-e	5	10	13	17	28	29 38	34	41	
K_4	7	11	19	27 36	37 52				
K_5-e	7	13	22	31 39	40 66				
K_5	9	16	30 34	43 67	112				
K_6-e	9	17	31 39	45 70	59 135				
K_6	11	21	37 55	119	205				
K_7-e	11	28	40 66	59 135	251				
K_7	13	28 34	51 88	204					

Table III. Two types of Ramsey numbers $R(G, H)$, includes all known nontrivial values.

The exact values in Table III above involving K_3-e are trivial, since $R(K_3-e, K_k) = R(K_3-e, K_{k+1}-e) = 2k-1$, for all $k \geq 2$. Other not shown bounds can be easily obtained by using Table I, an obvious generalization of inequality $R(k, l) \leq R(k-1, l) + R(k, l-1)$, and by monotonicity of Ramsey numbers, in this case $R(K_{k-1}, G) \leq R(K_k-e, G) \leq R(K_k, G)$.

For the following numbers it was established that the critical graphs are unique: $R(K_3, K_l-e)$ for $l=3$ [Tr], 6 and 7 [Ra1], $R(K_4-e, K_4-e)$ [FRS2], $R(K_5-e, K_5-e)$ [Ra3] and $R(K_4-e, K_7-e)$ [McR]. Wang, Wang and Yan in [WWY2] constructed cyclic graphs establishing $R(K_3, K_{13}-e) \geq 54$ and $R(K_3, K_{15}-e) \geq 69$. The upper bounds in [HZ2] were obtained by a reasoning generalizing the bounds for classical numbers in [HZ1].

G	H	K_4-e	K_5-e	K_6-e	K_7-e	K_8-e	K_9-e	$K_{10}-e$	$K_{11}-e$
K_3		CH2	Clan	FRS1	GH	Ra1	Ra1	MR6 MR6	WWY2 MR6
K_4-e		CH1	FRS2	McR	McR	Ea1 HZ2	Ex14	Ex14	
K_4		CH2	EHM1	Ex11 Ea1	Ex14 HZ2				
K_5-e		FRS2	CEHMS	Ex14 Ea1	Ex14 HZ2				
K_5		BH	Ex8 Ex8	Ea1 HZ2	HZ2				
K_6-e		McR	Ex14 Ea1	Ex14 HZ2	Ex14 HZ2				
K_6		McN	Ex14 Ea1	ShZ2	ShZ2				
K_7-e		McR	Ex14 HZ2	Ex14 HZ2	ShZ1				
K_7		Ea1 Ea1	Ex14 ShZ2	ShZ2					

References for Table III.

4. General Graph Numbers in Two Colors

This section includes data with respect to general graph results. We tried to include all nontrivial values and identities regarding exact results (or references to them), but only those out of general bounds and other results which, in our opinion, have a direct connection to the evaluation of specific numbers. If some small value cannot be found below, it may be covered by the cumulative data gathered in section 7, or be a special case of a general result listed in this section. Note that $B_1 = F_1 = C_3 = W_3 = K_3$, $B_2 = K_4 - e$, $P_3 = K_3 - e$, $W_4 = K_4$ and $C_4 = K_{2,2}$ imply other identities not mentioned explicitly.

Paths:

$$R(P_n, P_m) = n + \lfloor m/2 \rfloor - 1 \quad \text{for all } n \geq m \geq 2 \quad [\text{GeGy}]$$

Cycles:

$$R(C_3, C_3) = 6 \quad [\text{GG}]$$

$$R(C_4, C_4) = 6 \quad [\text{CH1}]$$

Result obtained independently in [Ros] and [FS1], new simple proof in [KR]:

$$R(C_n, C_m) = \begin{cases} 2n - 1 & \text{for } 3 \leq m \leq n, m \text{ odd}, (n, m) \neq (3, 3) \\ n - 1 + m/2 & \text{for } 4 \leq m \leq n, m \text{ and } n \text{ even}, (n, m) \neq (4, 4) \\ \max\{n - 1 + m/2, 2m - 1\} & \text{for } 4 \leq m < n, m \text{ even and } n \text{ odd} \end{cases}$$

Unions of cycles, $R(nC_p, mC_q)$, [MS, Den]

$$R(nC_3, mC_3) = 3n + 2m \quad \text{for } n \geq m \geq 1, n \geq 2 \quad [\text{BES}]$$

$$R(nC_4, mC_4) = 2n + 4m - 1 \quad \text{for } m \geq n \geq 1, (n, m) \neq (1, 1) \quad [\text{LiWa}]$$

Wheels:

$$R(W_3, W_5) = 11 \quad [\text{Clan}]$$

$$R(W_3, W_n) = 2n - 1 \quad \text{for all } n \geq 6 \quad [\text{BE2}]$$

All critical colorings for $R(W_3, W_n)$ for all $n \geq 3$ [RaJi]

$$R(W_4, W_5) = 17 \quad [\text{He3}]$$

$$R(W_5, W_5) = 15 \quad [\text{HM2, He2}]$$

$$R(W_4, W_6) = 19, R(W_5, W_6) = 17 \quad \text{and } R(W_6, W_6) = 17,$$

and all critical colorings (2, 1 and 2) for these numbers [FM]

Books:

$$R(B_1, B_n) = 2n + 3 \text{ for all } n > 1 \text{ [RS1]}$$

$$R(B_3, B_3) = 14 \text{ [RS1, HM2]}$$

$$R(B_2, B_5) = 16, R(B_3, B_5) = 17, R(B_5, B_5) = 21,$$

$$R(B_4, B_4) = 18, R(B_4, B_6) = 22, R(B_6, B_6) = 26,$$

in general $R(B_n, B_n) = 4n + 2$ for $4n + 1$ a prime power,

and some other general equalities and bounds for $R(B_n, B_m)$ [RS1].

Complete bipartite graphs:

$$R(K_{2,3}, K_{2,3}) = 10 \text{ [Bu4]}$$

$$R(K_{2,3}, K_{2,4}) = 12 \text{ [ER]}$$

$$R(K_{2,3}, K_{1,7}) = 13 \text{ [Par4]}$$

$$R(K_{2,3}, K_{3,3}) = 13 \text{ and } R(K_{3,3}, K_{3,3}) = 18 \text{ [HM3]}$$

$$R(K_{2,2}, K_{2,8}) = 15 \text{ and } R(K_{2,2}, K_{2,11}) = 18 \text{ [HM]}$$

$$R(K_{2,2}, K_{1,15}) = 20 \text{ [La2]}$$

$$R(nK_{1,3}, mK_{1,3}) = 4n + m - 1 \text{ for } n \geq m \geq 1, n \geq 2 \text{ [BES]}$$

Asymptotics for $K_{2,m}$ versus K_n [CLRZ]

$R(K_{1,n}, K_{1,m}) = n + m - \varepsilon$, where $\varepsilon = 1$ if both n and m are even and $\varepsilon = 0$ otherwise [Har1]. It is also a special case of multicolor numbers for stars obtained in [BuRo1].

$R(K_{2,n}, K_{2,n}) \leq 4n - 2$ for all $n \geq 2$, exact values 6, 10, 14, 18, 21, 26, 30, 33, 38, 42, 46, 50, 54, 57 and 62 of $R(K_{2,n}, K_{2,n})$ for $2 \leq n \leq 16$, respectively. The first open case is $65 \leq R(K_{2,17}, K_{2,17}) \leq 66$ [EHM2].

Triangle versus other graphs:

$$R(3, k) = \Theta(k^2 / \log k) \text{ [Kim]}$$

Explicit construction for $R(3, 4k + 1) \geq 6R(3, k + 1) - 5$, for all $k \geq 1$ [CCD]

Explicit triangle-free graphs with independence k on $\Omega(k^{3/2})$ vertices [Alon2, CPR]

$$R(K_3, K_7 - 2P_2) = R(K_3, K_7 - 3P_2) = 18 \text{ [SchSch2]}$$

$$R(K_3, K_3 + \bar{K}_m) = R(K_3, K_3 + \bar{C}_m) = 2m + 5 \text{ for } m \geq 212 \text{ [Zhou1]}$$

$R(K_3, G) = 2n(G) - 1$ for any connected G on at least 4 vertices and with at most $(17n(G) + 1)/15$ edges, in particular for $G = P_i$ and $G = C_i$, for all $i \geq 4$ [BEFRS1]

$R(K_3, G) \leq 2e(G) + 1$ for any graph G without isolated vertices [Sid3, GK]

$R(K_3, G) \leq n(G) + e(G)$ for all G , a conjecture [Sid2]

$R(K_3, K_n)$, see section 2

$R(K_3, K_n - e)$, see section 3

$R(K_3, G)$ for all connected G up to 9 vertices, see section 7

See [AKS, BBH1, BBH2, FL, Fra1, Fra2, Gri, Loc, KM1, RK3, RK4, She2, Stat, Yu1]

Paths versus other graphs:

Paths versus stars [Par2, BEFRS2]
 Paths versus trees [FS4]
 Paths versus books [RS2]
 Paths versus cycles [FLPS, BEFRS2]
 Paths versus K_n [Par1]
 Paths versus $K_{n,m}$ [Häg]
 Paths versus W_4 and W_5 [SuBa]
 Paths and cycles versus trees [FSS1]
 Sparse graphs versus paths and cycles [BEFRS2]
 Graphs with long tails [Bu2, BG]

Cycles versus complete graphs:

$R(C_4, K_3) = R(C_4, C_3) = 7$ [CS]
 $R(C_4, K_4) = 10$ [CH2]
 $R(C_4, K_5) = 14$ [GG] [He2]
 $R(C_4, K_6) = 18$ [Ex2] [RoJa1]
 $R(C_4, K_7) = 22$ [RT] [JR1]
 $R(C_4, K_8) = 26$ [RT]
 $30 \leq R(C_4, K_9)$, $34 \leq R(C_4, K_{10})$ [RT]
 C_4 versus K_n [CLRZ]

$R(C_5, K_3) = R(C_5, C_3) = 9$ [CS]
 $R(C_5, K_4) = 13$ [He2, JR2]
 $R(C_5, K_5) = 17$ [He2, JR2]
 $R(C_5, K_6) = 21$ [JR6]
 $R(C_5, K_7) = 25$ [Schi2]

$R(C_6, K_4) = 16$ [JR4]
 $R(C_6, K_5) = 21$ [JR4]
 Cycles versus K_n [BoEr, FS4, EFRS2, CLRZ]

$R(C_n, K_m) = (n-1)(m-1) + 1$, for $n \geq m^2 - 2$ [BoEr], for $n > 3 = m$ [FS1], for $n \geq 4 = m$ [YHZ], for $n \geq 5 = m$ [BJYHRZ], for $n \geq 6 = m$, and for $n \geq 7 = m$ with $n \geq m(m-2)$ [Schi1]. Since 1976, it was conjectured to be true for all $n \geq m \geq 3$, except $n = m = 3$ [FS4, EFRS2].

Cycles versus other graphs:

C_4 versus stars [Par3, Chen]
 C_4 versus trees [EFRS4, Bu6, Chen]
 C_4 versus $K_{m,n}$ [HM]
 C_4 versus all graphs on six vertices [JR3]

$R(C_4, B_n) = 7, 9, 11, 12, 13$ and 16 , for $2 \leq n \leq 7$, respectively [FRS6]

$R(C_4, B_8) = 17$ [Tse] (not 16 as claimed in [FRS6])

$R(C_4, G) \leq 2q + 1$ for any isolate-free graph G with q edges [RoJa2]

$R(C_4, G) \leq p + q - 1$ for any connected graph G on p vertices and q edges [RoJa2]

$R(C_5, W_6) = 13$ [ChvS]

$R(C_5, K_6 - e) = 17$ [JR2]

C_5 versus all graphs on six vertices [JR2]

$R(C_6, K_5 - e) = 17$ [JR4]

C_6 versus all graphs on five vertices [JR4]

$R(C_n, G) \leq 2q + \lfloor n/2 \rfloor - 1$, for $3 \leq n \leq 6$, for any isolate-free graph G with $q > 3$ edges.

It is conjectured that it also holds for other n [JR5, RoJa2].

Cycles versus paths [FLPS, BEFRS2]

Cycles versus stars [La1, Clark, see Par5]

Cycles versus trees [FSS1]

Cycles versus books [FRS6, FRS7, Zhou1]

Cycles versus wheels [Zhou2]

See also bipartite graphs for $K_{2,2} = C_4$

Stars versus other graphs:

Stars versus C_4 [Chen]

Stars versus W_4 and W_5 [SuBa]

Stars versus paths [Par2, BEFRS2]

Stars versus cycles [La1, Clark, see Par5]

Stars versus books [CRSPS, RS2]

Stars versus trees [Bu1, GV, ZZ]

Stars versus stripes [CL, Lor]

Stars versus $K_{2,n}$ [Par4]

Stars versus $K_{n,m}$ [Stev, Par3]

Stars versus $K_n - tK_2$ [Hua1, Hua2]

Stars versus $2K_2$ [MO]

Union of two stars [Gro2]

Books versus other graphs:

$R(B_3, K_4) = 14$ [He3]

$20 \leq R(B_3, K_5) \leq 22$ [He2]

Books versus paths [RS2]

Books versus trees [EFRS7]

Books versus stars [CRSPS, RS2]

Books versus cycles [FRS6, FRS7, Zhou1]

Books versus wheels [Zhou3]

Books versus $K_2 + C_n$ [Zhou3]
 Books and $(K_1 + tree)$ versus K_n [LR1]

Wheels versus other graphs:

W_4 and W_5 versus stars and paths [SuBa]
 $R(W_5, K_5 - e) = 17$ [He2][YH]
 $27 \leq R(W_5, K_5) \leq 29$ [He2]
 $R(W_6, C_5) = 13$ [ChvS]
 Wheels versus cycles [Zhou2]
 Wheels versus books [Zhou3]

Trees and Forests:

Trees, forests [EG, GRS, FSS1, GV, CsKo]
 Trees versus C_4 [EFRS4, Bu6, Chen]
 Trees versus paths [FS4]
 Trees versus paths and cycles [FSS1]
 Trees versus books [EFRS7]
 Trees versus stars [Bu1, GV, ZZ]
 Trees versus K_n [Chv]
 Trees versus $K_n + \bar{K}_m$ [RS2, FSR]
 Trees versus bipartite graphs [EFRS6]
 Trees versus almost complete graphs [GJ2]
 Trees versus small ($n(G) \leq 5$) connected G [FRS4]
 Linear forests, forests [BuRo2, FS3, CsKo]
 Forests versus K_n [Stahl]
 Forests versus almost complete graphs [CGP]

Mixed special cases:

$R(C_5 + e, K_5) = 17$ [He5]
 $R(W_5, K_5 - e) = 17$ [He2][YH]
 $20 \leq R(B_3, K_5) \leq 22$ [He2]
 $27 \leq R(W_5, K_5) \leq 29$ [He2]
 $25 \leq R(K_5 - P_3, K_5) \leq 28$ [He2]
 $26 \leq R(K_{2,2,2}, K_{2,2,2})$ ($K_{2,2,2}$ is an octahedron) [Ex8]

Mixed general cases:

Unicyclic graphs [Gro1, Köh]
 $K_{2,m}$ and C_{2m} versus K_n [CLRZ]
 $K_{2,n}$ versus any graph [RoJa2]
 nK_3 versus mK_3 [BES]
 nK_3 versus mK_4 [LorMu]

$R(nK_4, nK_4) = 7n + 4$ for large n [Bu7]
 Stripes [CL, Lor]
 Union of two stars [Gro2]
 Double stars* [GHK]
 Graphs with bridge versus K_n [Li]
 Fans $F_n = K_1 + nK_2$ versus K_m [LR2]
 $R(F_1, F_n) = R(K_3, F_n) = 4n + 1$ and bounds for $R(F_m, F_n)$ [GGS]
 Multipartite complete graphs [BEFRS3, EFRS4, FRS3, Stev]
 Multipartite complete graphs versus trees [EFRS8, BEFRSGJ]
 Disconnected graphs versus any graph [GJ1]
 Graphs with long tails [Bu2, BG]
 Brooms+ [EFRS3]

Other general results:

- [Wa1] $R(k, k) \leq 4R(k, k-2) + 2$.
- [Chv] $R(K_n, T_m) = (n-1)(m-1) + 1$ for any tree T on m vertices.
- [CH2] $R(G, H) \geq (\chi(G) - 1)(c(H) - 1) + 1$, where $\chi(G)$ is the chromatic number of G , and $c(H)$ is the size of the largest connected component of H .
- [AP] Constructive asymptotic lower bounds for $R(k, l)$.
- [BE1] $R(G, G) \geq \lfloor (4n(G) - 1)/3 \rfloor$ for any connected G .
- [BE2] Graphs yielding $R(K_n, G) = (n-1)(n(G) - 1) + 1$ and related results (see also [EFRS5]).
- [BES] Study of Ramsey numbers for multiple copies of graphs. See also [Bu1, LorMu].
- [Zeng] $R(nK_3, nG)$ for all isolate-free graphs G on 4 vertices.
- [Bu7] Study of Ramsey numbers for large disjoint unions of graphs, in particular $R(nK_k, nK_l) = n(k+l-1) + R(K_{k-1}, K_{l-1}) - 2$, for n large enough. See also [Bu8].
- [Bu2] Graphs H yielding $R(G, H) = (\chi(G) - 1)(n(H) - 1) + s(G)$, where $s(G)$ is a chromatic surplus of G , defined as the minimum number of vertices in some color class under all vertex colorings in $\chi(G)$ colors (such H 's are called G -good). This idea, initiated in [Bu2], is a basis of a number of exact results for $R(G, H)$ for large and sparse graphs H [BG, BEFRS2, BEFRS4, Bu5, FS, EFRS4, FRS3, BEFRSGJ, BF, LR4]. A survey of this area appeared in [FRS5].

* - A double star is a union of two stars with their centers joined by an edge.

+ - A broom is a star with a path attached to its center.

- [BEFS] Bounds for the difference between consecutive Ramsey numbers.
- [Par3] Relations between some Ramsey graphs and block designs. See also [Par4].
- [Bra3] $R(G, H) > h(G, d)n(H)$ for all nonbipartite G and almost every d -regular H , for some h unbounded in d .
- [CSRT] $R(G, G) \leq c_d n(G)$ for all G , where constant c_d depends only on the maximum degree d in G . The constant was improved in [GRR].
- [ChenS] $R(G, G) \leq c_d n$ for all d -arrangeable graphs G on n vertices. The constant was improved in [Eaton].
- [EFRS9] Study of graphs G for which there exists a constant C such that for all H with no isolates $R(G, H) \leq Ce(H)$.
- [LRS] $R(G, G) < 6n$ for all n -vertex graphs G , in which no two vertices of degree at least 3 are adjacent. This improves the result $R(G, G) \leq 12n$ in [Alon1].
- [RoJa2] $R(K_{2,k}, G) \leq kq + 1$, for $k \geq 2$, for isolate-free graphs G with $q \geq 2$ edges.
- [FSS1] Discussion of the conjecture that $R(T_1, T_2) \leq n(T_1) + n(T_2) - 2$ holds for all trees T_1, T_2 .
- [FM] $R(W_6, W_6) = 17$ and $\chi(W_6) = 4$. This gives a counterexample $G = W_6$ to the Erdős conjecture (see [GRS]) $R(G, G) \geq R(K_{\chi(G)}, K_{\chi(G)})$.
- [LR3] Bounds on $R(H + \bar{K}_n, K_n)$ for general H .
- [-] Special cases of multicolor results listed in section 5.
- [-] See also surveys listed in section 7.

5. Multicolor Graph Numbers

The only known value of a multicolor classical Ramsey number:

$$R_3(3) = R(3,3,3) = R(3,3,3; 2) = 17 \quad \text{[GG]}$$

2 critical colorings [KS, LayMa]

Bounds for multicolor classical numbers:

$$51 \leq R_4(3) = R(3,3,3,3) \leq 62 \quad \text{[Chu1] [FKR]}$$

$$162 \leq R_5(3) \leq 307 \quad \text{[Ex10] [FKR, Ea1]}$$

$$538 \leq R_6(3) \leq 1838 \quad \text{[FreSw] [Ea1]}$$

$$1682 \leq R_7(3) \leq 12861 \quad \text{[FreSw] [Ea1]}$$

$128 \leq R(4,4,4) \leq 236$	[HI] [Ea2]
$458 \leq R(4,4,4,4)$	[Mat]
$942 \leq R(4,4,4,4,4)$	[Mat]
$385 \leq R(5,5,5)$	[Mat]
$1833 \leq R(5,5,5,5)$	[Mat]
$4711 \leq R(5,5,5,5,5)$	[Mat]
$1070 \leq R(6,6,6)$	[Mat]
$3433 \leq R(6,6,6,6)$	[Mat]
$3211 \leq R(7,7,7)$	[Mat]
$12841 \leq R(7,7,7,7)$	[Mat]
$30 \leq R(3,3,4) \leq 31$	[Ka2] [PR1, PR2]
$45 \leq R(3,3,5) \leq 57$	[Ex2, KLR] [Ea2]
$60 \leq R(3,3,6)$	[Rob3, Rob4]
$74 \leq R(3,3,7)$	[WSL]
$110 \leq R(3,3,9)$	[SLZL]
$141 \leq R(3,3,11)$	[Rob3]
$174 \leq R(3,3,12)$	[WSL]
$194 \leq R(3,3,13)$	[WSL]
$55 \leq R(3,4,4) \leq 79$	[KLR] [Ea2]
$80 \leq R(3,4,5) \leq 161$	[Ex12] [Ea2]
$91 \leq R(3,3,3,4) \leq 153$	[Rob2] [Ea2]
$433 \leq R(3,3,3,11)$	implicit in [Rob2]
$144 \leq R(3,3,4,4)$	[Gi1+Ka2+Ea1]

The best published upper bound $R_4(3) \leq 64$ by Sánchez-Flores [San] improved a very old bound $R_4(3) \leq 65$ obtained by Folkman [Fo] in 1974. In [PR1] it is conjectured that $R(3,3,4) = 30$, and the results in [PR2] eliminate some cases which could give $R(3,3,4) = 31$. The upper bounds marked [Ea1] and [Ea2] are easy implications of basic inequalities. Lower bounds for higher numbers of the form $R(3, k, l)$ and $R(3, 3, 3, k_1, \dots, k_r)$ can be computed using inequalities from [Rob2, Rob3] (see the following subsections), for example $433 \leq R(3,3,3,11)$.

Giraud [Gi1] presented a reasoning, which produces lower bounds for $(k+1)$ -color numbers provided certain k -color cyclic graphs are given. As an example, by using this method, one can obtain $144 \leq R(3,3,4,4)$ from a cyclic $(3,3,4)$ -coloring on 29 points.

Multicolor special cases:

$R_3(C_4) = 11$	[BS, see also Clap]
$R_3(C_5) = 17$	[YR1]
$R_3(C_6) = 12$	[YR2]
$R_3(C_7) = 25$	[FSS2]
$18 \leq R_4(C_4) \leq 21$	[Ex2] [Ir]
$27 \leq R_5(C_4) \leq 29$	[LaWo1]
$R(C_4, C_4, K_3) = 12$	[Schu]
$R(C_4, K_3, K_3) = 17$	[ER]
$13 \leq R(C_3, C_4, C_5)$	[Rao]
$R(K_{1,3}, C_4, K_4) = 16$	[KM2]
$R(K_4 - e, K_4 - e, P_3) = 11$	[Ex7]
$28 \leq R_3(K_4 - e) \leq 30$	[Ex7] [Piw3]
$R(C_4, C_4, C_4, T) = 16$ for $T = P_4$ and $T = K_{1,3}$	[ER]
$25 \leq R(C_3, C_3, C_4, C_4)$	[Rao]

All colorings on at least 14 vertices for (K_3, K_3, K_3) , and all colorings for $(K_4 - e, K_4 - e, P_3)$ were found in [Piw3].

Multicolor results for complete graphs and cycles:

- General bounds for $R_k(G)$ [CH3].
- Bounds for $R_k(3)$ [Fre, Chu1, Chu2, ChGri, GrRö, Wan].
- $R(3, 3, 3, k_1, \dots, k_r) \geq 3R(3, 3, k_1, \dots, k_r) + R(k_1, \dots, k_r) - 3$ [Rob2].
- $R(3, k, l) \geq 4R(k, l - 2) - 3$ [Rob3].
- $R_k(4) \geq 3 \cdot 5^{k-1} + 1$ and $R_k(5) \geq 4 \cdot 6.48^{k-1} + 1$ for all k , and other general lower bounds on $R_k(n)$ [Song1].
- $R(C_n, C_n, C_n) \leq (4 + o(1))n$, with equality for odd n [Łuc]. It was conjectured in [BoEr] that for all odd n we have $R(C_n, C_n, C_n) = 4n - 3$.
- Formulas for $R(C_n, C_m, C_k)$ and $R(C_n, C_m, C_k, C_l)$ for n sufficiently large [EFRS1].
- $R_k(C_4) \leq k^2 + k + 1$ for all $k \geq 1$, $R_k(C_4) \geq k^2 - k + 2$ for all $k - 1$ which is a prime power [Ir, Chu2, ChGra], and $R_k(C_4) \geq k^2 + 2$ for odd prime power k [LaWo1]. The latter was extended to all prime power k in [Ling, LaMu]. For small k some specific constructions were published: $R_3(C_4) = 11$ [BS] and $R_4(C_4) \geq 18$ [Ex2].

Other general multicolor results:

- Formulas for $R_k(G)$ for G being P_3 , $2K_2$ and $K_{1,3}$ for all k , and for P_4 if k is not divisible by 3 [Ir]. Wallis [Wall] showed $R_6(P_4)=13$, which already implied $R_{3t}(P_4)=6t+1$, for all $t \geq 2$. Independently, the case $R_k(P_4)$ for $k \neq 3^m$ was completed by Lindström in [Lind], and later Bierbrauer proved $R_{3^m}(P_4)=2 \cdot 3^m + 1$ for all $m \geq 1$.
- Bounds for the bipartite graphs $R_k(K_{s,t})$, in particular for $K_{2,2}=C_4$ and for $K_{2,t}$ [ChGra, AFM].
- $tk^2+1 \leq R_k(K_{2,t+1}) \leq tk^2+k+2$, where upper bound is general, and lower bound holds when both t and k are prime powers [LaMu].
- Monotone paths and cycles [Lef].
- Formulas for $R(P_{n_1}, \dots, P_{n_k})$, except few cases [FS2].
- Formulas for $R(S_1, \dots, S_k)$, where S_i 's are arbitrary stars [BuRo1].
- Formulas for $R(S_1, \dots, S_k, K_n)$, where S_i 's are arbitrary stars [Jac].
- Formulas for $R(S_1, \dots, S_k, nP_2)$, where S_i 's are arbitrary stars [CL2].
- Formulas for $R(S_1, \dots, S_k, T)$, where S_i 's are stars and T is a tree [ZZ].
- Formulas for $R(pP_3, qP_3, rP_3)$ and $R(pP_4, qP_4, rP_4)$ [Scob].
- Cockayne and Lorimer [CL1] found the exact formula for $R(n_1P_2, \dots, n_kP_2)$, and later Lorimer [Lor] extended it to a more general case of $R(K_m, n_1P_2, \dots, n_kP_2)$. Still more general cases of the latter, with multiple copies of the complete graph and forests, were studied in [Stahl, LorSe, LorSo].
- If G is connected and $R(K_k, G)=(k-1)(n(G)-1)+1$, in particular if G is any tree, then $R(K_{n_1}, \dots, K_{n_k}, G) = (R(K_{n_1}, \dots, K_{n_k})-1)(n(G)-1)+1$ [BE2]. A generalization for connected G_1, \dots, G_n in place of G appeared in [Jac].
- Study of $R(S, G_1, \dots, G_k)$ for large sparse S [EFRS1, Bu3].
- Constructive bound $R(G_1, \dots, G_{t^{n-1}}) \geq t^n + 1$ for some families of decompositions of K_t^n [LaWo1, LaWo2].
- Bounds for trees $R_k(T)$ and forests $R_k(F)$ [EG, GRS, BB, GT, Bra1, Bra2, SwPr].
- See also surveys listed in section 7.

6. Hypergraph Numbers

The only known value of a classical Ramsey number for hypergraphs:

$$R(4,4;3) = 13 \quad \text{[MR1]}$$

more than 200000 critical colorings

Other hypergraph cases:

$$33 \leq R(4,5;3) \quad \text{[Ex13]}$$

$$63 \leq R(5,5;3) \quad \text{[Ea1]}$$

$$56 \leq R(4,4,4;3) \quad \text{[Ex8]}$$

$$34 \leq R(5,5;4) \quad \text{[Ex11]}$$

$$R(K_4 - t, K_4 - t; 3) = 7 \quad \text{[Ea4]}$$

$$R(K_4 - t, K_4; 3) = 8 \quad \text{[Sob, Ex1, MR1]}$$

$$14 \leq R(K_4 - t, K_5; 3) \quad \text{[Ex1]}$$

$$13 \leq R(K_4 - t, K_4 - t, K_4 - t; 3) \leq 17 \quad \text{[Ex1] [Ea1]}$$

The computer evaluation of $R(4,4;3)$ in [MR1] consisted of an improvement of the upper bound from 15 to 13, which followed an extensive theoretical study of this number in [Gi3, Is1, Sid1]. Exoo in [Ex1] announced the bounds $R(4,5;3) \geq 30$ and $R(5,5;4) \geq 27$ without presenting the constructions. The bound of $R(4,5;3) \geq 24$ was obtained by Isbell [Is2]. Shastri in [Sha] shows a weak bound $R(5,5;4) \geq 19$ (now 34 in [Ex11]), nevertheless his lemmas and those in [Ka3, Abb, GRS, HuSo] can be used to derive other lower bounds for higher numbers. Study of lower bounds on $R(p,q;4)$ can be found in [SYL, Song2].

Let $H^{(r)}(s,t)$ be the complete r -partite r -uniform hypergraph with $r-2$ parts of size 1, one part of size s , and one part of size t (for example for $r=2$ it is the same as $K_{s,t}$). For the multicolor numbers, Lazebnik and Mubayi [LaMu] proved that

$$tk^2 - k + 1 \leq R_k(H^{(r)}(2,t+1)) \leq tk^2 + k + r,$$

where the lower bound holds when both t and k are prime powers. For $H^{(r)}(s,t)$, more bounds are presented in [LaMu].

Lower bounds on $R_m(k;s)$ are discussed in [DLR, AW]. In [AS], it is shown that for some values of a, b the numbers $R(m,a,b;3)$ are at least exponential in m . Other theoretical results on hypergraph numbers are gathered in [GrRö, GRS].

7. Cumulative Data and Surveys

Cumulative data for two colors:

- [CH1] $R(G,G)$ for all graphs G without isolates on at most 4 vertices.
- [CH2] $R(G,H)$ for all graphs G and H without isolates on at most 4 vertices.
- [Clan] $R(G,H)$ for all graphs G on at most 4 vertices and H on 5 vertices, except five entries (now all solved).
- [He4] All critical colorings for $R(G,H)$, for isolate-free graphs G and H as in [Clan] above.
- [Bu4] $R(G,G)$ for all graphs G without isolates and with at most 6 edges.
- [He1] $R(G,G)$ for all graphs G without isolates and with at most 7 edges.
- [HM2] $R(G,G)$ for all graphs G on 5 vertices and with 7 or 8 edges.
- [He2] $R(G,H)$ for all graphs G and H on 5 vertices without isolates, except 7 entries (5 still open).
- [HoMe] $R(G,H)$ for $G = K_{1,3} + e$ and $G = K_4 - e$ versus all connected graphs H on 6 vertices, except $R(K_4 - e, K_6)$. The result $R(K_4 - e, K_6) = 21$ was claimed by McNamara [McN, unpublished].
- [FRS4] $R(G,T)$ for all connected graphs G on at most 5 vertices and all (except some cases) trees T .
- [FRS1] $R(K_3, G)$ for all connected graphs G on 6 vertices.
- [Jin] $R(K_3, G)$ for all connected graphs G on 7 vertices. Some errors in [Jin] were found by [SchSch1].
- [Brin] $R(K_3, G)$ for all connected graphs G on at most 8 vertices. The numbers for K_3 versus sets of graphs with fixed number of edges, on at most 8 vertices, were presented in [KM1].
- [BBH1] $R(K_3, G)$ for all connected graphs G on 9 vertices. See also [BBH2].
- [JR3] $R(C_4, G)$ for all graphs G on at most 6 vertices.
- [JR2] $R(C_5, G)$ for all graphs G on at most 6 vertices.
- [JR4] $R(C_6, G)$ for all graphs G on at most 5 vertices.

Chvátal and Harary [CH1, CH2] formulated several simple but very useful observations how to discover values of some numbers. All five missing entries in the tables of Clancy [Clan] have been solved. Out of 7 open cases in [He2] 2 have been solved, the bounds for 2 were improved, and the status of the other 3 did not change. Section 4 of this survey under "Mixed special cases" lists 4 of them (labeled [He2], 1 solved, 3 open). $R(4,5) = R(G_{19}, G_{23}) = 25$ is the second solved case. The other 2 open entries are K_5 versus K_5 (see section 2) and K_5 versus $K_5 - e$ (see section 3).

Cumulative data for three colors:

- [YR3] $R_3(G)$ for all graphs G with at most 4 edges and no isolates.
- [YR1] $R_3(G)$ for all graphs G with 5 edges and no isolates, except $K_4 - e$. The case of $R_3(K_4 - e)$ remains open (see section 5).
- [YY] $R_3(G)$ for all graphs G with 6 edges and no isolates, except 10 cases.
- [AKM] $R(F, G, H)$ for most triples of isolate-free graphs with at most 4 vertices. Some of the missing cases completed in [KM2].

Surveys:

- [Bu1] A general survey of results in Ramsey graph theory by S.A. Burr (1974)
- [Par5] A general survey of results in Ramsey graph theory by T.D. Parsons (1978)
- [Har2] Summary of progress by Frank Harary (1981)
- [ChGri] A general survey of bounds and values by F.R.K. Chung and C.M. Grinstead (1983)
- [JGT] Special volume of *the Journal of Graph Theory* (1983)
- [Rob1] Nice textbook-type review of Ramsey graph theory for newcomers (1984)
- [Bu6] What can we hope to accomplish in generalized Ramsey Theory ? (1987)
- [GrRö] Survey of asymptotic problems by R.L. Graham and V. Rödl (1987)
- [GRS] An excellent book by R.L. Graham, B.L. Rothschild and J.H. Spencer, second edition (1990)
- [FRS5] Survey of graph goodness results, i.e. conditions for the formula $R(G, H) = (\chi(G) - 1)(n(H) - 1) + s(G)$ (1991)
- [Neš] A chapter in *Handbook of Combinatorics* (1996)
- [Caro] Survey of zero-sum Ramsey theory (1996)

The surveys by S.A. Burr [Bu1] and T.D. Parsons [Par5] contain extensive chapters on general exact results in graph Ramsey theory. F. Harary presented the state of the theory in 1981 in [Har2], where he also gathered many references including seven to other survey papers. A decade ago, Chung and Grinstead in their survey paper [ChGri] gave less data than in this note, but included a broad discussion of different methods used in Ramsey computations in the classical case. S.A. Burr, one of the most experienced researchers in Ramsey graph theory, formulated in [Bu6] seven conjectures on Ramsey numbers for sufficiently large and sparse graphs, and reviewed the evidence for them found in the literature. Recently three of them have been refuted in [Bra3].

For newer extensive presentations see [GRS, GrRö, FRS5, Neš], though these focus on asymptotic theory not on the numbers themselves. Finally, this compilation could not pretend to be complete without mentioning a special volume of the *Journal of Graph Theory* [JGT] dedicated entirely to Ramsey theory. Besides a number of research papers, it includes historical notes and presents to us Frank P. Ramsey (1903-1930) as a person.

8. Concluding Remarks

This compilation does not include information on numerous variations of Ramsey numbers, nor related topics, like size Ramsey numbers, zero-sum Ramsey numbers, irredundant Ramsey numbers, induced Ramsey numbers, local Ramsey numbers, connected Ramsey numbers, chromatic Ramsey numbers, avoiding sets of graphs in some colors, coloring graphs other than complete, or the so called Ramsey multiplicities. Interested reader can find such information in the surveys listed in section 7 here.

The author apologizes for any omissions or other errors in reporting results belonging to the scope of this work. Suggestions for any kind of corrections and/or additions will be greatly appreciated.

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References

We mark the papers containing results obtained with the help of computer algorithms with stars. We identify two categories of such papers: marked with * involving some use of computers, where the results are easily verifiable with some computations, and those marked with **, where cpu intensive algorithms have to be implemented to replicate or verify the results. The first category contains mostly constructions done by algorithms, while the second mostly nonexistence results or claims of complete enumerations of special kinds of graphs.

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