Small Ramsey Numbers

Stanisław P. Radziszowski
Department of Computer Science
Rochester Institute of Technology
Rochester, NY 14623, spr@cs.rit.edu
http://www.cs.rit.edu/~spr

first version: June 11, 1994; revision #16: January 15, 2021
http://www.combinatorics.org

ABSTRACT: We present data which, to the best of our knowledge, includes all known nontrivial values and bounds for specific graph, multicolor and hypergraph Ramsey numbers, where the avoided graphs are complete or complete without one edge. Many results pertaining to other more studied cases are also presented. We give references to all cited bounds and values, as well as to previous similar compilations. We do not attempt complete coverage of asymptotic behavior of Ramsey numbers, but rather we concentrate on their specific values.

Mathematical Reviews Subject Number 05C55

Revisions

1993, February	preliminary version, RIT-TR-93-009 [Ra2]
1994, July 3	posted on the web at the *ElJC*
1994, November 7	*ElJC* revision #1
1995, August 28	*ElJC* revision #2
1996, March 25	*ElJC* revision #3
1997, July 11	*ElJC* revision #4
1998, July 9	*ElJC* revision #5
1999, July 5	*ElJC* revision #6
2000, July 25	*ElJC* revision #7
2001, July 12	*ElJC* revision #8
2002, July 15	*ElJC* revision #9
2004, July 4	*ElJC* revision #10
2006, August 1	*ElJC* revision #11
2009, August 4	*ElJC* revision #12
2011, August 22	*ElJC* revision #13
2014, January 12	*ElJC* revision #14
2017, March 3	*ElJC* revision #15
2021, January 15	*ElJC* revision #16

Table of Contents **2**

1. Scope and Notation **3**

2. Classical Two-Color Ramsey Numbers **4**

2.1 Values and bounds for $R(k, l)$, $k \leq 10$, $l \leq 15$ 4
2.2 Bounds for $R(k, l)$, higher parameters 7
2.3 General results on $R(k, l)$ 9

3. Two Colors: $K_n - e$, K_3, $K_{m,n}$ **12**

3.1 Dropping one edge from complete graph 12
3.2 Triangle versus other graphs 15
3.3 Complete bipartite graphs 16

4. Two Colors: Numbers Involving Cycles **20**

4.1 Cycles, cycles versus paths and stars 20
4.2 Cycles versus complete graphs 21
4.3 Cycles versus wheels 23
4.4 Cycles versus books 25
4.5 Cycles versus other graphs 26

5. General Graph Ramsey Numbers in Two Colors **27**

5.1 Paths 27
5.2 Wheels 27
5.3 Books 28
5.4 Trees and forests 30
5.5 Stars, stars versus other graphs 31
5.6 Paths versus other graphs 32
5.7 Fans, fans versus other graphs 33
5.8 Wheels versus other graphs 33
5.9 Books versus other graphs 34
5.10 Trees and forests versus other graphs 35
5.11 Cases for $n(G)$, $n(H) \leq 5$ 36
5.12 Miscellaneous cases 36
5.13 Multiple copies of graphs, disconnected graphs 37
5.14 General results for special graphs 38
5.15 General results for sparse graphs 39
5.16 General results 40

6. Multicolor Graph Ramsey Numbers **42**

6.1 Bounds for classical numbers 42
6.2 General results for complete graphs 44
6.3 Cycles 46
6.4 Paths, paths versus other graphs 51
6.5 Special cases 53
6.6 General results for special graphs 54
6.7 General results 55

7. Hypergraph Ramsey Numbers **57**

7.1 Values and bounds for numbers 57
7.2 Cycles and paths 58
7.3 General results for 3-uniform hypergraphs 60
7.4 General results 61

8. Cumulative Data and Surveys **62**

8.1 Cumulative data for two colors 62
8.2 Cumulative data for three colors 64
8.3 Electronic resources 64
8.4 Surveys 65

9. Concluding Remarks **67**

References **68-116**

1. Scope and Notation

There is vast literature on Ramsey type problems starting in 1930 with the original paper of Ramsey [Ram]. Graham, Rothschild and Spencer in their book [GRS] present an exciting development of Ramsey Theory. The subject has grown amazingly, in particular with regard to asymptotic bounds for various types of Ramsey numbers (see the survey papers [GrRö, Neš, ChGra2, Ros2]), but the progress on evaluating the basic numbers themselves has been unsatisfactory for a long time. In the last few decades, however, considerable progress has been obtained in this area, mostly by employing computer algorithms. The few known exact values and several bounds for different numbers are scattered among many technical papers. This compilation is a fast source of references for the best results known for specific numbers. It is not supposed to serve as a source of definitions or theorems, but these can be easily accessed via the references gathered here.

Ramsey Theory studies conditions when a combinatorial object contains necessarily some smaller given objects. The role of Ramsey numbers is to quantify some of the general existential theorems in Ramsey Theory.

Let $G_1, G_2, \ldots, G_m$ be graphs or s-uniform hypergraphs (s is the number of vertices in each edge). $R(G_1, G_2, \ldots, G_m; s)$ denotes the m-color **Ramsey number** for s-uniform graphs/hypergraphs, avoiding G_i in color i for $1 \le i \le m$. It is defined as the least integer n such that, in any coloring with m colors of the s-subsets of a set of n elements, for some i the s-subsets of color i contain a sub-(hyper)graph isomorphic to G_i (not necessarily induced). The value of $R(G_1, G_2, \ldots, G_m; s)$ is fixed under permutations of the first m arguments. If $s=2$ (standard graphs) then s can be omitted. If G_i is a complete graph K_k, then we may write k instead of G_i, and if $G_i = G$ for all i we may use the abbreviation $R_m(G; s)$ or $R_m(G)$. For $s=2$, $K_k - e$ denotes a K_k without one edge, and for $s=3$, $K_k - t$ denotes a K_k without one triangle (hyperedge).

The graph nG is formed by n disjoint copies of G, $G \cup H$ stands for vertex disjoint union of graphs, and the **join** $G+H$ is obtained by adding all of the edges between vertices of G and H to $G \cup H$. P_i is a **path** on i vertices, C_i is a **cycle** of length i, and W_i is a **wheel** with $i-1$ spokes, i.e. a graph formed by some vertex x, connected to all vertices of the cycle C_{i-1} (thus $W_i = K_1 + C_{i-1}$). $K_{n,m}$ is a complete n by m bipartite graph, in particular $K_{1,n}$ is a **star** graph. The **book** graph $B_i = K_2 + \overline{K}_i = K_1 + K_{1,i}$ has $i+2$ vertices, and can be seen as i triangular pages attached to a single edge. The **fan** graph F_n is defined by $F_n = K_1 + nK_2$. For a graph G, $n(G)$ and $e(G)$ denote the number of vertices and edges, respectively, and $\delta(G)$ and $\Delta(G)$ minimum and maximum degree of G. Finally, $\chi(G)$ denotes the chromatic number of G. In general, we follow the notation used by West [West].

Section 2 contains the data for the classical two color Ramsey numbers $R(k, l)$ for complete graphs, section 3 for the much studied two color cases of $K_n - e$, K_3, $K_{m,n}$, and section 4 for numbers involving cycles. Section 5 lists other often studied two color cases for general graphs. The multicolor and hypergraph cases are gathered in sections 6 and 7, respectively. Finally, section 8 gives pointers to cumulative data and to other surveys.

2. Classical Two-Color Ramsey Numbers

2.1. Values and bounds for $R(k, l)$, $k \leq 10$, $l \leq 15$

k \ l	3	4	5	6	7	8	9	10	11	12	13	14	15
3	6	9	14	18	23	28	36	40 42	47 50	53 59	60 68	67 77	74 87
4		18	25	36 41	49 61	59 84	73 115	92 149	102 191	128 238	138 291	147 349	158 417
5			43 48	58 87	80 143	101 216	133 316	149 442	183 633	203 848	233 1138	267 1461	275 1878
6				102 165	115 298	134 495	183 780	204 1171	262 1804	294 2566	347 3703	5033	401 6911
7					205 540	219 1031	252 1713	292 2826	405 4553	417 6954	511 10578	15263	22112
8						282 1870	329 3583	343 6090	457 10630	16944	817 27485	41525	873 63609
9							565 6588	581 12677	22325	38832	64864		
10								798 23556	45881	81123			1313

Table Ia. Known nontrivial values, lower bounds (2020) and upper bounds (2017) for two color Ramsey numbers $R(k, l) = R(k, l; 2)$, for $k \leq 10$, $l \leq 15$. For the best known upper bounds (2020) with $k \geq 4$ see Table Ib.

k \ l	4	5	6	7	8	9	10	11	12	13	14	15
3	GG	GG	Kéry	Ka2 GrY	GR McZ	Ka2 GR	Ex5 GoR1	Ex20 GoR1	Kol1 Les	Kol1 GoR1	Kol2 GoR1	Kol2 GoR1
4	GG	Ka1 MR4	Ex19 MR5	Ex3 Mac	ExT Mac	Ex16 Mac	HaKr1 Mac	ExT Spe4	SuLL Spe4	ExT Spe4	ExT Spe4	Tat Spe4
5		Ex4 AnM1	Ex9 HZ1	CaET HZ1	HaKr1 Spe4	Kuz Mac	ExT Mac	Kuz HW+	Kuz HW+	Kuz HW+	Kuz HW+	2.3.h HW+
6			Ka2 Mac	ExT HZ1	ExT Mac	Kuz Mac	Kuz Mac	Tat HW+	Kuz HW+	Kuz HW+	HW+	2.3.i HW+
7				She2 Mac	Tat HZ1	Kuz HZ2	Kuz Mac	XXER HW+	XSR2 HW+	XuXR HW+	HW+	HW+
8					BurR Mac	Kuz Ea1	Kuz HZ2	2.3.i HW+	HW+	XXER HW+	HW+	2.3.i HW+
9						She2 ShZ1	XSR2 Ea1	HW+	HW+	HW+		
10							She2 Shi2	HW+	HW+			2.3.i

References for Table Ia; all upper bounds for $k \geq 4$, $l \geq 6$ were improved in 2019 [AnM2], see Table Ib. HW+ abbreviates HWSYZH, as enhanced by Boza [Boza5], see 2.1.m.

k \ l	5	6	7	8	9	10	11	12	13	14	15
4	25	40	58	79	106	136	171	211	257	307	364
5	48	85	133	194	282	381	511	673	861	1082	1342
6		161	273	427	656	949	1352	1865	2510	3308	4305
7			497	840	1379	2134	3216				
8				1532	2683	4432	7647				

Table Ib: Upper bounds for $R(k, l)$, $k \geq 4$, $l \geq 5$. All of them were obtained by Angeltveit and McKay [AnM2] in 2019, except $R(4, 5)$ [MR4], and they improve over previously best known bounds reported in Table Ia.

We split the data into Table Ia with a separate table of references corresponding to it, and Table Ib of new upper bounds. In Table Ia, the known exact values appear as centered entries, lower bounds as top entries, and upper bounds as bottom entries. For some of the exact values two references are given when the lower and upper bound credits are different. In 2019, in a large computational project, Angeltveit and McKay [AnM2] obtained new upper bounds as reported in Table Ib. These, by using the classical recursive upper bound 2.3.a, lead to further improvements of other upper bounds on $R(k, l)$ for $k \geq 4$, $l \geq 6$. For example, the bounds $R(9,9) \leq 5366$ and $R(7,12) \leq 5081$ implied by 2.3.a, but not reported in Table Ib, improve over those listed in Table Ia.

(a) The task equivalent to that of proving $R(3, 3) \leq 6$ was the second problem in the Kürschák Mathematics Competitions in Hungary in 1947 [BaLiu]. It also was the second problem in Part I of the William Lowell Putnam Mathematical Competition held in March 1953 [Bush].

(b) Greenwood and Gleason [GG] established the initial values $R(3, 4) = 9$, $R(3, 5) = 14$ and $R(4, 4) = 18$ in 1955.

(c) Kéry [Kéry] proved that $R(3, 6) = 18$ in 1964, but only in 2007 an elementary and self-contained proof of this result appeared in English [Car].

(d) All of the critical graphs for the numbers $R(k, l)$ (graphs on $R(k, l) - 1$ vertices without K_k and without K_l in the complement) are known for $k = 3$ and $l = 3$, 4, 5 [Kéry], 6 [Ka2], 7 [RaK2, McZ], 8 [BrGS] and 9 [GoR1], and there are 1, 3, 1, 7, 191, 477142, and 1 of them, respectively. All $(3, k)$-graphs, for $k \leq 6$, were enumerated in [RaK2], and all (4,4)-graphs in [MR2]. There exists a unique critical graph for $R(4,4)$ [Ka2]. Until 2015, there were 350904 known critical graphs for $R(4, 5)$ [MR4], but the full set of such graphs was computed in 2016 [McK3], and there are 352366 of them.

(e) In [MR5], strong evidence is given for the conjecture that $R(5, 5) = 43$ and that there exist exactly 656 critical graphs on 42 vertices. The upper bound of 49 was established in 1997 [MR5]. Angeltveit and McKay improved it by 1 to 48 in 2016 [AnM1].

(f) The graphs constructed by Exoo in [Ex9, Ex12-Ex20, Ex22], and some others, are available electronically from http://cs.indstate.edu/ge/RAMSEY. Fujita [Fuj1] maintains a website with some lower bound constructions; in particular, it presents the bound $R(4,8) \geq 58$ obtained independently from Exoo.

(g) *Cyclic* (or *circulant*) graphs are often used for Ramsey graph constructions. Several cyclic graphs establishing lower bounds were given in the Ph.D. dissertation by J.G. Kalbfleisch in 1966, and many others were published in the next few decades (see [RaK1]). Harborth and Krause [HaKr1] presented all best lower bounds up to 102 from cyclic graphs avoiding complete graphs. In particular, no lower bound in Table Ia can be improved with a cyclic graph on less than 102 vertices, except possibly for $R(3, k)$ for $k \geq 13$. See also items 2.3.l and section 5.16.o [HaKr1]. Larger cyclic heuristic constructions for $R(3, k)$ were explored in [JiLTX1, JiLTX2]. Several best lower bounds from *distance colorings*, a slightly more general concept than circular graphs, are presented in [HaKr2].

(h) The claim that $R(5, 5) = 50$ posted on the web [Stone] is in error, and despite being shown to be incorrect more than once, this value is still being cited by some authors. The bound $R(3, 13) \geq 60$ [XieZ] cited in the 1995 version of this survey was shown to be incorrect in [Piw1]. Another incorrect construction for $R(3, 10) \geq 41$ was described in [DuHu].

(i) There are really only two general upper bound inequalities useful for small parameters, namely 2.3.a and 2.3.b. Stronger upper bounds for specific parameters were difficult to obtain, and they often involved massive computations, like those for the cases of (3,8) [McZ], (3,10) [GoR1], (4,5) [MR4], (4,6) and (5,5) [MR5, AnM1]. The bound $R(6,6) \leq 166$, only 1 more than in [Mac], is an easy consequence of a theorem in [Walk] (2.3.b) and $R(4,6) \leq 41$. Since 2020, we know that $R(6,6) \leq 161$ [AnM2].

(j) T. Spencer [Spe4], Mackey [Mac], and Huang and Zhang [HZ2], using the bounds for minimum and maximum number of edges in (4,5) Ramsey graphs listed in [MR3, MR5], were able to establish new upper bounds for several higher Ramsey numbers, improving on all of the previous longstanding best results by Giraud [Gi3, Gi5, Gi6].

(k) In Table Ia, only some of the higher bounds implied by 2.3.* are shown, and more similar bounds could be derived. In general, we show bounds beyond the contiguous small values if they improve on results previously reported in this survey or published elsewhere. Some easy upper bounds implied by 2.3.a are marked as [Ea1].

(l) In 2009, we have recomputed the upper bounds in Table Ia marked [HZ2] using the method from the paper [HZ2], because the bounds there relied on an overly optimistic personal communication from T. Spencer. Further refinements of this method are studied in [HZ3, ShZ1, Shi2]. The paper [Shi2] subsumes the main results of the manuscripts [ShZ1, Shi2]. All these bounds are now improved by the bounds in Table Ib obtained in [AnM2].

(m) In 2013, Boza [Boza5] using the method of [HWSYZH], which is abbreviated as HW+ in Table Ia, computed the bounds marked HW+ by starting from better upper bounds for smaller parameters. Most of the currently shown bounds are thus better than those originally listed in [HWSYZH, HZ3]. All these bounds are now improved by the bounds in Table Ib obtained in [AnM2].

(n) In 2015, Exoo and Tatarevic obtained several lower bound improvements marked [ExT] in Tables Ia and IIa by using some modifications of general circulant constructions, but especially related to the quadratic residues Paley graph Q_{101} and the cubic residues graph G_{127}. More bounds by Tatarevic are reported in [Tat]. In 2016, Kuznetsov [Kuz] obtained several further new lower bounds building up on circulant graphs. Also in 2015 and 2016, somewhat surprisingly, Kolodyazhny [Kol1, Kol2] improved four longstanding lower bounds on $R(3, k)$ in Table Ia.

(o) Some lower bounds in Table Ia, like for $R(6,8)$ or $R(8,8)$, may seem rather weak, yet they are not easy to improve. For comments on $R(8,8)$ see [ExT].

2.2. Bounds for $R(k, l)$, higher parameters

(a) The upper bounds in Tables Ia and IIa marked [GoR1, Les, Back1] were obtained mainly by deriving lower bounds for several cases of $e(3, k, n)$, which denotes the minimum number of edges in n-vertex triangle-free graphs with independence number less than k. The study of $e(3, k, n)$ was also the main tool for the results obtained in [GrY, GR, RaK2, RaK3, GoR2].

k \ l	15	16	17	18	19	20	21	22	23
3	74 Kol2 87 GoR1	82 Ex21 97 Back3	92 W1+ 109 Back1	99 Ex16 120 Back4	106 W1+ 132 Back3	111 Ex16 145 Les	122 W1+ 157 Back4	131 W2+ 171 Back2	139 XWCS 185 Back2
4	158 Tat	170 Tat	200 Lia+	205 2.3.e	213 2.3.h	234 Ex16	242 SLZL	314 LinCa	
5	275 2.3.h	293 ExT	388 XSR2	396 2.3.h	411 XSR2	424 XSR2	441 2.3.i	492 Ihr	521 2.3.i
6	401 2.3.i	434 SLLL	548 SLLL	614 SLLL	710 SLLL	878 SLLL	888 2.3.h	1070 SLLL	
7		629 2.3.i	729 2.3.i	797 2.3.i	908 SLLL		1214 SLLL		
8	873 2.3.i		1005 2.3.i	1049 2.2.h	1237 2.2.h		1617 2.3.i		

Table IIa. Known bounds for higher two-color Ramsey numbers $R(k, l)$, with references.
Lower and upper bounds are given for $k=3$, only lower bounds for $k \geq 4$;
Lia+, W1+ and W2+ abbreviate LiaWXS, WWY1 and WSLX2, respectively.

k \ l	24	25	26	27	28	29	30	31	32
3	143 W1+	154 W2+	159 W1+	172 LiLi	177 LiLi	190 LiLi	195 LiLi	206 LiLi	217 LiLi

k \ l	33	34	35	36	37	38	39	40	41
3	224 LiLi	230 Ji+	242 Ji+	252 Ji+	264 Ji+	272 Ji+	284 Ji+	294 Ji+	308 Ji+

k \ l	42	43	44	45	46	47	48	49	50
3	318 Ji+	332 Ji+	338 Ji+	354 Ji+	360 Ji+	380 Ji+	384 Ji+	402 Ji+	

Table IIb. Known lower bounds for higher Ramsey numbers $R(3, l)$ for $l \geq 24$.
W1+, W2+ and Ji+ abbreviate WSLX1, WSLX2 and JiLTX2, respectively.

k	11	12	13	14	15	16	17
lower bound	1597	1640	2557	2989	5485	5605	8917
reference	2.2.c	Tat	2.2.c	2.2.c	2.2.c	2.2.c	LuSL
k	18	19	20	21	22	23	24
lower bound	11005	17885	21725	30925	39109	49421	
reference	LuSL	LuSL	Ex23	Ex23	Ex23	Ex23	

Table IIc. Known lower bounds for diagonal Ramsey numbers $R(k, k)$ for $k \geq 11$;
All lower bounds for $k \geq 13$ are from Paley graphs, see also 2.2.c below.

(b) *Ramsey Calculus* [Back1], is an extensive manuscript by Backelin, which, among other goals, addresses the derivation of $e(3, k, n)$ and the corresponding realisers while avoiding reliance on computer assisted results as far as possible. It achieves the derivation of several lower bounds for $e(3, k+1, n)$ better than those in [GoR1, RaK3, RaK4] for n close to and above $13k/4$. Better lower bounds on $e(3, k, n)$ sometimes lead to better upper bounds on $R(3, l)$, like for $l = 18$ and $l = 20$ [Back4]. Further improvements to bounds on $e(3, k, n)$ were obtained in [Krü].

(c) The construction by Shearer [She2] (see also items 2.3.j, 6.2.k and 6.2.l), using the data obtained by Shearer [She4] for primes up to 7000, implies the lower bounds in Table IIc marked 2.2.c. An equivalent construction was studied by Mathon [Mat]. The first two bounds credited in Table IIc to [LuSL] also follow similarly from the data in [She4]. The same approach does not improve on the bound $R(12, 12) \geq 1639$ [XSR2], later increased to 1640 [Tat]. The bounds in [Ex23] were obtained by extending data for Paley graphs beyond [Sha4] and improving on [LiaWXCS].

(d) The lower bounds marked [XuXR], [XXER], [XSR2], 2.3.e, 2.3.h and 2.3.i need not be cyclic. Several of the Cayley colorings from [Ex16] are also non-cyclic. All other lower bounds listed in Table IIa/b were obtained by construction of circular graphs.

(e) The graphs establishing lower bounds marked 2.3.h can be constructed by using appropriately chosen graphs G and H with a common m-vertex induced subgraph, similarly as it was done in several cases in [XuXR].

(f) Yu [Yu2] constructed a special class of triangle-free cyclic graphs establishing several lower bounds for $R(3, k)$, for $k \geq 61$. All of these bounds can be improved by the inequalities in 2.3.c and data from Tables Ia and IIa/b.

(g) Unpublished bound $R(4, 22) \geq 314$ [LiSLW] improved over 282 given in [SuL]. [LinCa] obtained the same bound, and also $R(4, 25) \geq 458$. Not yet published bounds $R(3, 23) \geq 139$ [XWCS] and $R(4, 17) \geq 200$ [LiaWXS] improve over 137 and 182 obtained in [WSLX2] and [LuSS1], respectively. The bound $R(9, 17) \geq 1411$ is given in [XuXR]. Large cyclic heuristic constructions for $R(3, k)$ for $k < 50$ were explored in [JiLTX1, JiLTX2].

(h) Two special cases, $R(8, 18) \geq 1049$ and $R(8, 19) \geq 1237$, can be obtained by applying 2.3.i and 2.3.h below. In both cases we start with the 816-vertex graph G, witnessing $R(8, 13) \geq 817$, obtained by 2.3.i. Next, for properly chosen graphs H in the application of 2.3.h, we have large common subgraphs of G and H, namely the 101-vertex witness of $R(6, 6) \geq 102$ and the 204-vertex witness of $R(7, 7) \geq 205$, respectively.

(i) One can expect that the lower bounds in Tables IIa/b are weaker than those in Table Ia, especially smaller ones, in the sense that some of them should not be that hard to improve, in contrast to the bounds in Table Ia.

2.3. General results on $R(k, l)$

(a) $R(k, l) \leq R(k-1, l) + R(k, l-1)$, with strict inequality when both terms on the right hand side are even [GG]. There are obvious generalizations of this inequality for avoiding graphs other than complete.

(b) $R(k, k) \leq 4R(k, k-2) + 2$ [Walk].

(c) Explicit construction for $R(3, 3k+1) \geq 4R(3, k+1) - 3$, for all $k \geq 2$ [CleDa],
explicit construction for $R(3, 4k+1) \geq 6R(3, k+1) - 5$, for all $k \geq 1$ [ChCD].

(d) Explicit triangle-free graphs with independence k on $\Omega(k^{3/2})$ vertices [Alon2, CoPR]. For other constructive results in relation to $R(3, k)$ see [BrBH1, BrBH2, Fra1, Fra2, FrLo, GoR1, Gri, KlaM1, Loc, RaK2, RaK3, RaK4, Stat, Yu1]. See also 2.3.3 and 2.3.4 below.

(e) The study of bounds for the difference between consecutive Ramsey numbers was initiated in [BEFS], where the bound $R(k, l) \geq R(k, l-1) + 2k - 3$, for $k, l \geq 3$, was established by a construction. In 1980, Erdős and Sós (cf. [Erd2,ChGra2]) asked: If we set $\Delta_{k,l} = R(k, l) - R(k, l-1)$, then is it true that $\Delta_{k,k+1}/k \to \infty$ as $k \to \infty$? Only easy bounds on $\Delta_{k,l}$ are known, in particular for $k = 3$ we have $3 \leq \Delta_{3,l} \leq l$. For some discussion of the roadblocks on the latter see [XSR2, GoR2, ZhuXR]. It is also known that $R(3, k) \geq R(3, K_{k-1} - e) + 4$ [ZhuXR].

(f) A conjecture that $R(k, l) \geq R(k-1, l+1)$ for all $3 \leq k \leq l$ (called DC), its implications, evidence for validity, and related problems [LiaRX]. For the multicolor version of DC and its consequences see item 6.2.v.

(g) By taking a disjoint union of two critical graphs one can easily see that $R(k, p) \geq s$ and $R(k, q) \geq t$ imply $R(k, p+q-1) \geq s+t-1$. Xu and Xie [XuX1] improved this construction to yield better general lower bounds, in particular $R(k, p+q-1) \geq s+t+k-3$.

(h) For $2 \leq p \leq q$ and $3 \leq k$, if (k, p)-graph G and (k, q)-graph H have a common induced subgraph on m vertices without K_{k-1}, then $R(k, p+q-1) > n(G)+n(H)+m$. In particular, this construction implies the bounds $R(k, p+q-1) \geq R(k, p) + R(k, q)+k-3$ and $R(k, p+q-1) \geq R(k, p) + R(k, q)+p-2$ [XuX1, XuXR], with small improvements in some cases, such as using the term $k-2$ instead of $k-3$ in the first bound [XSR2].

(i) $R(2k-1, l) \geq 4R(k, l-1) - 3$ for $l \geq 5$ and $k \geq 2$, and in particular for $k=3$ we have $R(5, l) \geq 4R(3, l-1) - 3$ [XXER].

(j) If the quadratic residues Paley graph Q_p of prime order $p = 4t+1$ contains no K_k, then $R(k, k) \geq p+1$ and $R(k+1, k+1) \geq 2p+3$ [She2, Mat]. Data for larger p was obtained in [LuSL], and further for p up to 25000 in [Ex23]. See also 3.1.e, and items 6.2.k and 6.2.l for similar multicolor results.

(k) Study of Ramsey numbers for large disjoint unions of graphs [Bu1, Bu9], in particular $R(nK_k, nK_l) = n(k+l-1) + R(K_{k-1}, K_{l-1}) - 2$, for n large enough [Bu8].

(l) $R(k, l) \geq L(k, l) + 1$, where $L(k, l)$ is the maximal order of any cyclic $(k, l)-$graph. A compilation of many best cyclic bounds was presented in [HaKr1].

(m) The graphs critical for $R(k, l)$ are $(k-1)-$vertex connected and $(2k-4)-$edge connected, for $k, l \geq 3$ [BePi]. This was improved to vertex connectivity k for $k \geq 5$ and $l \geq 3$ in [XSR2].

(n) All Ramsey-critical $(k, l)-$graphs are Hamiltonian for $k \geq l-1 \geq 1$ and $k \geq 3$, except when $(k, l) = (3, 2)$ [XSR2].

(o) Two-color lower bounds can be obtained by using items 6.2.m, 6.2.n and 6.2.o with $r = 2$. Some generalizations of these were obtained in [ZLLS].

In the last seven items (1)-(7) of this section we only briefly mention some pointers to the literature dealing with asymptotics of Ramsey numbers. This survey was designed mostly for small, finite, and combinatorial results, but still we wish to give the reader some useful and representative references to more traditional papers studying the infinite.

(1) In 1947, Erdős gave a simple probabilistic proof that $R(k, k) > 2^{k/2}$ [Erd1]. In 1975, Spencer [Spe1] improved it to $R(k, k) > \sqrt{2}e^{-1}k2^{k/2}(1+o(1))$. More probabilistic asymptotic lower bounds were obtained in [Spe1, Spe2, AlPu].

(2) The limit of $R(k, k)^{1/k}$, if it exists, is between $\sqrt{2}$ and 4 [GRS, GrRö, ChGra2].

(3) In 1995, Kim obtained a breakthrough result by proving that $R(3, k) = \Theta(k^2/\log k)$ [Kim]. The best known lower and upper bounds constants are 1/4 [BohK2, BohK3] and 1 (implicit in [She1]), respectively. An independent proof of the lower bound constant 1/4 and a conjecture that it is the best possible are presented in [FizGM].

(4) Other asymptotic and general results on triangle-free graphs in relation to $R(3, k)$ can be found in [Boh, AlBK, AjKS, Alon2, CleDa, ChCD, CoPR, Gri, FrLo, Loc, She1, She3].

(5) Explicit constructions yielded the lower bounds $R(4, k) \geq \Omega(k^{8/5})$, $R(5, k) \geq \Omega(k^{5/3})$ and $R(6, k) \geq \Omega(k^2)$ [KosPR]. For the same cases of k classical probabilistic arguments give $\Omega((k/\log k)^{5/2})$, $\Omega((k/\log k)^3)$ and $\Omega((k/\log k)^{7/2})$, respectively [Spe2]. These were improved to $\Omega(k^{5/2}/(\log k)^2)$, $\Omega(k^3/(\log k)^{8/3})$ and $\Omega(k^{7/2}/(\log k)^{13/4})$, respectively, in [Boh, BohK1], and in general to $R(s, t) \geq \Omega(t^{(s+1)/2}/(\log t)^{(s^2-s-4)/(2s-4)})$, for fixed s and large t [BohK1].

(6) Explicit construction of a graph with clique and independence k on $2^{c\log^2 k/\log\log k}$ vertices was presented by Frankl and Wilson [FraWi], and further constructions by Chung [Chu3] and Grolmusz [Grol1, Grol2]. In 2012, the best explicit construction for large k by Barak et al. [BarRSW] improved over [FraWi] by giving such a graph on $2^{2^{(\log\log k)^c}}$ vertices for some $c > 1$, or equivalently, on n vertices, where $\log\log n = (\log\log k)^c$. This was improved to $\log\log n = (\log k)^d$, for a positive constant d, by Cohen [Coh] in 2016. Explicit constructions such as these are usually weaker than known probabilistic results.

(7) In 2009, Conlon [Con1] obtained the best until then upper bound for the diagonal case

$$R(k+1, k+1) \leq \binom{2k}{k} k^{-c\log k/\log\log k}.$$

In 2020, Sah [Sah] improved it to

$$R(k+1, k+1) \leq \binom{2k}{k} e^{-c(\log k)^2}.$$

Other asymptotic bounds can be found, for example, in [Chu3, McS, Boh, BohK1] (lower bound) and [Tho] (upper bound), and for many other bounds in the general case of $R(k, l)$ consult [Spe2, GRS, GrRö, Chu4, ChGra2, LiRZ1, AlPu, Kriv, ConFS7].

3. Two Colors: $K_n - e$, K_3, $K_{m,n}$

3.1. Dropping one edge from complete graph

This section contains known values and nontrivial bounds for the two color case when the avoided graphs are complete or have the form $K_k - e$, but not both are complete.

(a) The exact values in Table IIIa involving $K_3 - e$ are obvious, since one can easily see that $R(K_3 - e, K_k) = R(K_3 - e, K_{k+1} - e) = 2k - 1$ for all $k \geq 2$.

(b) More bounds (beyond those shown in Tables IIIa/b) can be easily obtained using Table Ia/b, an obvious generalization of the inequality $R(k, l) \leq R(k-1, l) + R(k, l-1)$, and by monotonicity of Ramsey numbers, in this case $R(K_{k-1}, G) \leq R(K_k - e, G) \leq R(K_k, G)$.

G \ H	K_3-e	K_4-e	K_5-e	K_6-e	K_7-e	K_8-e	K_9-e	$K_{10}-e$	$K_{11}-e$
K_3-e	3	5	7	9	11	13	15	17	19
K_3	5	7	11	17	21	25	31	37	42 45
K_4-e	5	10	13	17	28	30 32	36 46	41 63	 82
K_4	7	11	19	30 32	37 49	52 74	62 105	 136	 171
K_5-e	7	13	22	37	65	66 97	69 143		
K_5	9	16	30 33	43 62	65 102	81 175	121 275	 381	 511
K_6-e	9	17	37	45 70	66 124	83 218	 361	 551	
K_6	11	21	43 53	58 110	 205	 371	 620	 949	 1352
K_7-e	11	28	65	66 124	 247	 465	 807	 1331	 2142
K_7	13	28 29	65 82	80 191	 388	 746	 1325	 2134	 3216
K_8	15	36 39	69 121	 300	 657	 1345	 2556	 4432	 7647
K_9	17	41 56	75 167						
K_{10}	19	49 							

Table IIIa. Bounds on the Ramsey numbers $R(G, H)$, for complete or missing one edge graphs G and H, but not both complete. Known exact values appear as centered entries, lower bounds as top entries, and upper bounds as bottom entries.

G \ H	K_4-e	K_5-e	K_6-e	K_7-e	K_8-e	K_9-e	$K_{10}-e$	$K_{11}-e$
K_3	ChH2	Clan	FRS1	GrH	Ra1	Ra1	MPR GoR2	WWY2 GoR2
K_4-e	ChH1	FRS2	McR	McR	AnM2 LidP	VO LidP	Ex14 Ea1	Ea1
K_4	ChH2	EHM1	Boza6 LidP	Ex14 LidP	VO BZ7	VO BZ2	AnM2	AnM2
K_5-e	FRS2	CE+	VO	VO	Ea1 Ea1	Ea1 Ea1	HT+	HT+
K_5	BoH	Ex6 BZ7	Ea1 LidP	VO LidP	VO Ea1	VO BZ7	AnM2	AnM2
K_6-e	McR	VO	Ex14 HZ3	VO LidP	VO HT+	Ea1	HT+	
K_6	McN/ ShWR	VO BZ1	Ea1 BZ7	ShZ2	BZ7	BZ7	AnM2	AnM2
K_7-e	McR	VO	VO LidP	Ea1	Ea1	HT+	HT+	HT+
K_7	Ea1 LidP	Ea1 Ea1	Ea1 Ea1	BZ7	BZ7	BZ7	AnM2	AnM2
K_8	VO LidP	Ea1 Ea1	BZ7	BZ7	BZ7	BZ7	AnM2	AnM2
K_9	VO Ea1	Ea1 Ea1						
K_{10}	VO							

References for Table IIIa;
CE+ abbreviates ClEHMS, HT+ abbreviates HTHZ1 (see also 3.1.o below),
for some details on BZ1, BZ2 and BZ7 see item 3.1.f.

k	11	12	13	14	15	16
lower bound	42 WWY2	49 VO	55 GoR2	61 VO	69 WWY2	74 Ea1
upper bound	45 GoR2	53 GoR2	62 GoR2	71 GoR2	80 GoR2	91 GoR2

Table IIIb. Lower and upper bounds for $R(K_3, K_k - e)$ for $11 \le k \le 16$.

(c) Upper bounds for Ramsey numbers $R(K_k, K_l - e)$ marked [AnM2] in references for Table IIIa are trivially implied by the bounds on $R(K_k, K_l)$ in Table Ib. These were obtained in 2020 by Angeltveit and McKay using linear programming, and they improve over upper bounds in Table Ia. The upper bounds by Lidický and Pfender [LidP] use flag algebras.

(d) Two special exact values, and several other bounds were obtained by Van Overberghe [VO] in 2020. The surprisingly large exact values $R(K_5-e, K_6-e) = 37$ and $R(K_5-e, K_7-e) = 65$ exploit some previously known strongly regular graphs on 27, 36 and 64 vertices, namely the Schläfli graph, $NO^-(6,2)$ and $VO^-(6,2)$ (see the website by A. E. Brouwer [Brou] for a great collection of strongly regular graphs). A lower bound $R(K_4-e, K_{16}-e) \geq 82$ is also in [VO].

(e) If the quadratic residues Paley graph Q_p of prime order $p = 4t+1$ contains no K_k-e, then $R(K_{k+1}-e, K_{k+1}-e) \geq 2p+1$. In particular, $R(K_{14}-e, K_{14}-e) \geq 2987$ [LiShen]. This was generalized to K_k-F for some small graphs F instead of an edge $e\,(=K_2)$ [WaLi]. See also item 2.3.j.

(f) This item follows personal communication from Boza [Boza5]. The upper bounds marked [BZ1] were obtained until 2012, while ones marked [BZ2] are from 2013. Several other improvements were obtained by Boza [Boza7] in 2014, marked also as [BZ7]. They are implied by [Boza6], the previous work [Boza1, Boza3, BoPo], the method of [HZ3], and the bounds given in [GoR2]. The enumeration of all (K_6, K_4-e)-graphs [ShWR] is used in [BoPo].

(g) All (K_3, K_k-e)-graphs were enumerated for $k \leq 6$ [Ra1] and $k = 7$ [Fid2, GoR2]. Full sets of (K_l, K_k-e)-graphs were posted for the parameters (K_3, K_k-e) for $k \leq 7$, (K_4, K_k-e) for $k \leq 5$, and (K_5, K_k-e) for $k \leq 4$ ([Fid2], available until 2014), and other full and restricted families at [BrCGM, Fuj1].

(h) The number of (K_3, K_l-e)-critical graphs for $l = 4$, 5 and 8 is 4, 2 and 9, respectively [MPR]. There are 7 critical graphs for $R(K_3, K_9-e)$, and at least 40 such graphs for $R(K_3, K_{10}-e)$ [GoR2].

(i) The critical graphs are unique for: $R(K_3, K_l-e)$ for $l=3$ [Tr], 6 and 7 [Ra1], $R(K_4-e, K_4-e)$ [FRS2], $R(K_5-e, K_5-e)$ [Ra3] and $R(K_4-e, K_7-e)$ [McR].

(j) All of the critical graphs for the cases $R(K_4-e, K_4)$ [EHM1], $R(K_4-e, K_5)$ and $R(K_5-e, K_4)$ [DzFi1] are known, and there are 5, 13 and 6 of them, respectively. The unpublished value of $R(K_4-e, K_6)$ [McN] was confirmed in [ShWR], where in addition all 24976 critical graphs were found.

(k) It is known that $R(K_4, K_{12}-e) \geq 128$ [Shao] by using one color of the $(4,4,4;127)$-coloring defined in [HiIr].

(l) If $m \leq n$ then $R(K_4-e, K_{m+n+1}) \geq R(3, m+1) + R(3, n+1) + n$.
Study of the growth of $R(K_4-e, K_n)$ and its relationship to $R(K_3, K_n)$ [JiLSX].

(m) $R(K_k-e, K_k-e) \leq 4R(K_{k-2}, K_k-e) - 2$ [LiShen].
For a similar inequality for complete graphs see 2.3.b.

(n) Study of the cases $R(K_m, K_n-K_{1,s})$ and $R(K_m-e, K_n-K_{1,s})$, with several exact values for special parameters [ChaMR]. This study was extended to some cases involving $R(K_m-K_3)$ [MonCR].

(o) The upper bounds from [ShZ1, ShZ2] are subsumed by a later article [Shi2].

(p) The upper bounds in [HZ3] were obtained by a reasoning generalizing the bounds for classical numbers in [HZ2]. Several other results from section 2.3 apply, though checking in which situation they do may require looking inside the proofs whether they still hold for $K_n - e$. The upper bounds in the manuscript [HTHZ1] (abbreviated as HT+ in the references for Table IIIa) are based on [HZ3].

3.2. Triangle versus other graphs

(a) $R(3, k) = \Theta(k^2/\log k)$ [Kim].
For more comments on asymptotics see section 2.3.(3) and the items 3.2.p/q below.

(b) Explicit construction for $R(3, 3k+1) \geq 4R(3, k+1) - 3$, for all $k \geq 2$ [CleDa], explicit construction for $R(3, 4k+1) \geq 6R(3, k+1) - 5$, for all $k \geq 1$ [ChCD].

(c) Explicit triangle-free graphs with independence k on $\Omega(k^{3/2})$ vertices [Alon2, CoPR].

(d) $R(K_3, K_7 - 2P_2) = R(K_3, K_7 - 3P_2) = 18$ [SchSch2].

(e) $R(K_3, K_3 + \overline{K}_m) = R(K_3, K_3 + C_m) = 2m + 5$, for $m \geq 212$ [Zhou1].

(f) $R(K_3, K_2 + T_n) = 2n + 3$ for n-vertex trees T_n, for $n \geq 4$ [SonGQ],
$R(K_3, K_1 + nK_3) = 6n + 1$, for $n \geq 3$ [HaoLin].

(g) $R(K_3, G) = 2n(G) - 1$ for any connected G on at least 4 vertices and with at most $(17n(G) + 1)/15$ edges, in particular for $G = P_i$ and $G = C_i$, for all $i \geq 4$ [BEFRS1].

(h) $R(K_3, Q_n) = 2^{n+1} - 1$ for large n [GrMFSS], where Q_n is the n-dimensional hypercube. For related publications on the general case of $R(K_m, Q_n)$ see [FizGMSS, ConFLS] and item 5.15.n.

(i) Relations between $R(3, k)$ and graphs with large $\chi(G)$ [BiFJ],
further detailed study of the relation between $R(3, k)$ and the chromatic gap [GySeT].

(j) $R(K_3, G) \leq 2e(G) + 1$ for any graph G without isolated vertices [Sid3, GoK].

(k) $R(K_3, G) \leq n(G) + e(G)$ for all G, a conjecture [Sid2].

(l) $R(K_3, G)$ for all connected G up to 9 vertices [BrBH1, BrBH2].

(m) $R(K_3, G)$ for all graphs G on 10 vertices [BrGS], except 10 cases (three of which, including $G = K_{10} - e$, were solved [GoR2]). See also several items in section 8.1.

(n) $R(nK_3, nK_3) = 5n$ for $n \geq 2$, $R(mK_3, nK_3) = 3m + 2n$ for $m \geq n \geq 2$ [BES], and $R(c(nK_3), c(nK_3)) = 7n - 2$ for $n \geq 2$, where $c(nK_3)$ is any connected graph containing n vertex disjoint triangles [GySá3].

(o) Formulas for $R(nK_3, mG)$ for all G of order 4 without isolates [Zeng].

(p) For every positive constant c, and for Δ and n large enough, there exists n-vertex graph G with $\Delta(G) \leq \Delta$ for which $R(K_3, G) > cn$ [Bra3].

(q) $R(K_3, K_{k,k}) = \Theta(k^2/\log k)$ [LinLi2].

(r) For $R(K_3, K_n)$ see section 2, and for $R(K_3, K_n - e)$ see section 3.1.

(s) Since $B_1 = F_1 = C_3 = W_3 = K_3$, other sections apply. See also [Boh, AjKS, BrBH1, BrBH2, FrLo, Fra1, Fra2, BiFJ, Gri, GySeT, Loc, KlaM1, LiZa1, RaK2, RaK3, RaK4, She1, She3, Spe2, Stat, Yu1].

3.3. Complete bipartite graphs

Note: This subsection gathers information on Ramsey numbers where specific bipartite graphs are avoided in edge colorings of K_n (as everywhere in this survey), in contrast to the often studied bipartite Ramsey numbers, which are not covered in this survey, where the edges of complete bipartite graphs $K_{n,m}$ are colored.

3.3.1. Numbers

The following Tables IVa and IVb gather information mostly from the surveys by Lortz and Mengersen [LoM3, LoM4]. All cases involving $K_{1,2} = P_3$ are solved by a formula for $R(P_3, G)$, which holds for all isolate-free graphs G, derived in [ChH2]. All star versus star numbers are given below in the item 3.3.2.a and in section 5.5.

p, q / m, n	1, 2	1, 3	1, 4	1, 5	1, 6	2, 2	2, 3	2, 4	2, 5	3, 3	3, 4
2, 2	4 ChH2	6 ChH2	7 Par3	8 Par3	9 FRS4	6 ChH1					
2, 3	5 ChH2	7 FRS4	9 Stev	10 FRS4	11 FRS4	8 HaMe4	10 Bu4				
2, 4	6 ChH2	8 HaMe3	9 Stev	11 HaMe4	13 LoM4	9 HaMe4	12 ExRe	14 EHM2			
2, 5	7 ChH2	9 HaMe3	11 Stev	13 Stev	14 LoM4	11 HaMe4	13 LoM3	16 LoM1	18 EHM2		
2, 6	8 ChH2	10 HaMe3	11 Stev	14 Stev	15* Shao	12 HaMe4	14 LoM3	17 LoM3	20 LoM1		
3, 3	7 ChH2	8 HaMe3	11 LoM4	12 LoM4	13 LoM4	11 Lortz	13 HaMe3	16 LoM4	18 LoM4	18 HaMe3	
3, 4	7 ChH2	9 HaMe3	11 LoM4	13 LoM4	14 LoM4	11 Lortz	14 LoM4	17 Sh1+	20 VO-LidP	19-20 VO-LidP	25 VO-LidP
3, 5	9 ChH2	10 HaMe3	13 Sh1+	15 Sh1+	17 LidP	14 HaMe4	17* Shao	19-20 VO-LidP	≤23 LidP	21-24 VO-LidP	25-29 VO-LidP

Table IVa. Ramsey numbers $R(K_{m,n}, K_{p,q})$; unpublished result marked with *, Sh1+ abbreviates ShaXBP, $R(K_{3,5}, K_{2,5}) \geq 21$ is in [ShaoWX].

m n	2	3	4	5	6	7	8	9	10	11
6	12 HaMe4	14 LoM3	17 LoM3	20 LoM1	21 EHM2					
7	14 HaMe4	17 LoM3	19 LoM3	21 LoM3	24 LoM1	26 EMH2				
8	15 HaMe4	18 LoM3	20 LoM3	22-23 LoM3	25 VO-LoM3	28 LoM1	30 EMH2			
9	16 HaMe4	19 LoM3	22 LoM3	25* Shao	27* Shao	29* Shao	32 LoM1	33 EHM2		
10	17 HaMe4	21 LoM3	24 LoM3	27 LoM3	27-29 LoM3	29-31 VO-LoM3	33 VO-LoM3	36 LoM1	38 EHM2	
11	18 HaMe4	≥ 22 VO	≥ 25 VO	≥ 28 VO	≥ 29 VO	≥ 33 VO	35 VO-LoM3	37 VO-LoM3	40 LoM1	42 EHM2

Table IVb. Known Ramsey numbers $R(K_{2,n}, K_{2,m})$ for $6 \leq n \leq 11$, $2 \leq m \leq 11$; unpublished results improving over [LoM3] are marked with a *.

(a) The next few easily computed values of $R(K_{1,n}, K_{2,2})$, extending data in the first row of Table IVa, are 13, 14, 21 and 22 for n equal to 9, 10, 16 and 17, respectively. See function $f(n)$ in 3.3.2.c of the next subsection below.

(b) Note that for graph G to avoid $K_{1,n}$ is equivalent to $\delta(G) < n$. For general monotonicity we have, for example, that rows of Table IVb are nondecreasing, but we do not know if they are strictly increasing.

(c) Formula for $R(K_{1,n}, K_{k_1, k_2, \cdots, k_t, m})$ for m large enough, in particular for $t = 1$, $k_1 = 2$ with $n \leq 5$, $m \geq 3$ and $n = 6$, $m \geq 11$, for example $R(K_{1,5}, K_{2,7}) = 15$ [Stev].

(d) The values and bounds for higher cases of $R(K_{2,2}, K_{2,n})$ are 20, 22, 22, 24, 25, 26, 27/28, 28/29, 30 and 32 for $12 \leq n \leq 21$, respectively. All of them were given in [HaMe4], except those for $n = 14$, 15 and 18, which were obtained in [Dyb1]. More exact values for prime powers $\lceil \sqrt{n} \rceil$ and $\lceil \sqrt{n} \rceil + 1$ can be found in [HaMe4].

(e) The known values of $R(K_{2,2}, K_{3,n})$ are 15, 16, 17, 20 and 22 for $6 \leq n \leq 10$ [Lortz], and $R(K_{2,2}, K_{3,12}) = 24$ [Shao]. See Tables IVa and IVb for the smaller cases, and [HaMe4] for upper bounds and values for some prime powers $\lceil \sqrt{n} \rceil$.

(f) $R(K_{2,n}, K_{2,n})$ is equal to 46, 50, 54, 57 and 62 for $12 \leq n \leq 16$, respectively.
The first open diagonal case is $65 \leq R(K_{2,17}, K_{2,17}) \leq 66$ [EHM2].
The status of all higher cases for $n < 30$ is listed in [LoM1].

(g) $R(K_{1,4}, K_{4,4}) = R(K_{1,5}, K_{4,4}) = 13$ [ShaXPB]
$R(K_{1,4}, K_{1,2,3}) = R(K_{1,4}, K_{2,2,2}) = 11$ [GuSL]
$R(K_{1,7}, K_{2,3}) = 13$ [Par4, Par6]

$R(K_{1,15}, K_{2,2}) = 20$ [La2]
$R(K_{2,2}, K_{4,4}) = 14$ [HaMe4]
$R(K_{2,2}, K_{4,5}) = 15$ [Shao]
$R(K_{2,2}, K_{4,6}) = 16$ [Shao]
$R(K_{2,2}, K_{5,5}) = R(K_{2,3}, K_{3,5}) = 17$ [Shao]

(h) A number of general upper and lower bounds for $R(K_{s,t}, K_{s,t})$, in particular for small fixed s, and for some slightly off-diagonal cases were obtained in [LoM2]. They can be used to derive the upper bounds for the cases listed in (h) and (i) below.

(i) Several lower bounds of the form $R(K_{s,t}, K_{s,t}) \geq m$ from distance colorings, a slightly more general concept than circular graphs, were presented in [HaKr2] for the following triples (s, t, m): (3,6,38), (3,7,42), (3,8,43), (3,9,54), (4,5,42), (4,6,43), (4,7,54), (5,5,54).

(j) $33 \leq R(K_{3,5}, K_{3,5}) \leq 33$ [VO][LidP]
$33 \leq R(K_{4,4}, K_{4,4}) \leq 49$ [VO][LidP]

3.3.2. General results

(a) $R(K_{1,n}, K_{1,m}) = n+m-\varepsilon$, where $\varepsilon=1$ if both n and m are even and $\varepsilon=0$ otherwise [Har1]. It is also a special case of multicolor numbers for stars obtained in [BuRo1].

(b) $R(K_{1,3}, K_{m,n}) = m+n+2$ for $m, n \geq 1$ [HaMe3].

(c) $R(K_{1,n}, K_{2,2}) = f(n) \leq n + \lceil \sqrt{n} \rceil + 1$, with $f(q^2) = q^2+q+1$ and $f(q^2+1) = q^2+q+2$ for every q which is a prime power [Par3]. Furthermore, $f(n) \geq n + \sqrt{n} - 6n^{11/40}$ [BEFRS4]. For more bounds on $f(n)$ see [Par5, Chen, ChenJ, MoCa, WuSZR, ZhaBC1]. Summary of what is known and further progress are reported in two 2017 papers [ZhaCC2, ZhaCC3]. With $f(22) = 28$ obtained in [SunSh], the values of $f(n)$ are known for all $n \leq 22$. Also note item 4.3.e.

(d) $R(K_{1,n+1}, K_{2,2}) \leq R(K_{1,n}, K_{2,2}) + 2$ [Chen].

(e) $R(K_{2,\lambda+1}, K_{1,v-k+1})$ is either $v+1$ or $v+2$ if there exists a (v, k, λ)-difference set. This and other related results are presented in [Par4, Par5]. See also [GoCM, GuLi].

(f) Formulas and bounds on $R(K_{2,2}, K_{2,n})$, and bounds on $R(K_{2,2}, K_{m,n})$. In particular, we have $R(K_{2,2}, K_{2,k}) = n + k\sqrt{n} + c$, for $k = 2, 3, 4$, some prime powers $\lceil \sqrt{n} \rceil$ and $\lceil \sqrt{n} \rceil + 1$, and some $-1 \leq c \leq 3$ [HaMe4]. An improvement of the latter for some special cases of n was obtained in [Dyb1]. Asymptotics of $R(K_{2,2}, K_{n,n})$ is discussed in [LiuLi2], where in particular the lower bound $R(K_{2,2}, K_{n,n}) = \Omega(n^{3/2}/\log n)$ is presented. See also item 4.2.d.

(g) $R(K_{2,n}, K_{2,n}) \leq 4n-2$ for all $n \geq 2$, and the equality holds if and only if there exists a strongly regular $(4n-3, 2n-2, n-2, n-1)$-graph [EHM2].

(h) Conjecture that $4n-3 \leq R(K_{2,n}, K_{2,n}) \leq 4n-2$ for all $n \geq 2$. Many special cases are solved and several others are discussed in [LoM1].

(i) $R(K_{2,n-1}, K_{2,n}) \leq 4n-4$ for all $n \geq 3$, with the equality if there exists a symmetric Hadamard matrix of order $4n-4$. There are only 4 cases in which the equality is still open for $3 \le n \le 58$, namely 30, 40, 44 and 48 [LoM1].

(j) $R(K_{2,n-s}, K_{2,n}) \le 4n-2s-3$ for $s \ge 2$ and $n \ge s+2$, with the equality in many cases involving Hadamard matrices or strongly regular graphs. Asymptotics of $R(K_{2,n}, K_{2,m})$ for $m \gg n$ [LoM3].

(k) Some algebraic lower and upper bounds on $R(K_{s,n}, K_{t,m})$ for various combinations of n, m and $1 \le t, s \le 3$ [BaiLi, BaLX]. A general lower bound $R(K_{m,n}) \ge 2^m(n - n^{0.525})$ for large n [Dong].

(l) Upper bounds for $R(K_{2,2}, K_{m,n})$ for $m, n \ge 2$, with several cases identified for which the equality holds. Special focus on the cases for $m = 2$ [HaMe4].

(m) Let G be any isolate-free graph with p vertices and $q \ge 2$ edges. Then it holds that $R(K_{2,2}, G) \le 2q+1$, with the equality for $G = qK_2$ or $G = K_3$, and $R(K_{2,2}, G) \le 2p+q-2$. Some generalizations to $R(K_{2,k}, G)$ [JRB].

(n) Bounds for the numbers of the form $R(K_{k,n}, K_{k,m})$, specially for fixed k and close to the diagonal cases. Asymptotics of $R(K_{3,n}, K_{3,m})$ for $m \gg n$ [LoM2].

(o) $R(nK_{1,3}, mK_{1,3}) = 4n+m-1$ for $n \ge m \ge 1$, $n \ge 2$ [BES].

(p) Asymptotics for $K_{2,m}$ versus K_n [CaLRZ]. Upper bound asymptotics for $K_{k,m}$ versus K_n [LiZa1] and for some bipartite graphs K_n [JiSa].

(q) Special two-color cases apply in the study of asymptotics for multicolor Ramsey numbers for complete bipartite graphs [ChGra1].

4. Two Colors: Numbers Involving Cycles

4.1. Cycles, cycles versus paths and stars

Note: The paper *Ramsey Numbers Involving Cycles* [Ra4] is based on the revision #12 of this survey. It collects and comments on the results involving cycles versus any graphs, in two or more colors. It contains some more details than this survey, but only until 2009.

Cycles

(a) $R(C_3, C_3) = 6$ [GG, Bush],
$R(C_4, C_4) = 6$ [ChH1].

(b) $R(C_3, C_n) = 2n-1$ for $n \geq 4$, $R(C_4, C_n) = n+1$ for $n \geq 6$,
$R(C_5, C_n) = 2n-1$ for $n \geq 5$, and $R(C_6, C_6) = 8$ [ChaS].

(c) Result obtained independently in [Ros1] and [FS1], a new simpler proof in [KáRos]:

$$R(C_m, C_n) = \begin{cases} 2n-1 & \text{for } 3 \leq m \leq n,\ m \text{ odd},\ (m, n) \neq (3,3), \\ n-1+m/2 & \text{for } 4 \leq m \leq n,\ m \text{ and } n \text{ even},\ (m, n) \neq (4,4), \\ \max\{n-1+m/2,\ 2m-1\} & \text{for } 4 \leq m < n,\ m \text{ even and } n \text{ odd}. \end{cases}$$

(d) Characterization of all graphs critical for $R(C_4, C_n)$ [WuSR].

(e) $R(mC_3, nC_3) = 3n + 2m$ for $n \geq m \geq 1$, $n \geq 2$ [BES].

(f) $R(mC_4, nC_4) = 2n + 4m - 1$ for $m \geq n \geq 1$, $(n, m) \neq (1,1)$ [LiWa1].

(g) Formulas for $R(mC_4, nC_5)$ [LiWa2].

(h) Formulas and bounds for $R(nC_m, nC_m)$ [Den2, Biel1].

(i) Study of $R(S_1, S_2)$, where S_1 and S_2 are sets of cycles [Hans].
A conjecture generalizing 4.1.c stated in [Hans] was proved in [WaCh2].

(j) Unions of cycles, formulas and bounds for various cases including diagonal, different lengths, different multiplicities [MiSa, Den2], powers of cycles [AllBS], disjoint cycles versus K_n [Fuj2], and their relation to 2-local Ramsey numbers [Biel1].

Cycles versus paths

Result obtained by Faudree, Lawrence, Parsons and Schelp in 1974 [FLPS]:

$$R(C_m, P_n) = \begin{cases} 2n-1 & \text{for } 3 \leq m \leq n,\ m \text{ odd}, \\ n-1+m/2 & \text{for } 4 \leq m \leq n,\ m \text{ even}, \\ \max\{m-1+\lfloor n/2 \rfloor,\ 2n-1\} & \text{for } 2 \leq n \leq m,\ m \text{ odd}, \\ m-1+\lfloor n/2 \rfloor & \text{for } 2 \leq n \leq m,\ m \text{ even}. \end{cases}$$

For all n and m it holds that $R(P_m, P_n) \leq R(C_m, P_n) \leq R(C_m, C_n)$. Each of the two inequalities can become an equality, and, as derived in [FLPS], all four possible combinations of

$<$ and $=$ hold for an infinite number of pairs (m, n). For example, if both m and n are even, and at least one of them is greater than 4, then $R(P_m, P_n) = R(C_m, P_n) = R(C_m, C_n)$. For related generalizations see [BEFRS2].

Cycles versus stars

Only partial results for C_m versus stars are known. Lawrence [La1] settled the cases for odd m and for long cycles (see also [Clark, Par6]). The case for short even cycles is open, and it is related in particular to bipartite graphs. Partial results for $C_4 = K_{2,2}$ are pointed to in subsections 3.3.1 and 3.3.2, especially in the item 3.3.2.c. The most known general exact result [La1] is:

$$R(C_m, K_{1,n}) = \begin{cases} 2n+1 & \text{for odd } m \le 2n+1, \\ m & \text{for } m \ge 2n. \end{cases}$$

Some new cases for even m not too small with respect to n were settled in 2016, in particular the exact values of $R(C_6, K_{1,n})$ for all $n \le 11$ were completed in [ZhaBC5]. The equality $R(C_6, K_{1,12}) = 17$ was obtained in [SunSh]. The progress on asymptotics for large even m, and exact values for large even m and n not too large were obtained in [AllŁPZ].

4.2. Cycles versus complete graphs

Since 1976, it was conjectured that $R(C_n, K_m) = (n-1)(m-1) + 1$ for all $n \ge m \ge 3$, except $n = m = 3$ [FS4, EFRS2]. Various parts of this conjecture were proved as follows: for $n \ge m^2 - 2$ [BoEr], for $n > 3 = m$ [ChaS], for $n \ge 4 = m$ [YHZ1], for $n \ge 5 = m$ [BolJY+], for $n \ge 6 = m$ [Schi1], for $n \ge m \ge 7$ with $n \ge m(m-2)$ [Schi1], for $n \ge 7 = m$ [ChenCZ1], and for $n \ge 4m+2$, $m \ge 3$ [Nik]. Open conjectured cases are marked in Table V by "conj."

In 2019, Keevash, Long and Skokan [KeeLS] proved the above conjecture for $n \ge C \log m / \log\log m$ for some absolute constant $C \ge 1$, and furthermore that for any $\varepsilon > 0$ and $n > n(\varepsilon)$, for the lower bound it holds that $R(C_n, K_m) > m \log m \gg (n-1)(m-1) + 1$ for all $3 \le n \le (1-\varepsilon)\log m / \log\log m$.

(a) The first column in Table V gives data from the first row in Table I.

(b) Joint credit [He2/JR4] in Table V refers to two cases in which Hendry [He2] announced the values without presenting the proofs, which later were given in [JR4]. The special cases of $R(C_6, K_5) = 21$ [JR2] and $R(C_7, K_5) = 25$ were solved independently in [YHZ2] and [BolJY+]. The double pointer [JaBa/ChenCZ1] refers to two independent papers, similarly as [JaAl/ZZ3], except that in the latter case [ZZ3] refers to an unpublished manuscript. For joint credits marked in Table V with "-", the first reference is for the lower bound and the second for the upper bound.

(c) Erdős et al. [EFRS2] asked what is the minimum value of $R(C_n, K_m)$ for fixed m, and they suggested that it might be possible that $R(C_n, K_m)$ first decreases monotonically, then attains a unique minimum, then increases monotonically with n. If so, then the results in [KeeLS] stated above imply that this transition of behavior happens at $n = \Theta(\log m / \log\log m)$.

	C_3	C_4	C_5	C_6	C_7	C_8	C_9	...	C_n for $n \ge m$
K_3	6 GG-Bush	7 ChaS	9 ...	11	13	15	17	... ...	$2n-1$ ChaS
K_4	9 GG	10 ChH2	13 He4/JR4	16 JR2	19 YHZ1	22 ...	25	... ...	$3n-2$ YHZ1
K_5	14 GG	14 Clan	17 He2/JR4	21 JR2	25 YHZ2	29 BolJY+	33 ...	... ...	$4n-3$ BolJY+
K_6	18 Kéry	18 Ex2-RoJa1	21 JR5	26 Schi1	31 ...	36	41	... ...	$5n-4$ Schi1
K_7	23 Ka2-GrY	22 RaT-JR1	25 Schi2	31 CheCZN	37 CheCZN	43 JaBa/Ch+	49 Ch+	... ...	$6n-5$ Ch+
K_8	28 GR-McZ	26 RaT	29 LidP	36 ChenCX	43 ChenCZ1	50 JaAl/ZZ3	57 BatJA	... ...	$7n-6$ conj.
K_9	36 Ka2-GR	30 RaT-LaLR	33-36 LidP	41 LidP	49-58 LidP		65 conj.	... ...	$8n-7$ conj.
K_{10}	40-42 Ex5-GoR1	36 LaLR						...	$9n-8$ conj.
K_{11}	47-50 Ex20-GoR1	40-44 VO-LaLR						...	$10n-9$ conj.

Table V. Known Ramsey numbers $R(C_n, K_m)$;
Ch+ abbreviates ChenCZ1, for comments on joint credits see 4.2.b.

(d) There exist constants $c_1, c_2 > 0$ such that $c_1(m^{3/2}/\log m) \le R(C_4, K_m) \le c_2(m/\log m)^2$. The lower bound, obtained by Bohman and Keevash ([BohK1] in 2010, see also 4.2.j/k below) improved over an almost 40 years old bound $c(m/\log m)^{3/2}$ by Spencer [Spe2], using the probabilistic method. The upper bound was reported in a paper by Caro, Li, Rousseau and Zhang [CaLRZ], who in turn give the credit to an unpublished work by Szemerédi from 1980. A refined upper bound, $R(C_4, K_m) \le (1+o(1))(m/\log m)^2$, was presented by Liu and Li [LiuLi2] in 2021.

(e) Erdős, in 1981, in the Ramsey problems section of the paper [Erd3] formulated a challenge by asking for a proof of $R(C_4, K_m) < m^{2-\varepsilon}$, for some $\varepsilon > 0$. To date, no such proof is known.

(f) Enumeration of all (C_n, K_4)-graphs for $n \le 7$ [JaNR].

(g) A theta graph θ_n is obtained from the cycle C_n by adding one edge between some of its nonadjacent vertices. Summary of what is known about $R(\theta_n, K_k)$, and an additional result for $k = 6$, are collected in [BaJBJ].

(h) Let $C_{\leq n}$ be the set of cycles of length at most n, and let the *girth* $g(G)$ be the length of the shortest cycle in graph G. Probabilistic lower bound asymptotics for $R(C_{\leq n}, K_m)$ [Spe2] currently is the same as for $R(C_n, K_m)$, for fixed n. However, there are clear differences already for girth 4 and 5 and small m: Backelin [Back1, Back2] found that $R(C_{\leq 4}, K_m) = 6, 8, 11, 15, 18$ for $m = 3, 4, 5, 6, 7$, and that $R(C_{\leq 5}, K_m) = 5, 8, 10, 13, 15$, also for $m = 3, 4, 5, 6, 7$, respectively.

(i) Erdős et al. [EFRS2] proved various facts about $R(C_{\leq n}, K_m)$, and in particular that it is equal to $2m-1$ for $n \geq 2m-1$, and to $2m$ for $m < n < 2m-1$. The upper asymptotics for $R(C_{\leq n}, K_m)$ is implied in the study of independence number in graphs with odd girth n [Den1]. The following close to the diagonal exact values were obtained in [WuSL]: $R(C_{\leq n}, K_n)$ is equal to $2n$ and $2n+1$ for odd n and even n, respectively, and $R(C_{\leq n}, K_{n+1}) = 2n+3$ for odd $n \geq 5$ and even $n \geq 16$.

(j) $R(C_{\geq n}, K_{m_1, \ldots, m_k}) = (k-1)(n-1) + m_1$ for $m_1 \leq \cdots \leq m_k$, $5m_{k-1} + 3m_k \leq n$ [PoSu1]. The same equality holds for $R(C_n, K_{m_1, \ldots, m_k})$ for large m_i's and very large n [PoSu2].

(k) The best known lower bound asymptotics $R(C_n, K_m) = \Omega(m^{(n-1)/(n-2)}/\log m)$, for fixed n and large m, was obtained by Bohman and Keevash [BohK1]. Note that for $n = 4$ it gives the lower bound in 4.2.d above. See also [Spe2, FS4, AlRö] for previous results.

(l) Upper bound asymptotics [BoEr, FS4, EFRS2, CaLRZ, Sud1, LiZa2, AlRö, DoLL2].

4.3. Cycles versus wheels

Note: In this survey the wheel graph $W_n = K_1 + C_{n-1}$ has n vertices,
while some authors use the definition $W_n = K_1 + C_n$ with $n+1$ vertices.
For the cases involving $W_3 = C_3$ versus C_m see sections 3.2 and 4.2.

(a) $R(C_3, W_n) = 2n-1$ for $n \geq 6$ [BuE3]. All critical graphs have been enumerated.
The critical graphs are unique for $n = 3, 5$, and for no other n [RaJi].

(b) $R(C_4, W_n) = 14, 16, 17$ for $n = 11, 12, 13$, respectively [Tse1],
$R(C_4, W_n) = 18, 19, 20, 21$ for $n = 14, 15, 16, 17$, respectively [DyDz2], and several higher values and bounds, including 9 cases of n between 18 and 44 [WuSR, WuSZR].

(c) $R(C_4, W_n) \leq n + \lceil (n-1)/3 \rceil$ for $n \geq 7$ [SuBUB], which was improved to
$R(C_4, W_n) \leq n + \sqrt{n-2} + 1$ for $n \geq 11$ [DyDz2].

(d) $R(C_4, W_{q^2+1}) = q^2 + q + 1$ for prime power $q \geq 4$ [DyDz2],
exact values of $R(C_4, W_{q^2+2})$ and $R(C_4, W_{q^2-i})$ for special q and small i [WuSZR].

(e) $R(C_4, W_n) = R(C_4, K_{1,n-1})$ for $n \geq 7$ [ZhaBC1, ZhaBC2].

(f) Tight bounds on $R(C_4, W_n)$ for $46 \leq n \leq 93$ [NoBa].

(g) $R(C_7, W_n) = 2n-1$ for $n = 9, 10, 11$ [ZhaZZ].

	C_3	C_4	C_5	C_6	C_7	C_8	C_m	for
W_4	9 GG	10 ChH2	13 He4	16 JR2	19 YHZ1	22 ...	$3m-2$ ...	$m \geq 4$ YHZ1
W_5	11 Clan	9 Clan	9 He2	11 JR2	13 SuBB2	15 ...	$2m-1$ ...	$m \geq 5$ SuBB2
W_6	11 BuE3	10 JR3	13 ChvS	16 SuBB2	19 SuBB2	22 ...	$3m-2$ ...	$m \geq 4$ SuBB2
W_7	13 BuE3	9 Tse1	13 LuLL	11 LuLL	13 LuLL		$2m-1$ ...	$m \geq 10$ Ch1
W_8	15 BuE3	11 Tse1	15 LuLL	16 LuLL	19 Ch2	22 ...	$3m-2$ ...	$m \geq 6$ Ch2
W_9	17 BuE3	12 Tse1	17 LuLL	13 LuLL	17 LuLL		$2m-1$	$m \geq 13$ Ch1
W_{10}	19 BuE3	13 Tse1		16 Z1	19 Z2		$3m-2$	$m \geq 9$ Ch2
W_n for	... $2n-1$ $n \geq 6$ BuE3		$2n-1$ $n \geq 19$ Zhou2		$2n-1$ $n \geq 29$ Zhou2		large wheels	cycles

Table VI. Ramsey numbers $R(W_n, C_m)$ for $n \leq 10$, $m \leq 8$;
Ch1, Ch2, Z1, Z2 abbreviate ChenCMN, ChenCNZ, ZhaBC5, ZhaZZ, respectively.

(h) $R(W_n, C_m) = 2n-1$ for odd m with $n \geq 5m-6$ [Zhou2].
The range of n was extended in [ZhaZC].

(i) $R(W_n, C_m) = 3m-2$ for even $n \geq 4$ with $m \geq n-1$, $m \neq 3$, was conjectured by Surahmat et al. [SuBT1, SuBT2, Sur]. Parts of this conjecture were proved in [SuBT1, ZhaCC1, Shi5, ZhaBC2, ZhaZC], and the proof was completed in [ChenCNZ].

(j) Conjecture that $R(W_n, C_m) = 2m-1$ for odd $n \geq 3$ and all $m \geq 5$ with $m > n$ [Sur]. It was proved for $2m \geq 5n-7$ [SuBT1], and improved to $2m \geq 3n-1$ in [ChenCMN]. For further progress see also [Shi5, ZhaBC2,Sanh, RaeZ, Alw].

(k) Observe apparently four distinct situations with respect to parity of m and n.

(l) Cycles are Ramsey unsaturated for some wheels [AliSur],
see also comments on [BaLS] in item 5.16.e.

(m) Study of cycles versus generalized wheels $W_{k,n}$ [Sur, SuBTB, Shi5, ZhaBC2, BieDa].

4.4. Cycles versus books

	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_m	for
B_2	7 RoS1	7 F*al*6	9 C*al*	11 F*al*8	13 ...	15	17	19	21	$2m-1$ ...	$m \geq 4$ F*al*8
B_3	9 RoS1	9 F*al*6	10 F*al*8	11 JR2	13 Shi5	15 F*al*8	17 ...	19	21	$2m-1$ ...	$m \geq 6$ F*al*8
B_4	11 RoS1	11 F*al*6	11 F*al*8	12 S*al*1	13 S*al*1	15 Shi5	17 Shi5	19 F*al*8	21 ...	$2m-1$ ...	$m \geq 7$ F*al*8
B_5	13 RoS1	12 F*al*6	13 F*al*8	14 S*al*1	15 S*al*1	15 S*al*2	17 S*al*2	19 Shi5	21 Shi5	$2m-1$ ...	$m \geq 8$ F*al*8
B_6	15 RoS1	13 F*al*6	15 F*al*8	16 S*al*2	17 S*al*2	18 S*al*2	18 S*al*2		21 Shi5	$2m-1$ ...	$m \geq 11$ Shi5
B_7	17 RoS1	16 F*al*6	17 F*al*8	16 S*al*2	19 S*al*2	20 S*al*2	21 S*al*2			$2m-1$	$m \geq 13$ Shi5
B_8	19 RoS1	17 Tse1	19 F*al*8	17 S*al*2	19 S*al*2	22 S*al*2	≥ 23 S*al*2			$2m-1$	$m \geq 14$ Shi5
B_9	21 RoS1	18 Tse1	21 F*al*8	18 S*al*2			≥ 25 S*al*2	≥ 26 S*al*2		$2m-1$	$m \geq 16$ Shi5
B_{10}	23 RoS1	19 Tse1	23 F*al*8	19 S*al*2				≥ 28 S*al*2		$2m-1$	$m \geq 17$ Shi5
B_{11}	25 RoS1	20 Tse1	25 F*al*8							$2m-1$	$m \geq 19$ Shi5
B_n for	... $2n+3$ $n \geq 2$ RoS1	$\approx n$ some (c)	... $2n+3$ $n \geq 4$ F*al*8		$2n+3$ $n \geq 15$ F*al*8		$2n+3$ $n \geq 23$ F*al*8		$2n+3$ $n \geq 31$ F*al*8	large books	cycles

Table VII. Ramsey numbers $R(B_n, C_m)$ for $n, m \leq 11$;
et al. abbreviations: F*al*/FRS, C*al*/ChRSPS, S*al*1/ShaXBP, S*al*2/ShaXB.

(a) For the cases of $B_1 = K_3$ versus C_m see section 4.2.
The exact values for the cases $(3,7)$, $(4,8)$, $(4,9)$, $(5,10)$, $(5,11)$ were obtained independently in [S*al*1, S*al*2]/[ShaXBP, ShaXB] using computer algorithms.

(b) $R(C_4, B_{12}) = 21$ [Tse1], $R(C_4, B_{13}) = 22$, $R(C_4, B_{14}) = 24$ [Tse2].
$R(C_4, B_8) = 17$ [Tse2] (it was reported incorrectly in [FRS7] to be 16).

(c) $q^2 + q + 2 \leq R(C_4, B_{q^2-q+1}) \leq q^2 + q + 4$ for prime power q [FRS7]. B_n is a subgraph of B_{n+1}, hence likely $R(C_4, B_n) = n + O(\sqrt{n})$ (compare to $R(C_4, K_{2,n})$ in section 3.3).

(d) $R(B_n, C_m) = 2n + 3$ for odd $m \geq 5$ with $n \geq 4m - 13$ [FRS9].

(e) $R(B_n, C_m) = 2m - 1$ for $n \geq 1$, $m \geq 2n + 2$ [FRS9]. The range of m was extended to $m \geq 2n - 1 \geq 7$ in [ShaXB], and to $m > (6n + 7)/4$ in [Shi5].

(f) Close to the diagonal we have $R(B_n, C_n) \geq 3n-2$ and $R(B_{n-1}, C_n) \geq 3n-4$ for $n \geq 3$ [ShaXB], and for all sufficiently large n it holds [LinP]:

$$R(B_n, C_m) = \begin{cases} 3m-2 & \text{for } 9n/10 \leq m \leq n, \\ 3n-2 & \text{if } m = n+1, \\ 3n & \text{for } n+2 \leq m \leq 10n/9. \end{cases}$$

(g) More theorems on $R(B_n, C_m)$ in [FRS7, FRS9, NiRo4, Zhou1].

(h) Cycles versus some generalized books $B_n^{(k)} = nK_1 + K_k$ [Shi5]. Exact asymptotics for odd cycles versus $B_n^{(k)}$ [LiuLi1], and for general cases close to the diagonal [LinP].

4.5. Cycles versus other graphs

(a) C_4 versus stars [Par3, Par4, Par5, BEFRS4, Chen, ChenJ, GoMC, MoCa, WuSZR, SunSh]. For several exact results see $K_{2,2}$ in Tables IVa and IVb, and for general results see items 3.3.1.a, 3.3.2.c, 3.3.2.d and 4.3.e.

(b) C_4 versus unions of stars [HaABS, Has, HaJu]

(c) C_4 versus trees [EFRS4, Bu7, BEFRS4, Chen]

(d) C_4 versus all graphs on six vertices [JR3]

(e) C_4 versus various types of complete bipartite graphs, see [LiuLi2] and section 3.3.

(f) $R(C_4, G) \leq 2q+1$ for any isolate-free graph G with $q \geq 2$ edges, and the equality holds for $G = qK_2$ or $G = K_3$ [RoJa2, JRB].

(g) $R(C_4, G) \leq 2p+q-2$ for any isolate-free graph G on p vertices and $q \geq 2$ edges [JRB].

(h) $R(C_5, K_6-e) = 17$ [JR4]

(i) $R(C_5, K_4-e) = 9$ [ChRSPS]

(j) C_5 versus all graphs on six vertices [JR4]

(k) $R(C_6, K_5-e) = 17$ [JR2]

(l) C_6 versus all stars up to $K_{1,12}$ [ZhaBC5, SunSh]
C_6 versus all graphs on five vertices [JR2]

(m) $R(C_{2m+1}, G) = 2n-1$ for sufficiently large sparse graphs G on n vertices, in particular $R(C_{2m+1}, T_n) = 2n-1$ for all $n > 1512m+756$, for n-vertex trees T_n [BEFRS2].
The range of n for trees was extended to $n \geq 25(2m+1)$ in [Bren2].

(n) $R(C_n, G) \leq 2q + \lfloor n/2 \rfloor - 1$, for $3 \leq n \leq 5$, for any isolate-free graph G with $q > 3$ edges. It is conjectured that it also holds for other n [RoJa2].

(o) Cycles versus trees [BEFRS2, FSS2]

(p) Cycles versus fans [Shi5]

(q) Exact asymptotics of odd cycles versus generalized fans [LiuLi1]

(r) Monotone paths and cycles [Lef]

(s) Cycles versus $K_{n,m}$ and multipartite complete graphs [BoEr, PoSu1, PoSu2]

(t) Cycles versus generalized books and wheels [Shi5, Sur, SuBTB]

(u) Cycles versus special graphs of the form $K_n + G$ with small $n \leq 3$ and sparse G [Shi5]

5. General Graph Numbers in Two Colors

This section includes data with respect to general graph results. We tried to include all nontrivial values and identities regarding exact results, or references to them, but only those out of general bounds and other results which, in our opinion, may have a direct connection to the evaluation of specific numbers. If some small value cannot be found below, it may be covered by the cumulative data gathered in section 8, or be a special case of a general result listed in this section. Note that $P_2 = K_2$, $B_1 = F_1 = C_3 = W_3 = K_3$, $B_2 = K_4 - e$, $P_3 = K_3 - e$, $W_4 = K_4$ and $C_4 = K_{2,2}$ imply other identities not mentioned explicitly.

5.1. Paths

$R(P_m, P_n) = n + \lfloor m/2 \rfloor - 1$ for all $n \geq m \geq 2$ [GeGy]

Classification of $R(P_m, P_n)$-critical graphs [Hook]

Stripes mP_2 [CocL1, CocL2, Lor]

Disjoint unions of paths (also called linear forests) [BuRo2, FS2]

Monotone paths [CaYZ], ordered path powers [Mub2]

5.2. Wheels

Note: In this survey the wheel graph $W_n = K_1 + C_{n-1}$ has n vertices,
while some authors use the definition $W_n = K_1 + C_n$ with $n+1$ vertices.

(a) $R(W_3, W_n) = 2n - 1$ for all $n \geq 6$ [BuE3],
All critical colorings for $R(W_3, W_n)$ for all $n \geq 3$ [RaJi].

(b) The graph $3K_{m-1}$ is a witness of $3m - 2 \leq R(W_m, W_n)$ for all even n, and the graph $2K_{m-1}$ is a witness of $2m - 1 \leq R(W_m, W_n)$ for all m and n. In Table VIII, the lower bounds without a credit are implied by these inequalities.

(c) $R(W_n, W_n) \leq 8n - 10$ for even n, and $R(W_n, W_n) \leq 6n - 8$ for odd n [MaoWMS].

(d) All critical colorings (2, 1 and 2) for $R(W_n, W_6)$, for $n = 4, 5, 6$ [FM].

(e) $R(W_6, W_6) = 17$, $R(4,4) = 18$ and $\chi(W_6) = 4$ give a counterexample $G = W_6$
to the Erdős conjecture (Erd2, see also [GRS]) that $R(G, G) \geq R(K_{\chi(G)}, K_{\chi(G)})$.

(f) The value $R(W_5, W_5) = 15$ was given in the Hendry's table [He2] without a proof.
Later the proof was published in [HaMe2].

m \ n	3	4	5	6	7	8	9	10
3	6	9 GG	11 Clan	11 BuE3	13 BuE3	15 BuE3	17 BuE3	19 BuE3
4		18 GG	17 He3	19 FM	21 VO-LidP	22-26 LidP	≥25	
5			15 He2	17 FM	13-16 LidP	17 VO-LidP	≥17	≥21 VO
6				17 FM	19 LidP	22-26 LidP	≥25	
7					19 VO-LidP	19-21 LidP	≥19 VO	≥21 VO
8						22-25 LidP	≥25	
9							≥21 VO	
10								≥28

Table VIII. Ramsey numbers $R(W_m, W_n)$ for $m \le n \le 10$.

5.3. Books, $B_n = K_2 + \bar{K}_n$

(a) $R(B_m, B_n) \le R(B_{m+1}, B_n)$ and $R(B_m, B_n) = R(B_n, B_m)$ hold for all $m, n \ge 1$.

(b) $R(B_1, B_n) = 2n+3 \le R(B_2, B_n)$ for all $n > 1$ [RoS1].

(c) $R(B_2, B_n) \le 2n+6$ for all $n > 1$ [RoS1], $R(B_2, B_n) \le 2n+5$ for $12 \le n \le 22$, $R(B_2, B_n) \le 2n+4$ for $23 \le n \le 37$, $R(B_2, B_n) = 2n+3$ for $n \ge 38$ [FRS8].

(d) There are 4 Ramsey-critical graphs for $R(B_2, B_3)$, a unique graph for $R(B_3, B_4)$ [ShaXBP], 3 for $R(B_2, B_6)$ and 65 for $R(B_2, B_7)$ [BlLR].

(e) Unpublished result $R(B_2, B_6) = 17$ [Rou] was confirmed in [BlLR].

(f) $R(B_n, B_n) = 4n+2$ for $4n+1$ a prime power.
If $4n+1$ is not the sum of two integer squares, then $R(B_n, B_n) \le 4n+1$ [RoS1].

(g) If $2(m+n)+1 > (n-m)^2/3$, then $R(B_m, B_n) \le 2(m+n+1)$ and $R(B_{n-1}, B_n) \le 4n-1$. Furthermore, if $n = 2 \pmod 3$ then $R(B_{n-2}, B_n) \le 4n-3$ [RoS1].

(h) Strongly regular graphs often provide good lower bounds. If there exists a strongly regular graph with the parameters (v, k, λ, μ), then $R(B_{\lambda+1}, B_{v-2k+\mu-1}) \ge v+1$. The lower bounds for a number of specific larger cases, like $R(B_{62}, B_{65}) = 256$ [RoS1] or $254 \le R(B_{37}, B_{88}) \le 255$ [Par6], are implied by the existence of a strongly regular graph with suitable parameters. 12 exact values of $R(B_m, B_n)$, beyond Table IXa, where

m \ n	1	2	3	4	5	6	7	8	9	10	11
1	6	7 ChH2	9 Clan	11 RoS1	13 RoS1	15 RoS1	17 RoS1	19 RoS1	21 RoS1	23 RoS1	25 RoS1
2		10 ChH1	11 Clan	13 Rou	16 RoS1	17 Rou	18 BlLR	19-22 5.3.bc	21-24 5.3.bc	23-26 5.3.bc	28 FRS8
3			14 RoS1	15 Sh1+	17 RoS1	≤19 LidP					
4				18 RoS1	≤19 5.3.g	22 RoS1			27-28 5.3.hg		
5					21 RoS1	≤23 5.3.g					
6						26 5.3.f	≤27 5.3.g	≤29 5.3.g			
7							30 5.3.f	≤31 5.3.g		36 5.3.hg	37-38 5.3.hg
8								≤33 5.3.f	≤35 5.3.g		
9									38 5.3.f	≤39 5.3.g	≤41 5.3.g

Table IXa. Ramsey numbers $R(B_m, B_n)$ for $m \leq 9$ and $1 \leq m \leq n \leq 11$;
See more details of items 5.3.b/c/f/g/h below, their further use leads to bounds not listed in the table. Sh1+ abbreviates ShaXBP.

m	n	$R(B_m B_n)$	v	k	λ	μ
11	11	46	45	22	10	11
14	17	64	63	30	13	15
23	26	100	99	48	22	24
22	37	120	119	54	21	27
29	38	136	135	64	28	32
34	37	144	143	70	33	35
47	50	196	195	96	46	48
46	58	210	209	100	45	50
56	56	226	225	112	55	56
38	82	244	243	110	37	60
62	65	256	255	126	61	63
69	71	281	280	135	70	60

Table IXb. Exact values of $R(B_m, B_n)$ from strongly regular (v, k, λ, μ)-graphs on up to 280 vertices, using 5.3.g/h [NiRo3]. It includes only the cases beyond Table IXa, and excludes the cases of $m = n$ for $4n + 1$ prime power, as in 5.3.f.

this lower bound meets the upper bound in 5.3.g were collected by Nikiforov and Rousseau [NiRo3], and they are presented in Table IXb. For a great collection of strongly regular graphs see the website by A. E. Brouwer [Brou].

(i) $R(B_m, B_n) = 2n+3$ for all $n \geq cm$ for some $c < 10^6$ [NiRo2, NiRo3].

(j) $R(B_n, B_n) = (4+o(1))n$ [RoS1, NiRS].

(k) For generalized books $B_n^{(k)} = nK_1 + K_k$, Conlon proved that $R(B_n^{(k)}, B_n^{(k)}) = 2^k n + o_k(n)$ [Con4]. A simplified proof, better control of the error term, and a proof that all extremal colorings for this Ramsey problem are quasirandom are in the follow-up paper [ConFW]. This more than answered some old questions by Erdős and others.

(l) Other general equalities and bounds involving $R(B_m, B_n)$ can be found in [RoS1, FRS8, Par6, NiRo2, NiRo3, NiRS, LiRZ2].

5.4. Trees and forests

In this subsection T_n and F_n denote an n-vertex tree and forest, respectively.

(a) $R(T_n, T_n) \leq 4n+1$ [ErdG]. Note that if T_n were a set of all n-vertex trees, then one might say that $R(T_n, T_n) = n$, since for every graph G at least one of G or $\overline{G}$ is connected, and thus it contains an n-vertex spanning tree.

(b) $R(T_n, T_n) \geq \lfloor (4n-1)/3 \rfloor$ [BuE2], see also section 5.15.

(c) Conjecture that $R(T_n, T_n)$ is at most $2n-2$ for even n and $2n-3$ for odd n [BuE2]. Note that this is the same as asking if $R(T_n, T_n) \leq R(K_{1,n-1}, K_{1,n-1})$. Zhao [Zhao] proved that $R(T_n, T_n) \leq 2n-2$ and thus confirmed the conjecture for even n. Independently, Ajtai et al. [AjKSS] announced a full proof for large n. This recent progress subsumes some of the results pointed to in items (d)-(m) below.

(d) For general discussion of related problems see [Bu7, FSS2, ChGra2], in particular of the conjecture that $R(T_m, T_n) \leq n+m-2$ holds for all trees [FSS2].

(e) If $\Delta(T_m) = m-2$ and $\Delta(T_n) = n-2$ then the exact values of $R(T_m, T_n)$ are known, and they are between $n+m-5$ and $n+m-3$ depending on n and m. In particular, we have $R(T_n, T_n) = 2n-5$ for even n and $R(T_n, T_n) = 2n-4$ for odd n [GuoV].

(f) Examples of families T_m and T_n (including P_n) for which $R(T_m, T_n) = n+m-c$, $c = 3, 4, 5$ [SunZ1], extending the results in [GuoV].

(g) View the tree T as a bipartite graph with parts t_1 and t_2, $t_2 \geq t_1$, then define $b(T) = \max\{2t_1 + t_2 - 1, 2t_2 - 1\}$. Then the bound $R(T, T) \geq b(T)$ holds always, $R(T, T) = b(T)$ holds for many classes of trees [EFRS3, GeGy], and asymptotically [HaŁT], but cases for inequality have been found [GrHK].

(h) Comments in [BaLS] about some conjectures on Ramsey saturation of non-star trees, which would imply that $R(T_n, T_n) \leq 2n-2$ holds for sufficiently large n.

(i) Formulas for $R(T_m, T_n)$ for some subcases of when T_m and T_n satisfy $\Delta(T_m) = m-3$ and $\Delta(T_n) \geq n-3$ [SunWW].

(j) $R(T_m, K_{1,n}) \le m+n-1$, with equality for $(m-1) \mid (n-1)$ [Bu1].

(k) $R(T_m, K_{1,n}) = m+n-1$ for sufficiently large n for almost all trees T_m [Bu1]. Many cases were identified for which $R(T_m, K_{1,n}) = m+n-2$ [Coc, ZhZ1], see also [Bu1].

(l) $R(T_m, K_{1,n}) \le m+n$ if T_n is not a star and $(m-1) \nmid (n-1)$,
some classes of trees and stars for which the equality holds [GuoV].

(m) In a sequence of papers [SunZ1, SunZ2, SunW, SunWW], Zhi-Hong Sun et al. obtain several exact results for $R(S, T)$, where the trees S and T have high maximum degree $\Delta \ge n-3$, or one of them has high maximum degree and the other is a path.

(n) Formulas for some cases of brooms [EFRS3], where broom is a star with a path attached to its center. These results were extended to all diagonal cases for brooms [YuLi]. Note that a tree T_n with $\Delta(T_n) = n-2$ is a broom, and this case is listed in 5.4.e.

(o) $R(F_n, F_n) > n + \log_2 n - O(\log\log n)$ [BuE2], forests are tight for this bound [CsKo].

(p) Forests, linear forests (unions of paths) [BuRo2, FS3, CsKo].

(q) Extensive tables of $R(T_m, T_n)$ for $6 \le m, n \le 8$, for many concrete pairs of trees, which were obtained through an adiabatic quantum optimization algorithm [RanMCG].

(r) Tristars and fountains [BroNN].

(s) Paths versus trees [FSS2], see also other parts of this survey involving special graphs,
in particular sections 5.5, 5.6, 5.10, 5.12 and 5.15.

5.5. Stars, stars versus other graphs

$R(K_{1,n}, K_{1,m}) = n+m-\varepsilon$, where $\varepsilon = 1$ for even n and m, and $\varepsilon = 0$ otherwise [Har1]. This is also a special case of multicolor numbers for stars 6.6.e obtained in [BuRo1].

$R(K_{1,n}, K_m) = n(m-1)+1$ by Chvátal's theorem [Chv].

Stars versus C_4 [Par3, Par4, Par5, BEFRS4, Chen, ChenJ], until 2002
Stars versus C_4 [GoMC, MoCa, WuSZR, ZhaBC1, ZhaCC2, ZhaCC3], since 2004
Stars versus $K_{2,n}$ [Par4, GoMC]
Stars versus $K_{n,m}$ [Stev, Par3, Par4]
See also section 3.3

$R(K_{1,4}, B_4) = 11$ [RoS2]
$R(K_{1,4}, K_{1,2,3}) = R(K_{1,4}, K_{2,2,2}) = 11$ [GuSL]

Stars versus paths [Par2, BEFRS2]
Stars versus cycles [La1, Clark, ZhaBC5, SunSh], see also [Par6] and section 4.1
Stars versus $2K_2$ [MeO]
Stars versus stripes mP_2 [CocL1, CocL2, Lor]
Stars versus bistars [AlmHS]
Stars versus kipas [LiZB]

Stars versus W_5 and W_6 [SuBa1]
$nK_{1,m}$ versus W_5 [BaHA]

Stars versus W_9 [Zhang2, ZhaCZ1]
Stars versus wheels [HaBA1, ChenZZ2, Kor, LiSch, HagMa]
Stars versus books [ChRSPS, RoS2]
Stars versus fans [ZhaBC3]
Stars versus trees [Bu1, Cheng, Coc, GuoV, SunZ1, SunZ2, SunWW, ZhZ1]
Stars versus $K_n - tK_2$ [Hua1, Hua2]
Stars versus almost all connected graphs on 6 vertices [LoM7]

Union of two stars [Gros2]
Asymptotics for double stars [NoSZ]
Double stars versus $K_{2,q}$ and sK_2 versus $K_s + C_n$ [SuAUB]
Unions of stars versus C_4 and W_5 [HaABS, Has, HaJu]
Unions of stars versus wheels [BaHA, HaBA2, SuBAU1]

5.6. Paths versus other graphs

Note: for cycles versus P_n see section 4.1.

P_3 versus all isolate-free graphs [ChH2]
Paths versus stars [Par2, BEFRS2]
Paths versus trees [FS4, FSS2, SunZ1, SunZ2, SunWW]
Paths versus books [RoS2]
Paths versus K_n [Par1]
Paths versus $2K_n$ [SuAM, SuAAM]
Paths versus $K_{n,m}$ [Häg]
Paths versus some balanced complete multipartite graphs [Pokr]
Paths versus W_5 and W_6 [SuBa1]
Paths versus W_7 and W_8 [Bas]
Paths versus wheels [BaSu, ChenZZ1, SaBr3, Zhang1]
Paths versus wheels, the last piece completed [LiNing2]
$R(P_n, mW_4) = 2n + m - 2$ [Sudar1]
Paths versus beaded wheels [AliBT2]
Paths versus sunflower graphs [AliTJ]
Paths versus powers of paths [Pokr, AllBS]
Paths versus fans [SaBr2]
Paths versus $K_1 + P_m$ [SaBr1, SaBr4]
Paths versus kipas [LiZBBH]
Paths versus $K_1 + F$, where F is a linear forest [LiNing1]
Paths versus Jahangir graphs [SuTo]
Paths and cycles versus trees [FSS2]

Powers of paths [AllBS]
Unions of paths [BuRo2]
Paths and unions of paths versus tK_n [Sudar2]
Paths and unions of paths versus Jahangir graphs [AliBas, AliBT1, AliSur]

Paths and unions of paths versus $K_{2m} - mK_2$ [AliBB]
Goodness of paths for tK_n [Sudar3]
Goodness of paths, results on graphs H for which P_n is H-good [PoSu1]
Sparse graphs versus paths and cycles [BEFRS2]
Graphs with long tails [Bu2, BuG]
Long paths versus other good graphs [PeiLi, PeiCLY]
Paths versus generalized wheels [BieDa]
Monotone paths [Lef, CaYZ] and monotone cycles [Lef]

5.7. Fans, fans versus other graphs

The fan graph F_n is defined by $F_n = K_1 + nK_2$.

$R(F_1, F_n) = R(K_3, F_n) = 4n+1$ for $n \geq 2$, and bounds for $R(F_m, F_n)$ [LiR2, GuGS]
$R(F_2, F_n) = 4n+1$ for $n \geq 2$ and $R(F_m, F_n) \leq 4n+2m$ for $n \geq m \geq 2$ [LinLi1]
$9n/2 - 5 \leq R(F_n, F_n) \leq 11n/2 + 6$ for all $n \geq 1$ [ChenYZ]
$R(K_4, F_n) = 6n+1$ for $n \geq 3$ [SuBB3]
$R(K_5, F_n) = 8n+1$ for $n \geq 5$ [ZhaCh]
$R(K_6, F_n) = 10n+1$ for $n \geq 6$ [KaOS]
A conjecture that $R(K_m, F_n) = 2mn - 2n + 1$ for $n \geq m \geq 4$ [SuBB3]
Fans versus paths, formulas for a number of cases including $R(P_6, F_n)$ [SaBr2].
Missing case $R(P_6, F_4) = 12$ solved in [Shao].
$R(F_m, K_n) \leq (1+o(1))n^2/\log n$ [LiR2]
Fans versus cycles [Shi5]
Exact asymptotics of odd cycles versus generalized fans [LiuLi1]
Fans versus wheels [ZhaBC4, MengZZ]
Fans versus trees and stars [ZhaBC3, Bren1]
Fans versus unicyclic graphs [Bren1]
Lower bounds on $R(F_2, K_n)$ from cyclic graphs for $n \leq 9$ [Shao]

5.8. Wheels versus other graphs

Notes: In this survey the wheel graph $W_n = K_1 + C_{n-1}$ has n vertices, while some authors use the definition $W_n = K_1 + C_n$ with $n+1$ vertices. For cycles versus W_n see section 4.3. Consider also similarity of wheels to other graphs, like fans, kipas [LiZBBH], sunflower [AliTJ], and Jahangir graphs [SuTo].

$R(W_5, K_5 - e) = 17$ [He2][YH]
$R(W_5, K_5) = 27$ [He2][RaST]
$33 \leq R(W_5, K_6) \leq 36$ [ShaoWX, LidP]
$45 \leq R(W_5, K_7) \leq 50$ [VO, LidP]
$34 \leq R(W_6, K_6) \leq 40$ [VO, LidP]
$43 \leq R(W_6, K_7) \leq 55$ [VO, LidP]

W_5 and W_6 versus stars and paths [SuBa1]
W_5 versus $nK_{1,m}$ [BaHA]
W_5 versus unions of stars [Has]
W_5 versus theta graphs θ_n [JaBVR]
W_5 and W_6 versus trees [BaSNM]
W_7 and W_8 versus paths [Bas]

W_7 versus trees T_n with $\Delta(T_n) \geq n-3$, other special trees T, and T_n for $n \leq 8$ [ChenZZ3, ChenZZ5, ChenZZ6]

W_7 and W_8 versus trees [ChenZZ4, ChenZZ5]
W_9 versus stars [Zhang2, ZhaCZ1, ZhaCC4, ZhaCC5]
W_9 versus trees of high maximum degree [ZhaCZ2]
W_{2n} versus trees of high maximum degree [HafBa]
$R(C_4, W_n) = R(C_4, K_{1,n-1})$ for $n \geq 7$ [ZhaBC1].

Wheels versus stars [HaBA1, ChenZZ2, Kor, LiSch, HagMa]
Wheels W_n, for even n, versus star-like trees [SuBB1]
Wheels versus paths [BaSu, ChenZZ1, SaBr3, Zhang1]
Wheels versus paths, the last piece completed [LiNing2]
Wheels versus fans and wheels [ZhaBC4, MengZZ]
Wheels versus some trees [RaeZ, ZhuZL]
Wheels versus books [Zhou3]
Wheels versus unions of stars [BaHA, HaBA2, SuBAU1]
Wheels versus linear forests (disjoint unions of paths) [SuBa2]
Some cases of wheels versus $K_n - K_{1,s}$ [ChaMR]
Generalized wheels versus cycles [Shi5, BieDa]
Generalized wheels versus trees [WaCh]
Upper asymptotics for $R(W_n, K_m)$ [Song5, SonBL]
Upper asymptotics for generalized wheels versus K_n [Song9]

5.9. Books versus other graphs

Note: for cycles versus B_n see section 4.4.

$R(B_3, K_4) = 14$ [He3]
$R(B_3, K_5) = 20$ [He2][BaRT]
$R(B_4, K_{1,4}) = 11$ [RoS2]

Cyclic lower bounds for $R(B_m, K_n)$ for $m \leq 7$, $n \leq 9$ and for $R(B_3, K_n - e)$ for $n \leq 7$ [Shao, ShaoWX]

$R(T_n, B_m) = 2n-1$ for all $n \geq 3m-3$ [EFRS7]

Books versus paths [RoS2]
Books versus stars [ChRSPS, RoS2] Books versus trees [EFRS7, ZhaCZ]
Books versus K_n [LiR1, Sud2]
Books versus wheels [Zhou3]

Books versus $K_2 + C_n$ [Zhou3]
Books and (K_1 + *tree*) versus K_n [LiR1]
Generalized books $K_3 + qK_1$ versus cycles [Shi5]
Generalized books $K_r + qK_1$ versus $K_1 + C_4$ [LinLiu]
Generalized books $K_r + qK_1$ versus K_n [NiRo1, NiRo4]

5.10. Trees and forests versus other graphs

In this subsection T_n and F_n denote n-vertex tree and forest, respectively.

$R(T_n, K_m) = (n-1)(m-1)+1$ [Chv]

$R(C_{2m+1}, T_n) = 2n-1$ for all $n > 1512m + 756$, for n-vertex trees T_n [BEFRS2].
The range of n was extended to $n \geq 25(2m+1)$ in [Bren2].

$R(T_n, B_m) = 2n-1$ for all $n \geq 3m-3$ [EFRS7]

$R(F_{nk}, K_m) = (n-1)(m-2)+nk$ for all forests F_{nk} consisting of k trees with n vertices each, also exact formula for all other cases of forests versus K_m [Stahl]

Exact results for almost all small ($n(G) \leq 5$) connected graphs G versus all trees [FRS4]

Trees versus stars [Bu1, Cheng, Coc, GuoV, ZhZ1]
Trees versus paths [FS4, FSS2]
Trees versus C_4 [EFRS4, Bu7, BEFRSS5, Chen]
Trees versus cycles [FSS2, EFRS6]
Trees versus books [EFRS7, ZhaCZ]
Trees versus fans [ZhaBC3]
Trees versus W_5 and W_6 [BaSNM]
Trees versus W_7 and W_8 [ChenZZ4, ChenZZ5]
Some trees versus wheels [RaeZ, ZhuZL]
Trees versus wheels [ZhaBC4]

Trees T_n with $\Delta(T_n) \geq n-3$, other special trees T,
and T_n for $n \leq 8$ versus W_7 [ChenZZ3, ChenZZ5, ChenZZ6]

Trees T_n with $\Delta(T_n) \geq n-4$ versus W_9 [ZhaCZ2]

Trees T_n with large $\Delta(T_n)$ versus W_{2m} [HafBa]

Star-like trees versus odd wheels [SuBB1, ChenZZ3]
Trees versus $K_n + \overline{K}_m$ [RoS2, FSR]
Trees versus generalized wheels [WaCh]
Trees versus bipartite graphs [BEFRS4, EFRS6]
Trees versus almost complete graphs [GoJa2]
Trees versus multipartite complete graphs [EFRS8, BEFRSGJ]

$R(T, G)$ for most non-star trees T and $n(G) \leq 6$ [LoM8], see item 8.1.q

Linear forests versus $3K_3$ and $2K_4$ [SuBAU2]
Linear forests versus $2K_m$ [SuAAM]

Linear forests versus tK_n [Sudar2, Sudar3]
Linear forests versus wheels [SuBa2]
Forests versus almost complete graphs [ChGP]
Forests versus complete graphs [BuE1, Stahl, BaHA]

Goodness of bounded degree trees [BalPS]

Study of graphs G for which all or almost all trees are G-good [BuF, BEFRSGJ], see also section 5.15 and 5.16, item [Bu2], for the definition and more pointers.

See also various parts of this survey for special trees, and section 5.4.

5.11. Cases for $n(G),\ n(H) \le 5$

Clancy [Clan], in 1977, presented a table of $R(G, H)$ for all isolate-free graphs G with $n(G) = 5$ and H with $n(H) \le 4$, except 5 entries. All five of the open entries have been solved as follows:

$R(B_3, K_4) = 14$	[He3]
$R(K_5, K_4 - e) = 16$	[BoH]
$R(W_5, K_4) = 17$	[He2]
$R(K_5 - e, K_4) = 19$	[EHM1]
$R(K_5, K_4) = R(4, 5) = 25$	[MR4]

An interesting case in [Clan] is:

$R(K_4, K_5 - P_3) = R(K_4, K_4 + e) = R(4, 4) = 18$

Hendry [He2], in 1989, presented a table of $R(G, H)$ for all graphs G and H on 5 vertices without isolates, except 7 entries. Five of the open entries have been solved:

$R(K_5, K_4 + e) = R(4, 5) = 25$	[Ka1][MR4]
$R(K_5, K_5 - P_3) = 25$	[Ka1][Boza2, CalSR]
$R(K_5, B_3) = 20$	[He2][BaRT]
$R(K_5, W_5) = 27$	[He2][RaST]
$R(W_5, K_5 - e) = 17$	[He2][YH]

The still open cases for K_5 versus $K_5 - e$ and K_5 are:

$30 \le R(K_5, K_5 - e) \le 33$	[Ex6][Boza7]
$43 \le R(K_5, K_5) \le 48$	[Ex4][AnM1]

All critical colorings for the case $R(C_5 + e, K_5) = 17$ were found by Hendry [He5].

5.12. Miscellaneous cases

$R(P, P) \ge 19$, where P is the 10-vertex Petersen graph [HaKr2]

$R(Q_3, Q_3) = 13$, where Q_3 is the 8-vertex 3-dimensional cube graph [LidP]

$30 \le R(K_{2,2,2}, K_{2,2,2}) \le 31$, where $K_{2,2,2}$ is the octahedron [HaKr2, LidP]

Unicyclic graphs [Gros1, Köh, KrRod]
$K_{2,m}$ and C_{2m} versus K_n [CaLRZ]
$K_{2,n}$ versus any isolate-free graph [RoJa2, JRB]
Union of two stars [Gros2]
Double stars* [GrHK, BahS, NoSZ]
Formulas for some cases of brooms+ [EFRS3], extended to all diagonal cases [YuLi]
Graphs with bridge versus K_n [Li1]
Multipartite complete graphs [BFRS, FRS3, Stev]
Multipartite complete graphs versus trees [EFRS8, BEFRSGJ]
Multipartite complete graphs versus sparse graphs [EFRS4]
Graphs with long tails [Bu2, BuG]

5.13. Multiple copies of graphs, disconnected graphs

(a) $2K_2$ versus isolate-free graphs [ChH2],
nK_2 versus isolate-free graphs [FSS1].

(b) nK_2 versus mK_2, in particular $R(nK_2, nK_2) = 3n - 1$ for $n \ge 1$ [CocL1, CocL2, Lor]

(c) $R(nK_3, nK_3) = 5n$ for $n \ge 2$, $R(mK_3, nK_3) = 3m + 2n$ for $m \ge n \ge 2$ [BES].

(d) Let $c(nK_k)$ denote the set of connected graphs containing n vertex disjoint K_k's. Then:
$R(c(nK_3), c(nK_3)) = 7n - 2$ for $n \ge 2$ [GySá3], and
$R(c(nK_k), c(nK_k)) = (k^2 - k + 1)n - k + 1$ for $k \ge 4$ and $n \ge R(k, k)$ [Rob].

(e) nK_3 versus mK_4 [LorMu]

(f) $nK_{1,m}$ versus W_5 [BaHA]

(g) $R(nK_4, nK_4) = 7n + 4$ for large n [Bu8]

(h) Stripes mP_2 [CocL1, CocL2, Lor]

(i) $R(G, H)$ for all disconnected isolate-free graphs H on at most 6 vertices versus all G on at most 5 vertices, except 3 cases [LoM5]. Missing cases were completed in [KroMe].

(j) $R(F, G \cup H) \le \max\{ R(F, G) + n(H), R(F, G) \}$ [Par6]

(k) $R(mG, nH) \le (m-1)n(G) + (n-1)n(H) + R(G, H)$ [BES]

(l) Formulas for $R(nK_3, mG)$ for all isolate-free graphs G on 4 vertices [Zeng]

(m) Variety of results for numbers of the form $R(nG, mH)$ [Bu1, BES, HaBA2, SuBAU1, SuAUB, Sudar2, Sudar3].

(n) Disjoint unions of paths (linear forests) [BuRo2, FS2],
Linear forests versus $3K_3 \cup 2K_4$ [SuBAU2]

---

* double star is a union of two stars with their centers joined by an edge

+ broom is a star with a path attached to its center

(o) Forests versus K_n [Stahl, BaHA] and W_n [BaHA]. Generalizations to forests versus other graphs G in terms of $\chi(G)$ and the chromatic surplus of G [Biel4], and for linear forests versus $2K_n$ [SuAM].

(p) Disconnected graphs versus other graphs [BuE1, GoJa1]

(q) See section 4.1 for cases involving unions of cycles

(r) See also [Bu9, BuE1, LorMu, MiSa, Den2, Biel1, Biel2]

5.14. General results for special graphs

(a) $R(K_m^p, K_n^q) = R(K_m, K_n)$ for $m, n \geq 3$, $m+n \geq 8$, $p \leq m/(n-1)$ and $q \leq n/(m-1)$, where K_s^t is a K_s with additional vertex connected to it by t edges [BEFS]. Some applications can be found in [BlLR].

(b) $R(K_{2,k}, G) \leq kq+1$ for $k \geq 3$, for isolate-free graphs G with $q \geq 2$ edges [RoJa2, JRB].

(c) $R(W_6, W_6) = 17$ and $\chi(W_6) = 4$ [FM]. This gives a counterexample $G = W_6$ to the Erdős conjecture (see [GRS]) $R(G,G) \geq R(K_{\chi(G)}, K_{\chi(G)})$, since $R(4,4) = 18$.

(d) $R(G+K_1, H) \leq R(K_{1, R(G, H)}, H)$ [BuE1].

(e) $R(\bar{K}_2+G, \bar{K}_2+G) \leq 4R(G, \bar{K}_2+G) - 2$ [LiShen].

(f) For arbitrary fixed graphs G and H, if n is sufficiently large then we have $R(K_2+G, K_1+nH) = (k+1)mn+1$, where $k = \chi(G)$ and $m = |V(H)|$ [LiR2].

(g) Study of $R(G+K_1, nH+K_1)$ [LinLD]. Further lower bounds based on the Paley graphs, in particular for $R(K_3+\bar{K}_n, K_3+\bar{K}_n)$ [LinLS].

(h) $R(K_{p+1}, B_q^r) = p(q+r-1)+1$ for generalized books $B_q^r = K_r + qK_1$, for sufficiently large q [NiRo1]. Formula for $R(K_1+C_4, B_q^r)$ for sufficiently large q [LinLiu].

(i) Study of the cases $R(K_m, K_n - K_{1,s})$ and $R(K_m - e, K_n - K_{1,s})$, with several exact values for special parameters [ChaMR]. This study was extended to some cases involving $R(K_m - K_3)$ [MonCR].

(j) Study of $R(T+K_1, K_n)$ for trees T [LiR1]. Asymptotic upper bounds for $R(T+K_2, K_n)$ [Song7], see also [SonGQ].

(k) Bounds on $R(H+\bar{K}_n, K_n)$ for general H [LiR3]. Also, for fixed k and m, as $n \to \infty$, $R(K_k+\bar{K}_m, K_n) \leq (m+o(1))\, n^k/(\log n)^{k-1}$ [LiRZ1].

(l) Asymptotics of $R(H+\bar{K}_n, K_n)$. In particular, the order of magnitude of $R(K_{m,n}, K_n)$ is $n^{m+1}/(\log n)^m$ [LiTZ]. Upper asymptotics for $R(K_s+K_{m,n}, K_k)$ [Song9].

(m) Study of the largest k such that if the star $K_{1,k}$ is removed from K_r, $r = R(G, H)$, any edge 2-coloring of the remaining part still contains monochromatic G or H, as for K_r, for various special G and H [HoIs].

(n) Let G'' be a graph obtained from G by deleting two vertices with adjacent edges. Then $R(G, H) \leq A+B+2+2\sqrt{(A^2+AB+B^2)/3}$, where $A = R(G'', H)$ and $B = R(G, H'')$ [LiRZ2].

5.15. General results for sparse graphs

(a) $R(K_n, T_m) = (n-1)(m-1)+1$ for any tree T_m on m vertices [Chv].

(b) Graphs yielding $R(K_n, G) = (n-1)(n(G)-1)+1$, called Ramsey n-good [BuE3], and related results [EFRS5]. An extensive survey and further study of n-goodness appeared in [NiRo4], 2009. More results on goodness of bounded degree trees [BalPS], 2016, and paths [PoSu1], 2017.

(c) $R(C_{2m+1}, G) = 2n-1$ for sufficiently large sparse graphs G on n vertices, little more complicated formulas for P_{2m+1} instead of C_{2m+1} [BEFRS2].

(d) $R(G,G) \le c_d n(G)$ for all G, where constant c_d depends only on the maximum degree d in G [ChRST]. The constant was improved in [GRR1, FoxSu1]. Tight lower and upper bounds for bipartite G [GRR2, Con2, ConFS7, ConFS8]. Further improvements of the constant c_d in general were obtained in [ConFS4], and for graphs with bounded bandwidth in [AllBS].

(e) Study of L-sets, which are sets of pairs of graphs whose Ramsey numbers are linear in the number of vertices. Conjecture that Ramsey numbers grow linearly for d-degenerate graphs (graph is d-degenerate if all its subgraphs have minimum degree at most d) [BuE1]. Progress towards this conjecture was obtained by several authors, including [KoRö1, KoRö2, KoSu, FoxSu1, FoxSu2]. Further progress was obtained in 2016 in relation to the chromatic number [Lee].

(f) $R(G,G) \le c_d n$ for all d-arrangeable graphs G on n vertices, in particular with the same constant for all planar graphs [ChenS]. The constant c_d was improved in [Eaton]. An extension to graphs not containing a subdivision of K_d [RöTh].

(g) Conjecture that $R(G,G) \le 12n(G)$ for all planar G, for sufficiently large n [AllBS].

(h) Ramsey numbers grow linearly for degenerate graphs versus some sparser graphs, arrangeable graphs, crowns, graphs with bounded maximum degree, planar graphs, and graphs without any topological minor of a fixed clique [Shi3].

(i) Ramsey number is *linear* in a class of graphs X if $R_X(p, q) \le c(p+q)$ for some constant c and all p, q, where we color the edges of graphs in X. A conjecture that this linearity holds for X if and only if the co-chromatic number is bounded in X [AtLZ]. Discussion of various old and new classes of Ramsey linear graphs [NeOs].

(j) Study of graphs G, called *Ramsey size linear,* for which there exists a constant c_G such that for all H with no isolates $R(G, H) \le c_G e(H)$ [EFRS9]. An overview and further results were given in [BaSS].

(k) $R(G, G) < 6n$ for all n-vertex graphs G, in which no two vertices of degree at least 3 are adjacent [LiRS]. This improves the result $R(G, G) \le 12n$ in [Alon1]. In an early paper by Burr and Erdős [BuE1] it was proved that if any two points of degree at least 3 are at distance at least 3 then $R(G, G) \le 18n$.

(l) $R(G_{a,b}, G_{a,b}) = (3/2+o(1))ab$, where $G_{a,b}$ is the rectangular $a\times b$ grid graph. Other similar results follow for bipartite planar graphs with bounded degree and grids of higher dimension [MoSST].

(m) $R(Q_n, Q_n) \leq 2^{2.62n+o(n)}$, for the n-dimensional hypercube Q_n with 2^n vertices [Shi1]. This bound can also be derived from a theorem in [KoRö1]. An improvement was obtained in [Shi4], a further one to $R(Q_n, Q_n) \leq 2^{2n+5}n$ in [FoxSu1], and another decrease of the upper bound to 2^{2n+6} in [ConFS8]. A lower bound construction for $12 \leq R(Q_3, Q_3)$ was presented in [HaKr2].

(n) $R(K_m, Q_n) = (m-1)(2^n - 1) + 1$ for every fixed m and sufficiently large n [FizGMSS]. This improves on the results in [ConFLS] and [GrMFSS]. The apparent contradiction with publication years is due to the timing of publication processes.

(o) Conjecture that $R(G, G) = 2n(G) - 1$ if G is unicyclic of odd girth [Gros1]. Further support for the conjecture was given in [Köh, KrRod].

(p) See also earlier subsections 5.* for various specific sparse graphs.

5.16. General results

(a) $R(G, H) \geq (\chi(G)-1)(c(H)-1)+1$, where $\chi(G)$ is the chromatic number of G, and $c(H)$ is the size of the largest connected component of H. [ChH2].

(b) $R(G, G) > (s\,2^{e(G)-1})^{1/n(G)}$, where s is the number of automorphisms of G. Hence $R(K_{n,n}, K_{n,n}) > 2^n$, see also item 6.7.l [ChH3].

(c) $R(G, G) \geq \lfloor (4n(G)-1)/3 \rfloor$ for any connected G, and $R(G, G) \geq 2n-1$ for any connected nonbipartite G. These bounds can be achieved for all $n \geq 4$ [BuE2].

(d) Graphs H yielding $R(G, H) = (\chi(G)-1)(n(H)-1)+s(G)$, where $s(G)$ is the chromatic surplus of G, defined as the minimum number of vertices in some color class under all vertex colorings in $\chi(G)$ colors (such H's are called G-good) [Bu2]. This idea is a basis of a number of exact results for $R(G, H)$ for large and sparse graphs H [BuG, BEFRS2, BEFRS3, Bu5, FaSi, EFRS4, FRS3, BEFSRGJ, BuF, LiR4, Biel2, SuBAU3, Song6, AllBS, PeiLi, PeiCLY, LiBie, BalPS, PoSu1, PoSu2, LinLiu]. Surveys of this area appeared in [FRS5, NiRo4].

(e) Graph G is Ramsey saturated if $R(G+e, G+e) > R(G, G)$ for every edge e in $\overline{G}$. The paper [BaLS] contains several theorems involving cycles, cycles with chords and trees on Ramsey saturated and unsaturated graphs, and also seven conjectures including one stating that almost all graphs are Ramsey unsaturated. Some classes of graphs were proved to be Ramsey unsaturated [Ho]. Special cases involving cycles and Jahangir graphs were studied in [AliSur].

(f) Relations between $R(3, k)$ and graphs with large $\chi(G)$ [BiFJ]. Further detailed study of the relation between $R(3, k)$ and the chromatic gap [GySeT].

(g) $R(G, H) > h(G, d)\,n(H)$ for all nonbipartite G and almost every d-regular H, for some h unbounded in d [Bra3].

(h) Lower asymptotics of $R(G, H)$ depending on the average degree of G and the size of H [DoLL1]. This continued the study initiated in [EFRS5], later much enhanced for both lower and upper bounds in [Sud3].

(i) Lower bound asymptotics of $R(G, H)$ for large dense H [LiZa1].

(j) A conjecture posed by Erdős in 1983 that there exists a constant c such that $R(G, G) \leq 2^{c\sqrt{e(G)}}$ for all isolate-free graphs G [Erd4]. Discussion of this conjecture and partial results, proof for bipartite graphs and progress in other cases are included in [AlKS]. In 2011, Sudakov [Sud4] completed the proof of this conjecture. An extension of the latter to some off-diagonal cases is presented in [MaOm1], and an improvement of the constant for bipartite graphs is given in [JoPe]. For the multicolor case see item 6.7.k.

(k) Lower bound on $R(G, K_n)$ depending on the density of subgraphs of G [Kriv]. This construction for $G = K_m$ produces a bound similar to the best known probabilistic lower bound by Spencer [Spe2]. Further lower and upper bounds on $R(G, K_n)$ in terms of n and $e(G)$ can be found in [Sud3].

(l) Upper bounds on $R(G, K_n)$ for dense graphs G [Con3].

(m) The graphs K_n and $K_n + K_{n-1}$ are Ramsey equivalent for $n \geq 4$, i.e. every graph arrows both of them or neither of them. This equivalence does not hold for $n = 3$, and every graph witnessing such nonequivalence contains K_6 [BlLi]. See references therein for history and further results on Ramsey equivalent and nonequivalent pairs of graphs.

(n) Relations between the cases of G or $G + K_1$ versus H or $H + K_1$ [BuE1].

(o) Study of cyclic graphs yielding lower bounds for Ramsey numbers. Exact formulas for paths and cycles, and values for small complete graphs and for graphs with up to five vertices [HaKr1].

(p) Relations between some Ramsey graphs and block designs [Par3, Par4].

(q) Lidický and Pfender used flag algebras to constrain the space of feasible Ramsey colorings of various types. This was implemented, and then led to a number of new upper bounds listed throughout this survey [LidP].

(r) Relations between the Shannon capacity of noisy communication channels and graph Ramsey numbers [Li2]. See also section 6 in [Ros2], and [XuR3].

(s) Given integer m and graphs G and H, determining whether $R(G, H) \leq m$ holds is NP−hard [Bu6]. Further complexity results related to Ramsey theory were presented by Burr in [Bu10].

(t) Ramsey arrowing is Π_2^p−complete, a rare natural example of a problem higher than NP in the polynomial hierarchy of computational complexity theory [Scha].

(u) Special cases of multicolor results listed in section 6.

(v) See also surveys listed in section 8.

6. Multicolor Ramsey Numbers

Until 2016, the only known value of a multicolor classical Ramsey number was:

$R_3(3) = R(3,3,3) = R(3,3,3;2) = 17$	[GG]
2 critical colorings (on 16 vertices)	[KaSt, LayMa]
2 colorings on 15 vertices	[Hein]
115 colorings on 14 vertices	[PR1]

Now, we know one more case, namely $R(3,3,4) = 30$. For some details see 6.1.c.

6.1. Bounds for classical numbers

General upper bound, implicit in [GG]:

$$R(k_1, \dots, k_r) \le 2 - r + \sum_{i=1}^{r} R(k_1, \dots, k_{i-1}, k_i - 1, k_{i+1}, \dots, k_r) \tag{a}$$

The inequality in (a) is strict if the right hand side is even and at least one of the terms in the summation is even. It is suspected that this upper bound is never tight for $r \ge 3$ and $k_i \ge 3$, except for $r = k_1 = k_2 = k_3 = 3$. However, only two parameter cases are known to improve over (a), namely $R_4(3) \le 62$ [FeKR], and $R(3,3,4) \le 31$ [PR1, PR2], $R(3,3,4) \le 30$ [CodFIM], for which (a) produces the bounds of 66 and 34, respectively.

Diagonal Cases

r \ m	3	4	5	6	7	8	9
3	17 GG	128 HiIr	454 Ex23	1106 Row3	3214 XuR1	6132 Row2	14081 Row3
4	51 Chu1	634 XXER	4073 Row3	21302 Row3	84623 Row3	168002 Row3	
5	162 Ex10	4176 Row1	38914 Row3				
6	538 FreSw	32006 Row1					
7	1682 FreSw	160024 Row1					
8	5288 Row3						

Table X. Known lower bounds for small parameter diagonal multicolor Ramsey numbers $R_r(m)$, with references.

A general construction of linear Ramsey graphs as described by Rowley [Row2, Row3] in 2020 leads to lower bounds in higher cases, such as $R_6(6) \ge 4515702$. Other lower bounds, implied by general constructions such as those in section 6.2, are not listed.

The most studied and intriguing open case is

$$\text{[Chu1]} \quad 51 \le R_4(3) = R(3,3,3,3) \le 62 \quad \text{[FeKR]}$$

The construction for $51 \le R_4(3)$ as described in [Chu1] is correct, but be warned of a typo found by Christopher Frederick in 2003 (there is a triangle (31,7,28) in color 1 in the displayed matrix). It was shown that the bound 51 cannot be improved by using group partitioning into disjoint union of symmetric product-free sets [Ana]. The inequality 6.1.a implies $R_4(3) \le 66$, Folkman [Fol] in 1974 improved this bound to 65, and Sánchez-Flores [Sán] in 1995 proved $R_4(3) \le 64$.

The upper bounds in $162 \le R_5(3) \le 307$, $538 \le R_6(3) \le 1838$, $1682 \le R_7(3) \le 12861$, $128 \le R_3(4) \le 230$ and $634 \le R_4(4) \le 6306$ are implied by 6.1.a (we repeat lower bounds from Table X just to see easily the ranges). All the latter and other upper bounds obtainable from known smaller bounds and 6.1.a can be computed with the help of a LISP program written by Kerber and Rowat [KerRo].

Off-Diagonal Cases

Three colors:

k \ m	4	5	6	7	8	9	10	11	12	13	14	15	16
3	30 Ka2	45 Ex2	61 ExT	85 Ex18	103 Ex18	129 Ex18	150 ExT	174 ExT	194 ExT	217 ExT	242 ExT	269 ExT	291 ExT
4	55 KrLR	89 Ex17	117 Ex17	152 ExT	193 6.2.g	242 ExT							
5	89 Ex17	139 Ex17	181 Ex17	241 6.2.g									

Table XI. Known nontrivial lower bounds for 3-color Ramsey numbers of the form $R(3, k, m)$, with references. See also 6.1.b/c/d below.

(b) In addition to Table XI, the bounds $303 \le R(3,6,6)$, $609 \le R(3,7,7)$ and $1689 \le R(3,9,9)$ were derived in [XXER] (used there for building other lower bounds for some diagonal cases). These three bounds were improved to 314, 623 and 1739, respectively, by Rowley [Row2].

(c) In several past revisions of this survey we wrote: "The other most studied, and perhaps the only open case of a classical multicolor Ramsey number, for which we can anticipate exact evaluation in the not-too-distance future is

$$\text{[Ka2]} \quad 30 \le R(3,3,4) \le 31 \quad \text{[PR1, PR2]}$$

In [PR1] it was conjectured that $R(3,3,4) = 30$, and the results in [PR2] eliminate some cases which could give $R(3,3,4) = 31$". Since 2016, we can write that $R(3,3,4) = 30$ due to the computations completed by Codish, Frank, Itzhakov and Miller [CodFIM].

(d) The upper bounds in the inequalities $45 \le R(3,3,5) \le 57$, $55 \le R(3,4,4) \le 77$ and $89 \le R(3,4,5) \le 158$ are implied by 6.1.a. We repeat lower bounds from Table XI to show explicitly the current ranges.

(e) In 2015, Exoo and Tatarevic obtained several lower bounds improvements which are marked as [ExT] in Table XI. The same paper improves also on several classical two-color cases in Table I, see also comments 2.1.n and 2.1.o.

Four colors:

$97 \le R(3,3,3,4) \le 149$	[Ex17], 6.1.a
$174 \le R(3,3,4,4) \le 450$	[Row2], 6.1.a
$381 \le R(3,4,4,4) \le 1577$	6.2.j, 6.1.a
$162 \le R(3,3,3,5)$	[XXER]
$513 \le R(3,3,3,10)$	6.2.g
$597 \le R(3,3,3,11)$	6.2.g
$693 \le R(3,4,5,5)$	[Row2]

Lower bounds for higher numbers can be obtained by using general constructive results from section 6.2 below. For example, the bounds $261 \le R(3,3,15)$ and $247 \le R(3,3,3,7)$ were not published explicitly but are implied by 6.2.g and 6.2.h, respectively.

6.2. General results for complete graphs

(a) $R(k_1, \ldots, k_r) \le 2 - r + \sum_{i=1}^{r} R(k_1, \ldots, k_{i-1}, k_i - 1, k_{i+1}, \ldots, k_r)$ [GG].

(b) $R_r(3) \ge 3R_{r-1}(3) + R_{r-3}(3) - 3$ [Chu1].

(c) $R_r(m) \ge c_m(2m-3)^r$, and some slight improvements of this bound for small values of m were described in [AbbH, Gi1, Gi2, Song2]. For $m = 3$, the best known lower bound is $R_r(3) \ge (3.199...)^r$ [XXER].

(d) $R_r(3) \le r!(e - e^{-1} + 3)/2 \approx 2.67\,r!$ [Wan] improved over the classical upper bound $3r!$ in [GG, GRS]. This was further improved to $R_r(3) \le r!(e - 1/6) + 1 \approx 2.55\,r!$ for all $r \ge 4$ [XuXC]. Drawing from the latter, further conditional upper bounds depending on the value of $R_4(3)$ were obtained in [Eli]. In particular, assuming that $R_4(3) = 51$, we have $R_r(3) \le r!(e - 5/8) + 1 \approx 2.09\,r!$ for all $r \ge 4$.

(e) The limit $L = \lim_{r\to\infty} R_r(3)^{1/r}$ exists, though it can be infinite [ChGri]. It is known that $3.199 < L$, as implied by (c) above. The lower bounds on the limits $\lim_{r\to\infty} R_r(k)^{1/r}$ for small fixed k are gathered in [Row1, Row3], see also 6.2.v. The best lower bounds for $R_r(k)$ from the k-th residue Paley graphs for $k = 3$ and $k = 4$ are described in [LocMc], though they are much weaker than those in Table X. For some older related results, mostly on the asymptotics of $R_r(3)$, see [AbbH, Fre, Chu2, GRS, GrRö].

(f) In 2020, the limit $\lim_{r\to\infty} R_r(3)^{1/r}$ was studied by Fox, Pach and Suk [FoxPS1] assuming a conjecture for multicolorings with bounded VC-dimension, and further for $\lim_{r\to\infty} R_r(k)^{1/r}$ when restricted to the so-called semi-algebraic colorings [FoxPS2].

(g) $R(3, k, l) \ge 4R(k, l-1) - 3$ for $k \ge 3$, $l \ge 5$, and in general for $r \ge 2$ and $k_i \ge 2$ it holds
$R(3, k_1, \ldots, k_r) \ge 4R(k_1 - 1, k_2, \ldots, k_r) - 3$ for $k_1 \ge 5$, and
$R(k_1, 2k_2 - 1, k_3, \ldots, k_r) \ge 4R(k_1 - 1, k_2, \ldots, k_r) - 3$ for $k_1 \ge 5$ [XuX2, XXER].

(h) $R(3, 3, 3, k_1, \ldots, k_r) \ge 3R(3, 3, k_1, \ldots, k_r) + R(k_1, \ldots, k_r) - 3$ [Rob2].
For $r+1$ colors, avoiding K_3 in the first r colors and avoiding K_m in the last color, $R(3, \ldots, 3, m) \le r!\, m^{r+1}$ [Sár1].

(i) $R(k_1, \ldots, k_r) \ge S(k_1, \ldots, k_r) + 2$, where $S(k_1, \ldots, k_r)$ is the generalized Schur number [AbbH, Gi1, Gi2]. In particular, the special case $k_1 = \ldots = k_r = 3$ has been widely studied [Fre, FreSw, Ex10, Rob3].

(j) $R(k_1, \ldots, k_r) \ge L(k_1, \ldots, k_r) + 1$, where $L(k_1, \ldots, k_r)$ is the maximal order of any cyclic $(k_1, \ldots, k_r)$–coloring, which can be considered a special case of Schur partitions defining (symmetric) Schur numbers. Many lower bounds for Ramsey numbers were established by cyclic colorings. The following recurrence can be used to derive lower bounds for higher parameters. For $k_i \ge 3$ [Gi2],

$$L(k_1, \ldots, k_r, k_{r+1}) \ge (2k_{r+1} - 3)L(k_1, \ldots, k_r) - k_{r+1} + 2.$$

(k) $R_r(m) \ge p+1$ and $R_r(m+1) \ge r(p+1)+1$ if there exists a K_m–free cyclotomic r–class association scheme of order p [Mat].

(l) If the quadratic residues Paley graph Q_p of prime order $p = 4t+1$ contains no K_k, then $R(s, k+1, k+1) \ge 4ps - 6p + 3$ [XXER].

(m) $R_r(pq+1) > (R_r(p+1)-1)(R_r(q+1)-1)$ [Abb1]

(n) $R_r(pq+1) > R_r(p+1)(R_r(q+1)-1)$ for $p \ge q$ [XXER]

(o) $R(p_1q_1+1, \ldots, p_rq_r+1) > (R(p_1+1, \ldots, p_r+1)-1)(R(q_1+1, \ldots, q_r+1)-1)$ [Song3]

(p) $R_{r+s}(m) > (R_r(m)-1)(R_s(m)-1)$ [Song2]

(q) $R(k_1, k_2, \ldots, k_r) > (R(k_1, \ldots, k_i) - 1)(R(k_{i+1}, \ldots, k_r) - 1)$ in [Song1], see [XXER].

(r) $R(k_1, k_2, \ldots, k_r) > (k_1 + 1)(R(k_2 - k_1 + 1, k_3, \ldots, k_r) - 1)$ [Rob4]

(s) Further lower bound constructions, though with more complicated assumptions, were presented in [XuX2, XXER].

(t) Grolmusz [Grol1] generalized the classical constructive lower bound by Frankl and Wilson [FraWi] (item 2.3.6) to more colors and to hypergraphs [Grol3] (item 7.4.m).

(u) $R(n, n, n) \le R(n-2, n, n) + 8R(n-1, n-1, n) - 6$ for $n \ge 3$ [HTHZ2].

(v) A conjecture that $R(k_1, k_2, \ldots, k_r) \ge R(k_1, k_2, \ldots, k_{r-2}, k_{r-1} - 1, k_r + 1)$ holds for all $k_r \ge k_{r-1} \ge 3$ (called DC), its implications, evidence for validity, and related problems

[LiaRX]. For two-color case see also item 2.3.f. If we set $L_k = \lim_{r \to \infty} R_r(k)^{1/r}$, then the limit L_k exists, finite or infinite, for every $k \geq 3$ [ChGri]. If DC holds, then all L_k's are finite or all of them are infinite [LiaRX]. See also 6.2.e.

(w) In 2020, Conlon and Ferber [ConFer] showed constructively that $R_3(k) > 2^{7k/8+o(k)}$ and $R_4(k) > 2^{k/2}3^{3k/8+o(k)}$, and they discussed more general best known lower and upper bounds on $R_r(k)$. An improvement to their construction by Wigderson [Wig] yields $R_r(k) \geq (2^{3r/8-1/4})^{k-o(k)}$, for any fixed $r \geq 2$.

(x) Exact asymptotics of a very special but important case is known, namely we have $R(3, 3, n) = \Theta(n^3 \text{poly}-\log n)$ [AlRö]. Generalizations to other parameters and more colors [HeWi]. For earlier results on general upper bounds and more asymptotics see [Chu4, ChGra2, ChGri, GRS, GrRö].

All lower bounds in (b) through (t) above are constructive. Item (h) generalizes (b), (o) generalizes both (m) and (q), and (q) generalizes (p). (n) is stronger than (m). Finally, we note that the construction in (o) with $q_1 = \ldots = q_i = 1 = p_{i+1} = \ldots = p_r$ is the same as (q).

6.3. Cycles

Note: The paper *Ramsey Numbers Involving Cycles* [Ra4] is based on the revision #12 of this survey. It collects and comments on the results involving cycles versus any graphs, in two or more colors. It contains some more details than this survey, but only until 2009.

6.3.1. Three colors

(a) One long cycle.

The first larger paper in this area by Erdős, Faudree, Rousseau and Schelp [EFRS1] appeared in 1976. It gives several formulas and bounds for $R(C_m, C_n, C_k)$ and $R(C_m, C_n, C_k, C_l)$ for large m. For three colors [EFRS1] includes:

$R(C_m, C_{2p+1}, C_{2q+1}) = 4m - 3$ for $p \geq 2$, $q \geq 1$,
$R(C_m, C_{2p}, C_{2q+1}) = 2(m+p) - 3$ and
$R(C_m, C_{2p}, C_{2q}) = m + p + q - 2$ for p, $q \geq 1$ and large m.

(b) Triple even cycles.

$R_3(C_{2m}) \geq 4m$ for all $m \geq 2$ [DzNS], see also 6.3.2.d/e/f. It was proven that $R(C_n, C_n, C_n) = (2 + o(1))n$ for even n [FiŁu1, GyRSS], which was improved to exactly $2n$, for large n, by Benevides and Skokan [BenSk]. In 2005, Dzido [Dzi1] conjectured that $R_3(C_{2m}) = 4m$ for all $m \geq 3$. The first open case is for $R_3(C_{10})$, known to be at least 20. A more general result holds for some off-diagonal cases [FiŁu1]:

$$R(C_{2\lfloor \alpha_1 n \rfloor}, C_{2\lfloor \alpha_2 n \rfloor}, C_{2\lfloor \alpha_3 n \rfloor}) = \\ (\alpha_1 + \alpha_2 + \alpha_3 + \max\{\alpha_1, \alpha_2, \alpha_3\} + o(1))n, \quad \textit{for all} \ \alpha_1, \alpha_2, \alpha_3 > 0.$$

The conjectured equality $R_3(C_{2m}) = 4m$, whenever true, implies $R_3(P_{2m+1}) = 4m+1$ [DyDR] (see also section 6.4). For general mixed-parity case see 6.3.1.d/e below.

$m\ n\ k$	$R(C_m, C_n, C_k)$	references	general results
3 3 3	**17**	GG	2 critical colorings [KaSt, LayMa]
3 3 4	17	ExRe	
3 3 5	21	Sun1+/Tse3	$5k-4$ for $k \geq 5$, $m=n=3$ [Sun1+]
3 3 6	26	Sun1+	
3 3 7	31	Sun1+	
3 4 4	12	Schu	
3 4 5	13	Sun1+/Rao/Tse3	
3 4 6	13	Sun1+/Tse3	
3 4 7	15	Sun1+/Tse3	
3 5 5	17	Tse3/LidP	
3 5 6	21	Sun1+	
3 5 7	25	Sun1+	
3 6 6	15-18	LidP	
3 6 7	21	Sun1+	
3 7 7			
4 4 4	**11**	BiaS	1000 critical colorings [Ra4]
4 4 5	12	Sun2+/Tse3	
4 4 6	12	Sun2+/Tse3	$k+2$ for $k \geq 11$, $m=n=4$ [Sun2+]
4 4 7	12	Sun2+/Tse3	values for $k=8$, 9, 10 are 12, 13, 13 [Sun2+]
4 5 5	13	Tse3	
4 5 6	13	Sun1+	
4 5 7	15	Sun1+	
4 6 6	11	Tse3	
4 6 7	13	Sun1+/Tse3	
4 7 7			
5 5 5	**17**	YR1	1701746176 critical colorings [Nar]
5 5 6	21	Sun1+	
5 5 7	25	Sun1+	
5 6 6	15-17	LidP	
5 6 7	21	Sun1+	
5 7 7			
6 6 6	**12**	YR2	$R_3(C_{2q}) \geq 4q$ for $q \geq 2$ [DzNS]
6 6 7	15	Sun1+	see 6.3.1.a for larger parameters
6 7 7			see 6.3.1.a for larger parameters
7 7 7	**25**	FSS3	$R_3(C_{2q+1}) = 8q+1$ for large q [KoSS1, KoSS2]
8 8 8	**16**	Sun/SunY	$R_3(C_{2q}) = 4q$ for large q [BenSk]

Table XII. Ramsey numbers $R(C_m, C_n, C_k)$ for $m, n, k \leq 7$ and $m=n=k=8$;
Sun1+ abbreviates SunYWLX, Sun2+ abbreviates SunYLZ2,
the work in [SunYWLX] and [SunYLZ2] is independent from [Tse3].

(c) Triple odd cycles.

Bondy and Erdős conjectured that $R(C_n, C_n, C_n) \leq 4n-3$ for all $n \geq 4$ (see for example [Erd2]). If true, then for all odd $n \geq 5$ we have $R(C_n, C_n, C_n) = 4n-3$. The first open case is for $R_3(C_9)$, known to be at least 33. Erdős [Erd3] and other authors credit this conjecture to Bondy and Erdős, often pointing to a 1973 paper [BoEr]. Interestingly, however, the conjecture is not mentioned in this paper.

Łuczak proved that $R(C_n, C_n, C_n) \leq (4+o(1))n$, with equality for odd n [Łuc]. The result $R_3(C_{2m+1}) = 8m+1$ for all sufficiently large m, or equivalently $R(C_n, C_n, C_n) = 4n-3$ for large odd n, was announced with an outline of the proof by Kohayakawa, Simonovits and Skokan [KoSS1], followed by the full proof in [KoSS2].

(d) Three mixed-parity cycles.

Ferguson [Ferg] shows that $R(C_m, C_n, C_k) = \max\{2m+n, 2n+m, (n+m)/2+k-2\}$, for all m, n, k sufficiently large, which generalizes and improves on all even case in [FiŁu1]. The reference [Ferg] consists of a Ph.D. thesis and three long arXiv preprints.

(e) Asymptotics for triples of cycles of mixed parity similar in form to (b) [FiŁu2].

(f) $R(C_3, C_3, C_k) = 5k-4$ for $k \geq 5$ [SunYWLX], and $R(C_4, C_4, C_k) = k+2$ for $k \geq 11$ [SunYLZ2]. All exceptions to these formulas for small k are listed in Table XII.

(g) Almost all of the off-diagonal cases in Table XII required the use of computers.

6.3.2. More colors

k \ m	3	4	5	6	7	8
3	17	11	17	12	25	16
4	51 62	18	33 77	18 20	49	20
5	162 307	27 29	65	26	97	28
6	538 1838	34 43	129		193	

Table XIII. Known values and bounds for $R_k(C_m)$ for small k, m;

(a) For the entries in the row $k = 3$ and in the column $m = 3$ in Table XIII, more details and all corresponding references are in sections 6.3.1 and 6.1, respectively. The lower bounds for $m = 5, 7$ are implied by 6.3.2.l. The bound $R_4(C_5) \leq 158$ follows from 6.3.2.k, and using a reasoning as in [Li4] and the equality $R_3(C_5) = 17$ one can obtain $R_4(C_5) \leq 137$. The bound $R_4(C_5) \leq 77$ was obtained in 2020 with the help of flag algebras [LidP]. The references to other cases with $k, m \geq 4$ can be found below in this section.

$R_4(C_4) = 18$	[Ex2] [SunYLZ1]
$33 \le R_4(C_5) \le 77$	[6.3.2.l] [LidP]
$18 \le R_4(C_6) \le 20$	[SunYJLS] [ZhaSW]
$27 \le R_5(C_4) \le 29$	[LaWo1]
$R_5(C_6) = 26$	[SunYJLS] [SunYW]
$34 \le R_6(C_4)$	[Ex22]
$24 \le R(C_3, C_4, C_4, C_4) \le 27$	[DyDz1] [XuR2]
$30 \le R(C_3, C_3, C_4, C_4) \le 36$	[DyDz1] [XuR2]
$49 \le R(C_3, C_3, C_3, C_4) \le 59$	[6.7.h] [LidP]
$18 \le R(C_4, C_6, C_6, C_6) \le 20$	[ZhaSW]
$18 \le R(C_4, C_4, C_6, C_6) \le 20$	[ZhaSW]
$R(C_4, C_4, C_4, C_6) = 19$	[ZhaSW]

(b) $R_k(C_4) \le k^2 + k + 1$ for all $k \ge 1$, $R_k(C_4) \ge k^2 - k + 2$ for all $k - 1$ which is a prime power [Ir, Chu2, ChGra1], and $R_k(C_4) \ge k^2 + 2$ for odd prime power k [LaWo1]. The latter was extended to any prime power k in [Ling, LaMu].

(c) Formulas for $R(C_m, C_n, C_k, C_l)$ for large m [EFRS1].

Bounds in (d)-(j) below cover different situations and each is interesting in some respect.

(d) $R_k(C_{2m}) \ge (k+1)m$ for odd k and $m \ge 2$, and
$R_k(C_{2m}) \ge (k+1)m - 1$ for even k and $m \ge 2$ [DzNS].

(e) $R_k(C_{2m}) \ge 2(k-1)(m-1) + 2$ [SunYXL].

(f) $R_k(C_{2m}) \ge k^2 + 2m - k$ for $2m \ge k+1$ and prime power k [SunYJLS].

(g) $R_k(C_{2m}) = \Theta(k^{m/(m-1)})$ for fixed $m = 2$, 3 and 5 [LiLih].

(h) $R_k(C_{2m}) \le 201km$ for $k \le 10^m/201m$ [ErdG].

(i) $R_k(C_{2m}) \le 2km + o(m)$ for all fixed $k \ge 2$ [ŁucSS].

(j) $R_k(C_{2m}) \le 2(k - c_k)m + o(m)$ for some small $c_k > 0$, for all fixed $k \ge 2$ [Sár2]. This was improved to an absolute constant $c = c_k = 1/4$ in [DavJR], and further to $c = 1/2$ in [KniSu]. See also 6.4.2.c.

(k) $R_k(C_5) < (18^k k!)^{1/2}/10$ [Li4].

(l) $2^k m < R_k(C_{2m+1}) \le (k+2)!(2m+1)$ [BoEr].
Better upper bound $R_k(C_{2m+1}) < 2(k+2)!\,m$ was obtained in [ErdG].
Still better upper bound $R_k(C_{2m+1}) \le (c^k k!)^{1/m}$, for some positive constant c,
if all Ramsey-critical colorings for C_{2m+1} are not far from regular, was obtained in [Li4].

(m) For each fixed $m \ge 3$, there exists a positive constant c such that for every $k \ge 3$, $R_k(C_{2m+1}) < c^{k-1}(k!)^{1/2+\delta}$, where δ is approaching 0 for large m [LinCh].

(n) $R_k(C_{2m+1}) \le k2^k(2m+1) + o(m)$ for all fixed $k \ge 4$ [ŁucSS].

(o) Conjecture that $R_k(C_{2m+1}) = 2^k m + 1$ for all $m \ge 2$ was credited by several authors to Bondy and Erdős [BoEr], though only lower bound, not the conjecture, is in this paper.

After more than 40 years, in 2016, Jenssen and Skokan [JenSk] posted on arXiv a proof of the conjecture for each fixed k with sufficiently large m. On the other hand, the work by Day and Johnson [DayJ] shows that the lower bound of the conjecture does not hold for each m and sufficiently large k.

(p) $R(C_n, C_{l_1}, \ldots, C_{l_k}) = 2^k(n-1)+1$ for all l_i's odd with $l_i > 2^i$, and sufficiently large n, and support for the conjecture that $R_k(C_n) = 2^{k-1}(n-1)+1$ for large odd n [AllBS].

(q) $R_l(C_{\le l+1}) = 2l+3$ for all odd $l \ge 3$. For even l we have: $R_4(C_{\le 5}) = 12$, $R_6(C_{\le 7}) = 12$, and $R_l(C_{\le l+1}) = 2l+3$ for $l = 8$, 10 and 12 [ZhuSWZ].

(r) Progress of asymptotic bounds for $R_k(C_n)$ [Bu1, GRS, ChGra2, Li4, LiLih, ŁucSS].

(s) Survey of multicolor cycle cases [Li3].

6.3.3. Cycles versus other graphs

(a) Some cases involving C_4:

$20 \le R(C_4, C_4, K_4) \le 21$	[DyDz1] [LidP]
$27 \le R(C_3, C_4, K_4) \le 32$	[DyDz1] [XSR1]
$52 \le R(C_4, K_4, K_4) \le 71$	[XSR1] [LidP]
$34 \le R(C_4, C_4, C_4, K_4) \le 48$	[DyDz1] [LidP]
$43 \le R(C_3, C_4, C_4, K_4) \le 76$	[DyDz1] [XSR1]
$87 \le R(C_4, C_4, K_4, K_4) \le 179$	[XSR1]
$R(K_{1,3}, C_4, K_4) = 16$	[KlaM2]
$R(C_4, C_4, K_4 - e) = 16$	[DyDz1]
$R(C_4, C_4, C_4, T) = 16$ for $T = P_4$ and $T = K_{1,3}$	[ExRe]

(b) Study of $R(C_n, K_{t_1}, \ldots, K_{t_k})$ and $R(C_n, K_{t_1, s_1}, \ldots, K_{t_k, s_k})$ for large n [EFRS1].

(c) $R(C_n, K_{t_1}, \ldots, K_{t_k}) = (n-1)(r-1)+1$ for $n \ge 4r+2$, where $r = R(K_{t_1}, \ldots, K_{t_k})$. This equality was obtained as a special case of more general results in [OmRa2]. Similar proof was presented later in [Mad]. Further, see items 6.6.f and 6.7.f.

(d) Study of asymptotics for $R(C_m, \ldots, C_m, K_n)$, in particular for any fixed number of colors $k \ge 4$ we have $R(C_4, C_4, \ldots, C_4, K_n) = \Theta(n^2/\log^2 n)$ [AlRö].

(e) Study of asymptotics for $R(C_{2m}, C_{2m}, K_n)$ for fixed m [AlRö, ShiuLL], in particular $R(C_4, C_4, K_n) = \Theta(n^2 \text{poly-}\log n)$ [AlRö].

(f) Study of the general upper bound on $R(C_4, \ldots, C_4, K_{1,n})$, which for 3 colors implies $R(C_4, C_4, K_{1,n}) \le n + 3 + \lceil \sqrt{4n+5} \rceil$ [ZhaCC4].

(g) Study of $R(C_4, K_{1,m}, P_n)$ [ZhZC, SunSh].

(h) Monotone paths and cycles [Lef].

(i) For combinations of C_3 and K_n see sections 2.2, 3.2, 4.2, 6.1 and 6.2.

6.4. Paths, paths versus other graphs

In 2007, Gyárfás, Ruszinkó, Sárközy and Szemerédi [GyRSS] established that for all n large enough we have

$$R(P_n, P_n, P_n) = 2n-2+(n \bmod 2).$$

Faudree and Schelp [FS2] conjectured that the latter holds for all $n \geq 1$. It is true for $n \leq 9$ (see (c) below), and the first open case is that for P_{10}. The conjectured equality $R(C_{2m}, C_{2m}, C_{2m}) = 4m$ (see 6.3.1.a), whenever true, implies the above for three paths P_{2m+1} [DyDR].

6.4.1. Three-color path and path-cycle cases

(a) $R(P_m, P_n, P_k) = m + \lfloor n/2 \rfloor + \lfloor k/2 \rfloor - 2$ for $m \geq 6(n+k)^2$ [FS2],
the equality holds asymptotically for $m \geq n \geq k$ with an extra term $o(m)$ [FiŁu1],
extensions of the range of m, n, k for which (a) holds were obtained in [Biel3].

(b) $R(P_3, P_m, P_n) = m + \lfloor n/2 \rfloor - 1$ for $m \geq n$ and $(m, n) \neq (3, 3), (4, 3)$ [MaORS2].

(c) $R_3(P_3) = 5$ [Ea1], $R_3(P_4) = 6$ [Ir],
$R(P_m, P_n, P_k) = 5$ for other $m-n-k$ combinations with $3 \leq m, n, k \leq 4$ [ArKM],
$R_3(P_5) = 9$ [YR1], $R_3(P_6) = 10$ [YR1], and $R_3(P_7) = 13$ [YY],
$R_3(P_8) = 14$, $R_3(P_9) = 17$ [DyDR].

(d) $R(P_4, P_4, P_{2n}) = 2n+2$ for $n \geq 2$,
$R(P_5, P_5, P_5) = R(P_5, P_5, P_6) = 9$,
$R(P_5, P_5, P_n) = n+2$ for $n \geq 7$,
$R(P_5, P_6, P_n) = R(P_4, P_6, P_n) = n+3$ for $n \geq 6$,
$R(P_6, P_6, P_{2n}) = R(P_4, P_8, P_{2n}) = 2n+4$ for $n \geq 14$ [OmRa1].

(e) $R(P_m, P_n, C_k) = 2n + 2\lfloor m/2 \rfloor - 3$ for large n and odd $m \geq 3$ [DzFi2],
improvements on the range of m, n, k [Biel3, Fid1].

(f) $R(P_3, P_3, C_m) = 5, 6, 6$, for $m = 3, 4$ [ArKM], 5,
$R(P_3, P_3, C_m) = m$ for $m \geq 6$ [Dzi2].
$R(P_3, P_4, C_m) = 7$ for $m = 3, 4$ [ArKM] and 5,
$R(P_3, P_4, C_m) = m+1$ for $m \geq 6$ [Dzi2].
$R(P_4, P_4, C_m) = 9, 7, 9$ for $m = 3, 4$ [ArKM] and 5 [Dzi2],
$R(P_4, P_4, C_m) = m+2$ for $m \geq 6$ [DzKP].

(g) $R(P_3, P_5, C_m) = 9, 7, 9, 7, 9$ for $m = 3, 4, 5, 6, 7$ [Dzi2, DzFi2],
$R(P_3, P_5, C_m) = m+1$ for $m \geq 8$ [DzKP].
A table of $R(P_3, P_k, C_m)$ for all $3 \leq k \leq 8$ and $3 \leq m \leq 9$ [DzFi2].

(h) $R(P_4, P_5, C_m) = 11, 7, 11, 11$, and $m+2$ for $m = 3, 4, 5, 7$ and $m \geq 23$,
$R(P_4, P_6, C_m) = 13, 8, 13, 13$, and $m+3$ for $m = 3, 4, 5, 7$ and $m \geq 18$ [ShaXSP].

(i) $R(P_3, P_n, C_4) = n+1$ for $n \geq 6$ [DzFi2],
$R(P_3, P_n, C_6) = n+2$ for $n \geq 6$,
$R(P_3, P_n, C_8) = n+3$ for $n \geq 7$ [Fid1],
$R(P_3, P_n, C_k) = 2n-1$, and
$R(P_4, P_n, C_k) = 2n+1$ for odd $k \geq 3$ and $n \geq k$ [DzFi2].

(j) $R(P_3, P_6, C_m) = m+2$ for $m \geq 23$,
$R(P_6, P_6, C_m) = R(P_4, P_8, C_m) = m+4$ for $m \geq 27$,
$R(P_6, P_7, C_m) = m+4$ for $m \geq 57$,
$R(P_4, P_n, C_4) = R(P_5, P_n, C_4) = n+2$ for $n \geq 5$ [OmRa1].

(k) $R(P_3, C_3, C_3) = 11$ [BuE3], $R(P_3, C_4, C_4) = 8$ [ArKM], $R(P_3, C_6, C_6) = 9$ [Dzi2],
$R(P_3, C_m, C_m) = R(C_m, C_m) = 2m-1$ for odd $m \geq 5$ [DzKP] (for $m = 5, 7$ [Dzi2]),

(l) $R(P_3, C_n, C_m) = R(C_n, C_m)$ for $n \geq 7$ and odd m, $5 \leq m \leq n$, and
some values and bounds on $R(P_3, C_n, C_m)$ in other cases [Fid1].

(m) $R(P_3, C_3, C_4) = 8$ [ArKM], $R(P_3, C_3, C_5) = 9$, $R(P_3, C_3, C_6) = 11$,
$R(P_3, C_3, C_7) = 13$, $R(P_3, C_4, C_5) = 8$, $R(P_3, C_4, C_6) = 8$,
$R(P_3, C_4, C_7) = 8$, $R(P_3, C_5, C_6) = 11$, $R(P_3, C_5, C_7) = 13$ and
$R(P_3, C_6, C_7) = 11$ [Dzi2].

(n) $R(P_4, C_3, C_5) = 13$, $R(P_4, C_4, C_5) = 10$, $R(P_4, C_4, C_6) = 9$,
$R(P_4, C_5, C_5) = 13$, $R(P_4, C_6, C_6) = 10$ [SunSh].

(o) A formula for $R(P_m, P_n, C_k)$ for k large enough and m, n satisfying some constraints. In addition, some cases involving tK_2 instead of C_k are derived as side results [KhoDz].

(p) Study of $R(P_n, C_4, K_{1,m})$ [ZhZC, SunSh].

(q) Formulas for $R(pP_3, qP_3, rP_3)$ and $R(pP_4, qP_4, rP_4)$ [Scob].

(r) $R(P_3, K_4-e, K_4-e) = 11$ [Ex7]. All colorings which can form any color neighborhood for the case $R_3(K_4-e)$ (see section 6.5) were found in [Piw2].

6.4.2. More colors

(a) $R_k(P_3) = k+1+(k \bmod 2)$, $R_k(2P_2) = k+3$ for all $k \geq 1$ [Ir].

(b) $R_k(P_4) = 2k + c_k$ for all k and some $0 \leq c_k \leq 2$. If k is not divisible by 3 then $c_k = 3 - k \bmod 3$ [Ir]. Wallis [Wall] showed $R_6(P_4) = 13$, which already implied $R_{3t}(P_4) = 6t+1$, for all $t \geq 2$. Independently, the case $R_k(P_4)$ for $k \neq 3^m$ was completed by Lindström in [Lind], and later Bierbrauer proved $R_{3^m}(P_4) = 2(3^m)+1$ for all $m > 1$. $R_3(P_4) = 6$ [Ir].

(c) $R_k(P_n) \leq (k - c_k)n + o(m)$ for some small $c_k > 0$, for all fixed $k \geq 2$ [Sár2]. This was improved to an absolute constant $c = c_k = 1/4$ in [DavJR], and further to $c = 1/2$ in [KniSu]. See also 6.3.2.j/o.

(d) Formula for $R(P_{n_1}, \ldots, P_{n_k})$ for large n_1 [FS2], and some extensions [Biel3]. Conjectures about $R(P_{n_1}, \ldots, P_{n_k})$ when all or all but one of n_i's are even [OmRa1].

(e) Formulas for $R(P_{n_1}, \ldots, P_{n_k}, C_m)$ for some cases, for large m [OmRa1].

(f) Formula for $R(n_1P_2, \ldots, n_kP_2)$, in particular $R(nP_2, nP_2, nP_2) = 4n-2$ [CocL1]. New proof with characterization of all critical graphs [XuYZ]. Note how close the latter is to $R(C_{2n}, C_{2n}, C_{2n}) = 4n$, and see an earlier item 6.3.1.b.

(g) Cockayne and Lorimer [CocL1] found the exact formula for $R(n_1P_2, \ldots, n_kP_2)$, and later Lorimer [Lor] extended it to a more general case of $R(K_m, n_1P_2, \ldots, n_kP_2)$. More general cases of the latter, with multiple copies of the complete graph, paths, stars and forests, were studied in [Stahl, LorSe, LorSo, GyRSS]. A special 3-color case $R(P_3, mP_2, nP_2) = 2m+n-1$ for $m \geq n \geq 3$ is given in [MaORS2], and some other cases in [KhoDz]. The general case of multicolor combinations of stars and stripes is completed in [OmRR]. Ramsey numbers for path-matchings and covering designs, generalizing $R(n_1P_2, \ldots, n_kP_2)$, are studied in [DeBGS].

(h) Multicolor cases for one large path or cycle involving small paths, cycles, complete and complete bipartite graphs [EFRS1].

(i) See sections 6.5 and 8.2, especially [ArKM, BoDD], for a number of cases for triples of small graphs.

6.5. Special cases

Denote $K_3+e = K_4 - P_3$.

$R_3(K_3+e) = R_3(K_3)$ [=17]	[YR3, ArKM]
$R(K_3+e, K_3+e, K_4-e) = R(K_3, K_3, K_4-e) = 17$	[ShWR]
$R(K_3+e, K_3+e, K_5-P_3) = R(K_3, K_3, K_4)$ [=30]	[ShWR]
If $R_4(K_3) = 51$ then $R_4(K_3+e) = 52$, and	
if $R_4(K_3) > 51$ then $R_4(K_3+e) = R_4(K_3)$	[ShWR]
$R_3(K_4-e) = 28$	[Ex7] [LidP]
$R(P_3, K_4-e, K_4-e) = 11$	[Ex7], all colorings [Piw2]
$R(P_3, K_4-e, K_4) = 17$	[ArKM]
$R(P_3, K_4, K_4) = 35$	[BuE3], special case of 6.7.d
$472 \leq R_3(K_6-e)$	[HeLD]
$1102 \leq R_3(K_7-e)$	[HeLD]
$21 \leq R(K_3, K_4-e, K_4-e) \leq 22$	[ShWR] [LidP]
$31 \leq R(K_3, K_4, K_4-e) \leq 40$	[VO] [LidP]
$33 \leq R(K_4, K_4-e, K_4-e) \leq 47$	[ShWR] [LidP]
$55 \leq R(K_4, K_4, K_4-e) \leq 94$	[Ea1] [LidP]

$R(C_4, P_4, K_4-e) = 11$	[ArKM]
$R(C_4, P_4, K_4) = 14$	[BoDD]
$R(C_4, C_4, K_4-e) = 16$	[DyDz1]
$R(C_4, K_3, K_4-e) = 17$	[BoDD]
$R(C_4, K_4-e, K_4-e) = 19$	[BoDD]
$29 \le R(C_4, K_4, K_4-e) \le 36$	[VO] [BoDD]
$52 \le R(C_4, K_4, K_4) \le 72$	[XSR1]

See also section 8.2 for pointers to cumulative data for three colors.

6.6. General results for special graphs

(a) Formulas for $R_k(G)$, where G is one of the graphs P_3, $2K_2$ and $K_{1,3}$, for all k, and for P_4 if k is not divisible by 3 [Ir]. For some details see section 6.4.2.b.

(b) $tk^2+1 \le R_k(K_{2,\,t+1}) \le tk^2+k+2$, where the upper bound is general, and the lower bound holds when both t and k are prime powers [ChGra1, LaMu].

(c) $(m-1)\lfloor (k+1)/2 \rfloor < R_k(T_m) \le 2km+1$ for any tree T_m with m edges [ErdG], see also [GRS]. The lower bound can be improved for special large k [ErdG, GRS]. The upper bound was improved to $R_k(T_m) < (m-1)(k+\sqrt{k(k-1)})+2$ in [GyTu].

(d) $k(\sqrt{m}-1)/2 < R_k(F_m) < 4km$ for any forest F_m with m edges [ErdG], see [GRS]. See also pointers in items (p) and (r) below.

(e) $R(S_1, \ldots, S_k) = n+\varepsilon$, where S_i's are arbitrary stars, $n = n(S_1)+\ldots+n(S_k) - 2k$, and we set $\varepsilon = 1$ if n is even and some $n(S_i)$ is odd, and $\varepsilon = 2$ otherwise [BuRo1]. See also [GauST, Par6]. Note that for graph G (here the set of edges in a given color), to avoid star $S = K_{1,\,n}$ is equivalent to have $\delta(G) < n$.

(f) Formula for $R(S_1, \ldots, S_k, K_n)$, where S_i's are arbitrary stars [Jaco]. It was generalized to a formula for $R(S_1, \ldots, S_k, K_{k_1}, \ldots, K_{k_r})$ expressed in terms of $R(k_1, \ldots, k_r)$ and star orders [BoCGR]. A much shorter proof of the latter was presented in [OmRa2]. Special cases for bistars [AlmHS], and bounds for stars and trees instead of stars [Bai].

(g) Formula for $R(S_1, \ldots, S_k, nK_2)$, where S_i's are arbitrary stars [CocL2], and a formula for $R(n_1K_2, \ldots, n_kK_2)$ [CocL1]. A new proof with characterization of all critical graphs [XuYZ]. See also cases involving P_2 in section 6.4.2.

(h) Formula for $R(S_1, \ldots, S_k, G)$, where S_i's are stars and G is a tree [ZhZ1], or G is a cycle or wheel [RaeZ], for G of some orders depending on stars. Extension of these results to larger ranges of orders of G, and for G being a path [Wang]. Special cases when S_i's are trees and G is a wheel [RaeZ].

(i) Formulas for $R(S_1, \ldots, S_k)$, where each S_i's is a star or m_iK_2 [ZhZ2, ErdG, OmRR], formula for the case $R(S, mK_2, nK_2)$ [GySá2].

(j) Formula for $R(F, K_{k_1}, \ldots, K_{k_r})$ in terms of $R(K_{k_1}, \ldots, K_{k_r})$ and the size and structure of any forest F [KamRa]. This corrects a claim in an earlier version of [AlmBCL]. The latter studies the concept of p-goodness.

(k) Bounds on $R_k(G)$ for unicyclic graphs G of odd girth.
Some exact values for special graphs G, for $k = 3$ and $k = 4$ [KrRod].

(l) For prime $p = 3q+1$, if the cubic residues Paley graph Q_p contains no $K_k - e$, then $R_3(K_{k+1} - e) > 3p$ [HeLD]. The cases $k = 5$ and $k = 6$ give two bounds listed in section 6.5. Also based off Paley graphs, several new lower bounds for $R_3(K_1 + G)$, and in particular for $R_3(B_n)$, were derived in [LinLS].

(m) $R_k(K_{3,3}) = (1 + o(1))k^3$ [AlRóS].

(n) Bounds on $R_k(K_{s,t})$, in particular for $K_{2,2} = C_4$ and $K_{2,t}$ [ChGra1, AxFM]. Asymptotics of $R_k(K_{s,t})$ for fixed k and s [DoLi, LiTZ]. Upper bounds on $R_k(K_{s,t})$ [SunLi].

(o) Exact asymptotics $R(K_{t,s}, K_{t,s}, K_{t,s}, K_m) = \Theta(m^t/\log^t m)$, for any fixed $t > 1$ and large $s \geq (t-1)!+1$ [AlRö].

(p) Variety of asymptotic results on $R(K_{2,s}, \ldots, K_{2,s}, K_m)$ [LeMu].

(q) Bounds on $R_k(G)$ for trees, forests, stars and cycles [Bu1].

(r) If T_n is the set of all n-vertex trees (and all monochromatic n-vertex trees are avoided), then $R_3(T_n) = 2n-2$ for even n, and $R_3(T_n) = 2n-1$ for odd n [GeGy].

(s) Bounds for trees $R_k(T)$ and forests $R_k(F)$ [ErdG, GRS, BierB, GyTu, Bra1, Bra2, SwPr].

(t) $R_3(G_{a,b}) = (2 + o(1))ab$, where $G_{a,b}$ is the rectangular $a \times b$ grid graph. Lower and upper bounds on $R_3(G)$ for graphs G with small bandwidth and bounded $\Delta(G)$ [MoSST].

(u) Study of the case $R(K_m, n_1P_2, \ldots, n_kP_2)$ [Lor]. Other similar results include $R(P_3, mK_2, nK_2) = 2m+n-1$ for $m \geq n \geq 3$ [MaORS2] and $R(S_n, nK_2, nK_2) = 3n-1$ [GySá2]. More general cases, with multiple copies of the complete graph, stars and forests, were investigated in [Stahl, LorSe, LorSo, GyRSS, OmRR]. See also section 6.4.2.

(v) See section 8.2, especially [ArKM, BoDD], for a number of cases for other small graphs, similar to those listed in sections 6.3 and 6.4.

6.7. General results

(a) In 2020, the limit $\lim_{r\to\infty} R_r(3)^{1/r}$ was studied by Fox, Pach and Suk [FoxPS1] assuming a conjecture for multicolorings with bounded VC-dimension, and further for $\lim_{r\to\infty} R_r(k)^{1/r}$ when restricted to the so-called semi-algebraic colorings [FoxPS2].

(b) Szemerédi's Regularity Lemma [Szem] states that the vertices of every large graph can be partitioned into similar size parts so that the edges between these parts behave almost randomly. This lemma has been used extensively in various forms to prove the upper bounds, including those studied in [BenSk, GyRSS, GySS1, HaŁP1+, HaŁP2+, KoSS1, KoSS2].

(c) $R(m_1 G_1, \ldots, m_k G_k) \leq R(G_1, \ldots, G_k) + \sum_{i=1}^{k} n(G_i)(m_i - 1)$, exercise 8.3.28 in [West].

(d) If G is connected and $R(K_k, G) = (k-1)(n(G)-1)+1$, in particular if G is any n-vertex tree, then $R(K_{k_1}, \ldots, K_{k_r}, G) = (R(k_1, \ldots, k_r) - 1)(n - 1) + 1$ [BuE3]. A generalization for connected $G_1, \ldots, G_n$ in place of G appeared in [Jaco].

(e) Conjecture that $R_3(H) \leq 2^{\Delta^{1+o(1)}} n$, where $\Delta = \Delta(H)$ [ConFS7].

(f) For connected graphs $G_1, \ldots, G_n$ with $s = R(G_1, \ldots, G_n)$ and $t = R(K_{k_1}, \ldots, K_{k_r})$, if $m \geq 2$ and $R(G_1, \ldots, G_n, K_m) = (s-1)(m-1)+1$, then $R(G_1, \ldots, G_n, K_{k_1}, \ldots, K_{k_r}) = (s-1)(t-1)+1$ [OmRa2]. This generalizes a result in [BoCGR]. The same result was presented much later in [Mad].

(g) If F, G, H are connected graphs then $R(F, G, H) \geq (R(F, G)-1)(\chi(H)-1) + \min\{R(F, G), s(H)\}$, where $s(G)$ is the chromatic surplus of G (see item [Bu2] in section 5.16). This leads to several formulas and bounds for F and G being stars and/or trees when $H = K_n$ [ShiuLL].

(h) $R(K_{k_1}, \ldots, K_{k_r}, G_1, \ldots, G_s) \geq (R(k_1, \ldots, k_r)-1)(R(G_1, \ldots, G_s)-1)+1$ for arbitrary graphs $G_1, \ldots, G_s$ [Bev]. This generalizes 6.2.q, but is a special case of 6.7.f.

(i) Constructive bound $R(G_1, \ldots, G_{t^{n-1}}) \geq t^n + 1$ for decompositions of K_{t^n} [LaWo1, LaWo2].

(j) $R(G_1, \ldots, G_k) \leq 32\Delta k^{\Delta} n$, where $n \geq n(G_i)$ and $\Delta \geq \Delta(G_i)$ for all $1 \leq i \leq k$,
$R(G_1, \ldots, G_k) \leq k^{2k\Delta q} n$, where $q \geq \chi(G_i)$ for all $1 \leq i \leq k$ [FoxSu1].

(k) $R_k(G) \leq k^{6e(G)^{2/3} k}$ for all isolate-free graphs G and $k \geq 3$ [JoPe].
For the original two-color conjecture, now a theorem, see item 5.16.j [Erd4].

(l) $R_k(G) > (sk^{e(G)-1})^{1/n(G)}$, where s is the number of automorphisms of G [ChH3]. Other general bounds for $R_k(G)$ [ChH3, Par6].

(m) Study of $R(G_1, \ldots, G_k, G)$ for large sparse G [EFRS1, Bu3].

(n) Study of asymptotics for $R(H, \ldots, H, K_m)$, in particular when H is a fixed bipartite graph, and for $R(C_n, \ldots, C_n, K_m)$ [AlRö]. See also sections 6.3.3.d/e.

(o) Relations between the Shannon capacity of noisy communication channels and graph Ramsey numbers. A lower bound construction for $R_k(m)$ implying that supremum of the Shannon capacity over all graphs with bounded independence cannot be achieved by any finite graph power [XuR3]. For some other links between Shannon capacity and Ramsey numbers see section 6 in [Ros2], and [Li2].

(p) See surveys listed in section 8.

7. Hypergraph Numbers

7.1. Values and bounds for numbers

The only known value of a classical Ramsey number for hypergraphs:

$R(4,4;3) = 13$ [MR1]

there are exactly 434714 critical colorings on 12 points, none of which extends to a 2-coloring of all triples in $K_{13}-t$ without monochromatic K_4 [McK2]

The computer evaluation of $R(4,4;3)$ in 1991 consisted of an improvement of the upper bound from 15 to 13. This result followed an extensive theoretical study of this number by several authors [Gi4, Isb1, Sid1].

(a) $35 \le R(4, 5;3)$ [Dyb2]
$63 \le R(4, 6;3)$ [Dyb3]
$88 \le R(5, 5;3)$ [Dyb3]
$79 \le R(4,4,4;3)$ [Dyb3]
$34 \le R(5, 5;4)$ [Ex11]
$163 \le R(5,5,5;3)$ [BudHR1]

The last bound can be much improved to $7570 \le R(5,5,5;3)$ by using $88 \le R(5, 5;3)$ and a general constructive result in [BrBH], which yields $R_k(5;3) \ge 87^{2^{k-2}}$.

(b) $R(K_4-t, K_4-t;3) = 7$ [Ea2]
$R(K_4-t, K_4;3) = 8$ [Sob, Ex1, MR1]
$R(K_4-t, K_5-t;3) = 12$ [LidP]
$14 \le R(K_4-t, K_5;3) \le 16$ [Ex1] [LidP]
$13 \le R(K_4-t, K_4-t, K_4-t;3) \le 14$ [Ex1] [LidP]

(c) The first bound on $R(4, 5;3) \ge 24$ was obtained by Isbell [Isb2]. Shastri [Shas] gave a weak bound $R(5, 5;4) \ge 19$ (now 34 in [Ex11]), nevertheless his lemmas, the stepping-up lemmas by Erdős and Hajnal (see [GRS, GrRö], also item 7.4.a), and others in [Ka3, Abb2, GRS, GrRö, HuSo, SonYL] can be used to derive better lower bounds for higher numbers.

(d) Several lower bound constructions for 3-uniform hypergraphs were presented in [HuSo]. Study of lower bounds on $R(p, q;4)$ can be found in [Song3] and [SonYL, Song4] (the latter two papers are almost the same in contents). Most of the concrete lower bounds in these papers can be easily improved by using the same techniques, but starting with better constructions for small parameters as listed above.

(e) $R(p, q;4) \ge 2R(p-1, q;4) - 1$ for $p, q > 4$, and
$R(p, q;4) \ge (p-1)R(p-1, q;4) - p + 2$ for $p \ge 5$, $q \ge 7$ [SonYL].
Lower bound asymptotics for $R(p, q;4)$ [SonLi].

(f) Recurrence relations in the form $R(p, q;r) \ge d(R(p-1, q;r)-1)+1$, where d depends on p, q and r, including the following: There exists $c \ge 25$, such that for k, $5 \le k \le c$, and any $p \ge k+2$ and $q \ge k+1$, we have $R(p, q;r) \ge (p-1)(R(p-1, q;r)-1)+1$ [Liu].

Such relations lead to the following bounds:

$R(5, 6; 4) \geq 67$, $R(6, 6; 4) \geq 133$, $R(7, 6; 4) \geq 661$,
$R(7, 7; 4) \geq 3961$, $R(8, 8; 4) \geq 194041$, $R(13, 6; 4) \geq 50689$,
$R(6, 6; 5) \geq 72$.

(g) $R(K_{1,1,c}, K_{1,1,c}; 3) = c+2$ for $2 \leq c \leq 4$, and a conjecture that this equality also holds for all $c \geq 5$ [MiPal].

(h) Lower bound asymptotics for $R(4, n; 3)$ [ConFS2], lower bound asymptotics for $R(5, n; 4)$ [MuSuk2, MuSuk3], and lower bound asymptotics for $R(6, n; 4)$ [MuSuk3].

(i) Lower and upper bounds on $R(K_4 - t, K_n; 3)$ [ErdH, MuSuk2]. Extensions to *r-half-graph B^r*, where $B^3 = K_4 - t$ [MuSuk2].

(j) Several constructive lower bounds for hypergraph numbers, including constructions which introduce a new color. In particular, they imply that $R_k(5;3)$ is equal to at least 82, 163, 131073, 262145 or 524289, for $k =$ 2, 3, 4, 5 and 6 colors, respectively [BudHR1]. Using 7.1.e and other known concrete lower bounds, $R(5,6;4) \geq 67$ and $R(4,4,5,5,5,5;3) \geq 17179869185$ are noted in [BudHP].

7.2. Cycles and paths

Definitions. $P_n^{r,s}$ is called an *s-path* in an r-uniform hypergraph H, if it consists of n hyperedges $\{e_1, ..., e_n\}$ in $E(H)$, such that $|e_i \cap e_{i+1}| = s$ for all $1 \leq i < n$, and all other vertices in e_j's are distinct [Peng]. An *s-cycle* $C_n^{r,s}$ is defined analogously. Several authors use the terms of *loose* paths and *loose* cycles, which are 1-path and 1-cycles, and *tight* paths and *tight* cycles, the latter most often for 3-uniform hypergraphs when they are 2-paths and 2-cycles, respectively. A 3-uniform *Berge* cycle is formed by n distinct vertices, such that all consecutive pairs of vertices are in an edge of the cycle, and all of the cycle edges are distinct. Berge cycles are not determined uniquely.

In the following items (b) to (i), when $r = 3$ or r is implied by the context, we write C_n and P_n for the r-uniform loose cycles and paths, $C_n^{r,1}$ and $P_n^{r,1}$, respectively. In other cases special comments are added.

Two colors

(a) Tetrahedron is formed by four triples on the set of four points. The Ramsey number of tetrahedron is $R(4, 4; 3) = 13$ [MR1].

(b) For loose cycles and paths, $R(C_3, C_3; 3) = 7$, $R(C_4, C_4; 3) = 9$, and for the r-uniform case we have in general $R(P_3, P_3; r) = R(P_3, C_3; r) = R(C_3, C_3; r) + 1 = 3r - 1$ and $R(P_4, P_4; r) = R(P_4, C_4; r) = R(C_4, C_4; r) + 1 = 4r - 2$, for $r \geq 3$. These results and discussion of several related cases were presented in [GyRa].

(c) $R(P_m, P_n; 3) = R(C_m, C_n; 3) + 1 = R(P_m, C_n; 3) = 2m + \lfloor (n+1)/2 \rfloor$, for all $m \geq n$, and $R(C_m, P_n; 3) = 2m + \lfloor (n-1)/2 \rfloor$, for $m > n$ [MaORS1, OmSh1].

(d) For loose cycles, $R(C_{2n}, C_{2n}; 3) > 5n-2$ and $R(C_{2n+1}, C_{2n+1}; 3) > 5n+1$, and asymptotically these lower bounds are tight [HaŁP1+]. Generalizations to r-uniform hypergraphs and graphs other than cycles appeared in [GySS1].

(e) For loose cycles, $R(C_n, C_n; r) = (r-1)n + \lfloor (n-1)/2 \rfloor$ for $n \geq 2$, $r \geq 8$ [OmSh2], and it also holds for $r=4$ [OmSh3]. Further extensions to off-diagonal cases as in (c) are obtained in [OmSh4]. Based on these results, it was conjectured that for $n \geq m \geq 3$ and $r \geq 3$, we have $R(C_n, C_m; r) = (r-1)n + \lfloor (m-1)/2 \rfloor$. In [Shah], the known cases of this conjecture are discussed, and it is shown that it holds for $r=5$ with large n.

(f) For tight cycles, $R(C_{3n}, C_{3n}; 3) \approx 4n$ and $R(C_{3n+i}, C_{3n+i}; 3) \approx 6n$ for $i=1$ or 2, and for tight paths $R(P_n, P_n; 3) \approx 4n/3$ [HaŁP2+]. Some related results are discussed in [PoRRS].

(g) Exact values for Ramsey numbers involving s-paths for even r and $s = r/2$, in particular for $P_n^{r,s}$ versus $P_3^{r,s}$ and $P_4^{r,s}$, when this value is $(n+1)s+1$ [Peng].

(h) For 3-uniform Berge cycles and two colors, $R(C_n, C_n; 3) = n$ for $n \geq 5$ [GyLSS].

(i) Lower and upper asymptotic bounds for $R(C_3^{3,1}, K_m; 3)$ and $R(C_3^{r,1}, K_m; r)$ [KosMV2].

(j) Lower and upper asymptotic bounds for $R(C_s, K_m; 3)$ for tight cycles C_s [MuR]. An improvement of the upper bound from the latter [Mub1].

(k) Gyárfás, Sárközy and Szemerédi proved that, for sufficiently large n, every 2-coloring of the edges of the complete 4-uniform hypergraph K_n contains a monochromatic 3-tight Berge cycle C_n [GySS2]. Exact formulas and bounds for Berge-K_n hypergraphs, including higher uniformity r [SaTWZ].

(l) Upper bounds on asymptotics of $R(C_n^{r,1}, K_m; r)$ for even and odd n [ColGJ]. Improvements of the results from the latter, in particular for the case of $n=5$ and $r=3$, and for general n [Mér].

(m) Summary of known values and ranges for hypergraph numbers for loose paths (and some other trees) versus complete hypergraphs, $R(P_m, K_n; 3)$, for $n \leq 10$ and odd m [BudP].

(n) Study of the growth rate of $R(P_m, K_n; r)$ for tight paths P_m with $m \geq r+3$, and links between the growth of $R(P_{r+1}, K_n; r)$ and $R(n, n; r)$ [MuSuk1]. The correct tower growth rate for ordered tight paths versus cliques [Mub2].

(o) Study of $R(G, nH; r)$ and $R(mG, nH; r)$ for loose/tight path, cycles and stars, including several exact results for large m or n [OmRa3]. The case of loose t-tight paths versus stars and some tripartite hypergraphs is explored in [BudHR2].

(p) Let F be the Fano plane, seen as a 3-uniform hypergraph of 7 hyperedges. If P_n and C_n are tight path and cycle on n vertices, respectively, then for sufficiently large n we have $R(P_n, F; 3) = 2n-1$ and $R(C_n, F; 3) = 2n-1$ [BalCSW].

More colors

(q) For loose cycles, $R_3(C_3; 3) = R(C_3, C_3, C_3; 3) = 8$, and in general for $k \geq 4$ colors Gyárfás and Raeisi established the bounds $k+5 \leq R_k(C_3; 3) \leq 3k$ [GyRa].

(r) For loose paths, we have $R_3(P_3;3) = 9$ and $10 \le R_4(P_3;3) \le 12$ [Jack]. This was improved to $R_k(P_3;3) = k+6$ for all $2 \le k \le 9$ [JacPR, PoRu], and extended to $k = 10$ [Pol]. The general upper bound $R_k(P_3;3) \le 2k + \sqrt{18k+1} + 2$ was obtained in [ŁuPo1], then improved to $R_k(P_3;3) \le 1.975k + 7\sqrt{k} + 2$ [ŁuPo2], and then further improved to $R_k(P_3;3) \le 1.546k$ for large k [BohZ]. For the *messy path* $M_3 = \{abc, bcd, def\}$, we have $R_k(M_3;3) \le 1.6k$ for large k [BohZ]. The general case $R_k(P_3;r)$ for loose path was asymptotically solved in [ŁuPR], though do not be confused by notation in this paper because their bounds are expressed in terms of r colors and k-uniform paths.

(s) For tight paths P_{m_i}, study of the growth rate of $R(P_{m_1}, \ldots, P_{m_k}, K_m; r)$ [MuSuk1].

(t) For 3-uniform Berge cycles, $R_3(C_n;3) = (1+o(1))5n/4$ [GySá1]. Some special cases for r-uniform hypergraphs with respect to Berge cycles were studied in [GyLSS].

(u) Study of Turán and Ramsey numbers of sets of minimal 3-uniform paths of length 4 for up to 4 colors [HanPR]. Minimality of path here means that there are no redundant edge intersections, in particular no vertex belongs to more than two edges.

7.3. General results for 3-uniform hypergraphs

(a) $2^{cn^2} < R(n, n;3) < 2^{2^n}$ is credited to Erdős, Hajnal and Rado (see [ChGra2] p. 30).

(b) For some a, b the numbers $R(m, a, b;3)$ are at least exponential in m [AbbS].

(c) Improved lower and upper asymptotics for $R(s, n;3)$ for fixed s and large n, proof of related Erdős and Hajnal conjecture on the growth of $R(4, n;3)$, and the lower bound $2^{n^{c\ln n}} < R(n, n, n;3)$ [ConFS2].

(d) The hedgehog H_t is a 3-uniform hypergraph with $t + t(t-1)/2$ vertices such that for every (i, j) with $1 \le i < j \le t$ there exists a unique vertex $k > t$ such that ijk is an edge, and H_t has no other edges. Conlon, Fox and Rödl studied the bounds on $R_k(H_t;3)$ for $2 \le k \le 4$ and large t [ConFR]. The hypergraphs H_t constitute the first family of hypergraphs whose Ramsey numbers show a strong dependence on the number of colors: their 2-color Ramsey numbers grow polynomially in t, while in the 4-color case they grow exponentially. $R_k(H_t;3) = O(t^2 \ln t)$ was obtained in [FoxLi].

(e) $R(G, G;3) \le cn(H)$ for some constant c depending only on the maximum degree of a 3-uniform hypergraph H [CooFKO1, NaORS]. Similar results were proved for r-uniform hypergraphs in [KüCFO, Ishi, CooFKO2, ConFS1], see also item 7.4.h.

(f) Asymptotic lower bounds for $R(K_{a,b,c}, K_{a,b,c};3)$, where $K_{a,b,c}$ is formed by all abc triples on sets of orders a, b, c [MiPal].

(g) If G is a 3-uniform H-free hypergraph, then G contains a complete or empty tripartite subgraph with parts of order $(\log n(H))^{c+1/2}$, where $c > 0$ depends only on H. Furthermore, for $k \ge 4$ no analogue of it can hold for k-uniform hypergraphs [ConFS5].

(h) Asymptotic or exact values of $R_k(H;3)$ when H is a bow $\{abc, ade\}$, kite $\{abc, abd\}$, tight path $P_3^{3,2} = \{abc, bcd, cde\}$, or windmill $\{abc, bde, cef, bce\}$, and a special case $R_6(kite;3) = 8$. General bounds $R_k(K_3;2) \le R_{4k}(K_4 - t;3) \le R_{4k}(K_3;2) + 1$ [AxGLM].

(i) Study of 3-uniform Berge-G graphs in r colors: asymptotic lower and upper bounds, and several exact values for small r with $G = K_3$ or $G = K_4$. Some asymptotics in the nonuniform case [AxGy]. This extends the results in 7.2.h [GyLSS] and 7.2.t [GySá1].

(j) Variety of general lower bound constructions for 3-uniform complete or complete missing one hyperedge hypergraphs from liftings of graphs, for two and more colors. For example, we have $R(K_{2s_1-1}-t, K_{2s_2-1}, K_{2s_3-1}; 3) \geq R(s_1, s_2, s_3)$ [BudHMP] and $R(K_5, K_{43}-t, K_{43}-t, K_{43}-t; 3) > 1257480$ [BudHLS].

(k) Upper bounds on $R_k(H; 3)$ for complete multipartite 3-uniform hypergraphs H, a 4-color case, and some other general and special cases [ConFS1, ConFS2, ConFS3]. $R_k(H; 3)$ ranges from $\sqrt{6k}\,(1+o(1))$ to double exponential in k [AxGLM].

7.4. General results

(a) If $R(n, n; r) > m$ then $R(2n+r-4, 2n+r-4; r+1) > 2^m$, for $n > r \geq 3$ (see [GRS] p. 106). This is the so-called stepping-up lemma, usually credited to Erdős and Hajnal. An improvement of the stepping-up lemma implying better lower bounds for a few types of hypergraph Ramsey numbers were obtained by Conlon, Fox and Sudakov [ConFS6].

(b) Lower bounds on $R_k(n; r)$ are discussed in [AbbW, DuLR].

(c) General lower bounds for large number of colors were given in an early paper by Hirschfeld [Hir], and some of them were later improved in [AbbL].

(d) Lower and upper asymptotics of $R(s, n; k)$ for fixed s [ConFS2, MuSuk2, MuSuk3].

(e) Exact and asymptotic results generalizing 7.2.d/e to r-uniform case for cycles, and 2- and 3-color cases for all r-uniform diamond matchings [GySS1].

(f) Exact formulas and bounds for Berge-K_n hypergraphs, including multiple colors [AxGy] and higher uniformity r [AxGy, SaTWZ]. Progress on the conjecture that every $(r-1)$-coloring of K_n^r, for fixed r and large n, contains a monochromatic Hamiltonian Berge cycle [MaOm2]. Determination of some cases of uniformity r, number of colors and G, for which the Ramsey number of Berge-G is superlinear [Gerbn]. Further study of multicolor Ramsey numbers for such Berge-G hypergraphs, with some equalities, were obtained in [GerMOV].

(g) Study of $R(G, nH; r)$ and $R(mG, nH; r)$ for loose/tight path and cycles (possibly with some additions), stars, r-partite hypergraphs, including several exact results for large m or n [OmRa3].

(h) $R(H, H; r) \leq cn(H)^{1+\varepsilon}$, for some constant $c = c(\Delta, r, \varepsilon)$ depending only on the maximum degree of H, r and $\varepsilon > 0$ [KoRö3]. The proofs of the linear bound $cn(H)$ were obtained independently in [KüCFO] and [Ishi], the latter including the multicolor case, and then without regularity lemma in [ConFS1]. More discussion of lower and upper bounds for various cases can be found in [ConFS1, ConFS2, ConFS3, CooFKO2].

(i) Let T_r be an r-uniform hypergraph with r edges containing a fixed $(r-1)$-vertex set S and the $(r+1)$-st edge intersecting all former edges in one vertex outside S. Then $R(T_r, K_t; r) = O(t^r/\log t)$ [KosMV1].

(j) Study of tree-star and tree-complete cases of Ramsey numbers for r-uniform hypergraphs. Several bounds and equalities for special cases [BudHR1]. This was posed and explored as a problem of which trees are Ramsey n-good hypergraphs [BudP]. Study of the Ramsey numbers of disjoint union of H-good hypergraphs [RaeK].

(k) Let $H^r(s, t)$ be the complete r-partite r-uniform hypergraph with $r-2$ parts of size 1, one part of size s, and one part of size t (for example, for $r=2$ it is the same as $K_{s,t}$). For the multicolor numbers, Lazebnik and Mubayi [LaMu] proved that

$$tk^2 - k + 1 \le R_k(H^r(2, t+1); r) \le tk^2 + k + r,$$

where the lower bound holds when both t and k are prime powers. For the general case of $H^r(s, t)$, more bounds are presented in [LaMu].

(l) $R_k(H; r)$ is polynomial in k when a fixed r-uniform H is r-partite, otherwise it is at least exponential in k [AxGLM].

(m) Grolmusz [Grol1] generalized the classical constructive lower bound by Frankl and Wilson [FraWi] (item 2.3.6) to more colors and to hypergraphs [Grol3].

(n) Lower and upper asymptotics, and other theoretical results on hypergraph numbers, are discussed in [GrRö, GRS, ConFS1, ConFS2, ConFS3, ConFS7, Song8, MuSuk1, MuSuk2, MuSuk3]. An extensive overview of progress and open problems in hypergraph Ramsey theory by Mubayi and Suk was compiled in 2018 [MuSuk4].

8. Cumulative Data and Surveys

8.1. Cumulative data for two colors

(a) $R(G, G)$ for all graphs G without isolates on at most 4 vertices [ChH1].

(b) $R(G, H)$ for all graphs G and H without isolates on at most 4 vertices [ChH2].

(c) $R(G, H)$ for all graphs G on at most 4 vertices and H on 5 vertices, except five entries [Clan], now all solved, see section 5.11. All critical colorings for the isolate-free graphs G and H studied in [Clan] were found in [He4].

(d) $R(G, G)$ for all graphs G without isolates and with at most 6 edges [Bu4].

(e) $R(G, G)$ for all graphs G without isolates and with at most 7 edges [He1].

(f) $R(G, G)$ for all graphs G on 5 vertices and with 7 or 8 edges [HaMe2].

(g) $R(G, H)$ for all graphs G and H on 5 vertices without isolates, except 7 entries [He2]. Only 2 cases are still open, see 5.11 and the paragraph at the end of this section.

(h) Tables of $R(G, H)$ for most connected graphs on up to 5 vertices and $R(G, G)$ for all isolate-free graphs with up to 7 edges [ReWi].

(i) $R(G, H)$ for all disconnected isolate-free graphs H on at most 6 vertices versus all G on at most 5 vertices, except 3 cases [LoM5]. Missing cases were completed in [KroMe].

(j) $R(G, H)$ for some G on 5 vertices versus all connected graphs on 6 vertices [LoM6].

(k) $R(G, H)$ for $G = K_{1,3} + e$ and $G = K_4 - e$ versus all connected graphs H on 6 vertices, except $R(K_4 - e, K_6)$ [HoMe]. The result $R(K_4 - e, K_6) = 21$ was claimed by McNamara [McN, unpublished], now confirmed in [ShWR].

(l) $R(G, H)$ for some graphs G with 4 vertices versus all graphs H with 7 vertices [Boza4].

(m) $R(G, T)$ for all connected graphs G with $n(G) \leq 5$, and almost all trees T [FRS4].

(n) $R(T_m, T_n)$ for $6 \leq m, n \leq 8$, for k-vertex trees T_k [RanMCG].

(o) $R(K_3, G)$ for all connected graphs G on 6 vertices [FRS1].

(p) $R(K_3, G)$ for all connected graphs G on 7 vertices [Jin].
Some errors in the latter were found [SchSch1].

(q) $R(S, G)$ for stars S versus almost all connected graphs G on 6 vertices [LoM7]. This was extended to $R(T, G)$ for most non-star trees T, in particular for all trees on at most 5 vertices versus all connected graphs G on at most 6 vertices [LoM8].

(r) Formulas for $R(nK_3, mG)$ for all G of order 4 without isolates [Zeng].

(s) $R(K_3, G)$ for all connected graphs G on at most 8 vertices [Brin]. The numbers for K_3 versus sets of graphs with fixed number of edges, on at most 8 vertices, were presented in [KlaM1].

(t) $R(K_3, G)$ for all connected graphs G on 9 vertices [BrBH1, BrBH2].

(u) $R(K_3, G)$ for all graphs G on 10 vertices, except 10 cases [BrGS].
Three of the open cases, including $G = K_{10} - e$, were solved [GoR2].

(v) $R(C_4, G)$ for all graphs G on at most 6 vertices [JR3].
This work was followed by two errata listed in the references.

(w) $R(C_5, G)$ for all graphs G on at most 6 vertices [JR4].

(x) $R(C_6, G)$ for all graphs G on at most 5 vertices [JR2].

(y) $R(K_{2,n}, K_{2,m})$ for all $2 \leq n, m \leq 10$ except 8 cases, for which lower and upper bounds are given [LoM3]. Further data for other complete bipartite graphs are gathered in section 3.3 and [LoMe4].

(z) All best lower bounds up to 102 from cyclic graphs. Formulas for best cyclic lower bounds for paths and cycles, and values for small complete graphs and for graphs with up to five vertices [HaKr1].

Chvátal and Harary [ChH1, ChH2] formulated several simple but very useful observations on how to discover values of some numbers. All five missing entries in the tables of Clancy [Clan] have been solved (section 5.11). Out of 7 open cases in [He2] 5 have been solved, including $R(4, 5) = R(G_{19}, G_{23}) = 25$ and other cases listed in section 5.11. The 2 cases still open are for K_5 versus K_5 (section 2.1) and K_5 versus $K_5 - e$ (section 3.1). Many extremal and other Ramsey graphs for various parameters are available at [BrCGM, McK1, Ex18, Fuj1], see section 8.3 below.

8.2. Cumulative data for three colors

(a) $R_3(G)$ for all graphs G with at most 4 edges and no isolates [YR3].

(b) $R_3(G)$ for all graphs G with 5 edges and no isolates, except $K_4 - e$ [YR1]. The case of $R_3(K_4 - e)$ remains open (see section 6.5).

(c) $R_3(G)$ for all graphs G with 6 edges and no isolates, except 10 cases [YY].

(d) $R(F, G, H)$ for many triples of isolate-free graphs with at most 4 vertices [ArKM]. Some of the missing cases completed in [KlaM2].

(e) Extension of [ArKM] to most triples of graphs with at most 4 vertices [BoDD].

(f) $R(P_3, P_k, C_m)$ for all $3 \le k \le 8$ and $3 \le m \le 9$ [DzFi2].

8.3. Electronic Resources

(a) W. Gasarch [Gas] maintains a website gathering over 60 pointers to literature on applications of Ramsey theory in computer science, and in particular logic, complexity theory and algorithms, http://www.cs.umd.edu/~gasarch/TOPICS/ramsey/ramsey.html.

(b) Many of the Ramsey graph constructions found by G. Exoo [Ex1-Ex23] are posted at http://cs.indstate.edu/ge/RAMSEY.

(c) G. Brinkmann, K. Coolsaet, J. Goedgebeur and H. Mélot, *House of Graphs: A database of interesting graphs* [BrCGM], http://hog.grinvin.org.

(d) B.D. McKay, presents some graphs related to classical Ramsey numbers [McK1], http://cs.anu.edu.au/people/bdm/data/ramsey.html.

(e) Set of Ramsey problems with comments and references by a team of students of Fan Chung, University of California San Diego [UCSD], *Erdős' Problems on Graphs, Ramsey Theory,* http://www.math.ucsd.edu/~erdosproblems/RamseyTheory.html (2010-2012).

(f) H. Fujita, collection of Ramsey graph constructions [Fuj1], http://opal.inf.kyushu-u.ac.jp/~fujita/ramsey.html, (2014-2017).

(g) M. Rubey, an electronic GUI resource for values of some small Ramsey numbers [Rub], http://www.findstat.org/StatisticsDatabase/St000479.

(h) S. Van Overberghe, Ramsey graph constructions associated with MS Thesis, Ghent University, Belgium, 2020 [VO], https://github.com/Steven-VO/circulant-Ramsey.

(i) A.E. Brouwer, Parameters of Strongly Regular Graphs [Brou], used mainly in 3.1.d and 5.3.h, https://www.win.tue.nl/~aeb/graphs/srg/srgtab.html.

8.4. Surveys

(1974) A general survey of results in Ramsey graph theory by S.A. Burr [Bu1]

(1978) A general survey of results in Ramsey graph theory by T.D. Parsons [Par6]

(1980) Survey of results and new problems on multiplicities and Ramsey multiplicities by S.A. Burr and V. Rosta [BuRo3]

(1981) Summary of progress by Frank Harary [Har2]

(1983) A survey of bounds and values by F.R.K. Chung and C.M. Grinstead [ChGri]

(1983) Special volume of the *Journal of Graph Theory* [JGT]

(1984) A review of Ramsey graph theory for newcomers by F.S. Roberts [Rob1]

(1987) An overview of progress so far and plans for the future, *What Can We Hope to Accomplish in Generalized Ramsey Theory?* by S. Burr [Bu7]

(1987) Survey of asymptotic problems by R.L. Graham and V. Rödl [GrRö]

(1990) *Ramsey Theory* by R.L. Graham, B.L. Rothschild and J.H. Spencer [GRS],
first edition 1980, second edition 1990, paperback of the second edition 2013.

(1991) Survey by R.J. Faudree, C.C. Rousseau and R.H. Schelp of graph goodness results, i.e. conditions for the formula $R(G, H) = (\chi(G) - 1)(n(H) - 1) + s(G)$ [FRS5]

(1996) A chapter in *Handbook of Combinatorics* by J. Nešetřil [Neš]

(1996) Survey of zero-sum Ramsey theory by Y. Caro [Caro]

(1997) Among 114 open problems and conjectures of Paul Erdős, presented and commented by F.R.K. Chung, 31 are concerned directly with Ramsey numbers [Chu4]. 216 references are given. An extended version of this work was prepared jointly with R.L. Graham [ChGra2] in 1998.

(2001) An extensive chapter on Ramsey theory in a widely used student textbook and researcher's guide of graph theory by D. West [West]

(2002) *Ramsey Theory and Paul Erdős* by R.L. Graham and J. Nešetřil [GrNe]

(2003) Special issue of *Combinatorics, Probability and Computing* [CoPC]

(2004) Dynamic survey of Ramsey theory applications by V. Rosta [Ros2]. A website maintained by W. Gasarch [Gas] gathers over 60 pointers to literature on applications of Ramsey theory in computer science.

(2009) History, results and people of Ramsey theory. The mathematical coloring book, mathematics of coloring and the colorful life of its creators by A. Soifer [Soi1].

(2010) *Hypergraph Ramsey Numbers* by D. Conlon, J. Fox and B. Sudakov [ConFS2].

(2011) *Ramsey Theory. Yesterday, Today and Tomorrow,* a special volume in the series *Progress in Mathematics* [Soi2]. A survey of Ramsey numbers involving cycles by the author is included in this volume [Ra4].

(2013) *Problems in Graph Theory from Memphis,* "a summary of problems and results coming out of the 20 year collaboration between Paul Erdős and the authors", by R.J. Faudree, C.C. Rousseau and R.H. Schelp [FRS6].

(2014) *Ramsey Theory on the Integers* by B. Landman and A. Robertson [LaRo], first edition 2004, second edition 2014.

(2015) *Recent Developments in Graph Ramsey Theory* by D. Conlon, J. Fox and B. Sudakov [ConFS7].

(2015) *Rudiments of Ramsey Theory,* a new edition of the classics by R.L. Graham and S. Butler [GrBu].

(2016) *On Some Open Questions for Ramsey and Folkman Numbers* [XuR4].

(2018) *A Survey of Hypergraph Ramsey Problems,* by D. Mubayi and A. Suk [MuSuk4].

(2018) *Ramsey Theory, Unsolved Problems and Results,* by Xiaodong Xu, Meilian Liang and Haipeng Luo [XuLL].

(2020) *An Introduction to Ramsey's Theorem* by A. Tripathi [Tri].

The surveys by S.A. Burr [Bu1] and T.D. Parsons [Par6] contain extensive chapters on general exact results in graph Ramsey theory. F. Harary presented the state of the theory in 1981 in [Har2], where he also gathered many references including seven to other early surveys of this area. More than three decades ago, Chung and Grinstead in their survey paper [ChGri] gave much less data than in this work, but they included a broad discussion of different methods used in Ramsey computations in the classical case. S. A. Burr, one of the most experienced researchers in Ramsey graph theory, formulated in [Bu7] seven conjectures on Ramsey numbers for sufficiently large and sparse graphs, and reviewed the evidence for them found in the literature. Three of them have been refuted in [Bra3].

For newer extensive presentations see [GRS, GrRö, FRS5, Neš, Chu4, ChGra2, ConFS7], though these focus on asymptotic theory not on the numbers themselves. A very welcome addition is the 2004 compilation of applications of Ramsey theory by V. Rosta [Ros2]. This survey could not be complete without recommending special volumes of the *Journal of Graph Theory* [JGT, 1983] and *Combinatorics, Probability and Computing* [CoPC, 2003], which, besides a number of research papers, include historical notes and present to us Frank P. Ramsey (1903-1930) as a person. Read a colorful book by A. Soifer [Soi1, 2009] on history and results in Ramsey theory, followed by a collection of essays and technical papers based on presentations from the 2009 Ramsey theory workshop at DIMACS [Soi2, 2011]. A 70-page long paper from 2015, entitled *Recent Developments in Graph Ramsey Theory,* by D. Conlon, J. Fox and B. Sudakov [ConFS7] documents in details what the title says.

The historical perspective and, in particular, the timeline of progress on prior best bounds, can be obtained by checking all the previous versions of this survey since 1994 at http://www.cs.rit.edu/~spr/ElJC/eline.html.

9. Concluding Remarks

This compilation does not include much information on numerous variations of Ramsey numbers, nor related topics, like

anti-Ramsey numbers,	bipartite Ramsey numbers,
chromatic Ramsey numbers,	complementary Ramsey numbers,
connected Ramsey numbers,	defective Ramsey numbers,
directed Ramsey numbers,	edge-chromatic Ramsey numbers,
edge-ordered Ramsey numbers,	Gallai-Ramsey numbers,
induced Ramsey numbers,	irredundant Ramsey numbers,
list Ramsey numbers,	local Ramsey numbers,
k-Ramsey numbers,	mixed Ramsey numbers,
multipartite Ramsey numbers,	online Ramsey numbers,
ordered Ramsey numbers,	ordered size Ramsey numbers,
oriented Ramsey numbers,	oriented size Ramsey numbers,
planar Ramsey numbers,	potential Ramsey numbers,
proper Ramsey numbers,	rainbow Ramsey numbers,
Ramsey game numbers,	Ramsey-Turán numbers,
restricted online Ramsey numbers,	restricted size Ramsey numbers,
semi-algebraic Ramsey numbers,	singular Ramsey numbers,
size Ramsey numbers,	size multipartite Ramsey numbers,
star-critical Ramsey numbers,	weakened Ramsey numbers,
zero-sum Ramsey numbers,	Ramsey equivalence,
Ramsey games,	avoiding sets of graphs in some colors,
coloring graphs other than complete,	or the so called Ramsey multiplicities.

Interested readers can find such information in some of the surveys listed in section 8 here.

Readers may be also interested in knowing that the US patent 6965854 B2 issued on November 15, 2005 claims a method of using Ramsey numbers in "Methods, Systems and Computer Program Products for Screening Simulated Traffic for Randomness." Check the original document at http://www.uspto.gov/patft if you wish to find out whether your usage of Ramsey numbers is covered by this patent.

Acknowledgements

In addition to the many individuals who helped to improve consecutive versions of this survey, the author would like to specially thank his Ramsey collaborator Xiaodong Xu. He has seen more revisions of this survey than one would wish. The author would also like to extend his thanks to Brendan McKay, Geoffrey Exoo and Heiko Harborth for their help in gathering data throughout the years. Those who contributed to the development and improvement of new revisions over the years are greatly appreciated.

The author apologizes for any omissions or other errors in reporting results belonging to the scope of this work. Suggestions for any kind of corrections or additions will be greatly appreciated and considered for inclusion in the next revision of this survey.

References

Out of 856 references gathered below, most appeared in about 100 different periodicals, among which most articles were published in: *Discrete Mathematics* 91, *Journal of Combinatorial Theory* (old, Series A and B) 61, *Journal of Graph Theory* 56, *Electronic Journal of Combinatorics* 47, *Ars Combinatoria* 32, *Journal of Combinatorial Mathematics and Combinatorial Computing* 31, *Graphs and Combinatorics* 28, *European Journal of Combinatorics* 25, *Australasian Journal of Combinatorics* 22, *Discrete Applied Mathematics* 21, *Combinatorica* 18, *Utilitas Mathematica* 18, *Combinatorics, Probability and Computing* 15, *Congressus Numerantium* 12, *SIAM Journal on Discrete Mathematics* 10, and *Discussiones Mathematicae Graph Theory* 9. There are 31 pointers to arXiv preprints. The results of 169 references depend on computer algorithms.

The references are ordered alphabetically by the last name of the first author, and where multiple papers have the same first author they are ordered by the last name of the second author, etc. We preferred that all work by the same author be in consecutive positions. Unfortunately, this causes that some of the abbreviations are not in alphabetical order. For example, [BaRT] is earlier on the list than [BaLS]. We also wish to explain a possible confusion with respect to the order of parts and spelling of Chinese names. We put them without any abbreviations, often with the last name written first as is customary in original. Sometimes this is different from the citations in other sources. One can obtain all variations of writing any specific name by consulting the authors database of *Mathematical Reviews* at http://www.ams.org/mathscinet/search, or *zbMATH* (formerly *Zentralblatt für Mathematik*) at http://www.zbmath.org/authors.

Papers containing results obtained with the help of computer algorithms have been marked with stars. We identify two such categories of papers: those marked with * involving some use of computers where the results are easily verifiable with some computations, and those marked with ** where cpu intensive algorithms have to be implemented to replicate or verify the results. The first category contains mostly constructions done by algorithms, while the second mostly nonexistence results or claims of complete enumerations of special classes of graphs.

A, Ba, Bl, Bu	page 68
Ca, Cl, D, E	page 75
Fa, Fi, Ga, Gu, Ha, He	page 82
I, J, K, La, Li, Lia, Lo	page 89
M, N, O, P, Q, Ra, Ro	page 97
Sa, Sh, Si, Su, Sun	page 104
T, U, V, W, X, Y, Z	page 110 - page 116

A

[Abb1] H.L. Abbott, Some Problems in Combinatorial Analysis, *Ph.D. thesis,* Department of Mathematical and Statistical Sciences, University of Alberta, Edmonton, 1965.

[Abb2] H.L. Abbott, A Theorem Concerning Higher Ramsey Numbers, in *Infinite and Finite Sets,* (A. Hajnal, R. Rado and V.T. Sós eds.) Vol. 1, 25-28, Colloq. Math. Soc. János Bolyai, Vol. 10, North-Holland, Amsterdam, 1975.

[AbbH] H.L. Abbott and D. Hanson, A Problem of Schur and Its Generalizations, *Acta Arithmetica,* **20** (1972) 175-187.

[AbbL] H.L. Abbott and Andy Liu, Remarks on a Paper of Hirschfeld Concerning Ramsey Numbers, *Discrete Mathematics,* **39** (1982) 327-328.

[AbbS] H.L. Abbott and M.J. Smuga-Otto, Lower Bounds for Hypergraph Ramsey Numbers, *Discrete Applied Mathematics,* **61** (1995) 177-180.

[AbbW] H.L. Abbott and E.R. Williams, Lower Bounds for Some Ramsey Numbers, *Journal of Combinatorial Theory,* Series A, **16** (1974) 12-17.

[-] Adiwijaya, see [SuAM, SuAAM].

[AjKS] M. Ajtai, J. Komlós and E. Szemerédi, A Note on Ramsey Numbers, *Journal of Combinatorial Theory,* Series A, **29** (1980) 354-360.

[AjKSS] M. Ajtai, J. Komlós, M. Simonovits and E. Szemerédi, Erdős-Sós Conjecture, *in preparation* (2013).

[AliBB] K. Ali, A.Q. Baig and E.T. Baskoro, On the Ramsey Number for a Linear Forest versus a Coctail Party Graph, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **71** (2009) 173-177.

[AliBas] K. Ali and E.T. Baskoro, On the Ramsey Numbers for a Combination of Paths and Jahangirs, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **65** (2008) 113-119.

[AliBT1] K. Ali, E.T. Baskoro and I. Tomescu, On the Ramsey Numbers for Paths and Generalized Jahangir Graphs $J_{s,\,m}$, *Bull. Math. Soc. Sci. Math. R. S. Roumanie (N.S.),* **51(99)** (2008) 177-182.

[AliBT2] K. Ali, E.T. Baskoro and I. Tomescu, On the Ramsey Number for Paths and Beaded Wheels, *Journal of Prime Research in Mathematics,* **5** (2009) 133-138.

[AliSur] K. Ali and Surahmat, A Cycle or Jahangir Ramsey Unsaturated Graphs, *Journal of Prime Research in Mathematics,* **2** (2006) 187-193.

[AliTJ] K. Ali, I. Tomescu and I. Javaid, On Path-Sunflower Ramsey Numbers, *Mathematical Reports,* Bucharest, **17** (2015) 385-390.

[AllBS] P. Allen, G. Brightwell and J. Skokan, Ramsey-Goodness - and Otherwise, *Combinatorica,* **33** (2013) 125-160.

[AllŁPZ] P. Allen, T. Łuczak, J. Polcyn and Yanbo Zhang, The Ramsey Number of a Long Even Cycle versus a Star, *preprint,* arXiv, http://arxiv.org/abs/2003.03310 (2020).

[AlmBCL] J. Alm, P. Bahls, K. Coffey and C. Langhoff, Generalizing p-Goodness to Ordered Graphs, *preprint,* arXiv, http://arxiv.org/abs/1412.3071 (2020).

[AlmHS] J.F. Alm, N. Hommowun and A. Schneider, Mixed, Multi-color, and Bipartite Ramsey Numbers Involving Trees of Small Diameter, *preprint,* arXiv, http://arxiv.org/abs/1403.0273 (2014).

[Alon1] N. Alon, Subdivided Graphs Have Linear Ramsey Numbers, *Journal of Graph Theory,* **18** (1994) 343-347.

[Alon2] N. Alon, Explicit Ramsey Graphs and Orthonormal Labelings, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #R12, **1** (1994), 8 pages.

[AlBK] N. Alon, S. Ben-Shimon and M. Krivelevich, A Note on Regular Ramsey Graphs, *Journal of Graph Theory,* **64** (2010) 244-249.

[AlKS] N. Alon, M. Krivelevich and B. Sudakov, Turán Numbers of Bipartite Graphs and Related Ramsey-Type Questions, *Combinatorics, Probability and Computing,* **12** (2003) 477-494.

[AlPu] N. Alon and P. Pudlák, Constructive Lower Bounds for Off-Diagonal Ramsey Numbers, *Israel Journal of Mathematics,* **122** (2001) 243-251.

[AlRö] N. Alon and V. Rödl, Sharp Bounds for Some Multicolor Ramsey Numbers, *Combinatorica,* **25** (2005) 125-141.

[AlRóS] N. Alon, L. Rónyai and T. Szabó, Norm-Graphs: Variations and Applications, *Journal of Combinatorial Theory,* Series B, **76** (1999) 280-290.

[Alw] R. Alweiss, Ramsey Numbers of Odd Cycles versus Larger Even Wheels, *Discrete Mathematics,* **341** (2018) 981-989.

[-] B.M.N. Alzaleq, see [BatJA, JaAl].

[Ana]* C.S. Anabanti, A Counterexample on a Group Partitioning Problem, *Birkbeck Mathematics Preprint Series,* #37, Birkbeck University of London, (2017), 7 pages.

[AnM1]** V. Angeltveit and B.D. McKay, $R(5, 5) \leq 48$, *Journal of Graph Theory,* **89** (2018) 5-13.

[AnM2]** V. Angeltveit and B.D. McKay, *personal communication* (2020).

[ArKM] J. Arste, K. Klamroth and I. Mengersen, Three Color Ramsey Numbers for Small Graphs, *Utilitas Mathematica,* **49** (1996) 85-96.

[-] H. Assiyatun, see [HaABS, HaBA1, HaBA2, BaHA, SuAAM, SuAUB, SuBAU1, SuBAU2, SuBAU3].

[AtLZ] A. Atminas, V. Lozin and V. Zamaraev, Linear Ramsey Numbers, *Proceedings of IWOCA 2018,* Singapore, LNCS 10979, Springer, (2018) 26-38.

[AxFM] M. Axenovich, Z. Füredi and D. Mubayi, On Generalized Ramsey Theory: the Bipartite Case, *Journal of Combinatorial Theory,* Series B, **79** (2000) 66-86.

[AxGy] M. Axenovich and A. Gyárfás, A Note on Ramsey Numbers for Berge-G Hypergraphs, *Discrete Mathematics,* **342** (2019) 1245-1252.

[AxGLM] M. Axenovich, A. Gyárfás, Hong Liu and D. Mubayi, Multicolor Ramsey Numbers for Triple Systems, *Discrete Mathematics,* **322** (2014) 69-77.

Ba - Bi

[BaRT]* A. Babak, S.P. Radziszowski and Kung-Kuen Tse, Computation of the Ramsey Number $R(B_3, K_5)$, *Bulletin of the Institute of Combinatorics and its Applications,* **41** (2004) 71-76.

[Back1] J. Backelin, Contributions to a Ramsey Calculus, *manuscript* 2000-2012.

[Back2] J. Backelin, *personal communication* (2013).

[Back3]* J. Backelin, Edge number report 1: state of the art estimates for $n \leq 43$, *preprint,* arXiv, http://arxiv.org/abs/1410.1843 (2014).

[Back4] J. Backelin, *personal communication* (2017).

[BahS] P. Bahls and T.S. Spencer, On the Ramsey Numbers of Trees with Small Diameter, *Graphs and Combinatorics,* **29** (2013) 39-44.

[-] P. Bahls, see also [AlmBCL].

[Bai] Bai Lufeng, Multi-color Ramsey Numbers for Trees versus Complete Graphs (in Chinese), *Mathematics in Practice and Theory,* **43** (2013) 252-254.

[BaiLi] Bai Lufeng and Li Yusheng, Algebraic Constructions and Applications in Ramsey Theory, *Advances in Mathematics,* **35** (2006) 167-170.

[BaLX] Bai Lufeng, Li Yusheng and Xu Zhiqiang, Algebraic Constructions and Applications in Ramsey Theory, *Journal of Mathematical Study (China),* **37** (2004) 245-249.

[-] Bai Lufeng, see also [SonBL].

[-] A.Q. Baig, see [AliBB].

[BaLS] P.N. Balister, J. Lehel and R.H. Schelp, Ramsey Unsaturated and Saturated Graphs, *Journal of Graph Theory,* **51** (2006) 22-32.

[BaSS] P.N. Balister, R.H. Schelp and M. Simonovits, A Note on Ramsey Size-Linear Graphs, *Journal of Graph Theory,* **39** (2002) 1-5.

[BalPS] I. Balla, A. Pokrovskiy and B. Sudakov, Ramsey Goodness of Bounded Degree Trees, *Combinatorics, Probability and Computing,* **27**(3) (2018) 289-309.

[BalCSW] J. Balogh, F.C. Clemen, J. Skokan and A.Z. Wagner, The Ramsey Number of Fano Plane Versus Tight Path, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P1.60, **27**(1) (2020), 16 pages.

[BaJBJ] A. Baniabedalruhman, M.M.M. Jaradat, M.S. Bataineh and A.M.M. Jaradat, The Theta-Complete Graph Ramsey Number $r(\theta_k, K_6)$; $k \geq 6$, *preprint,* (2020).

[-] A.M.M. Baniabedalruhman, see also [JaBa].

[-] Qiquan Bao, see [ShaXB, ShaXBP].

[BarRSW] B. Barak, A. Rao, R. Shaltiel and A. Wigderson, 2-Source Dispersers for $n^{o(1)}$ Entropy, and Ramsey Graphs Beating the Frankl-Wilson Construction, *Annals of Mathematics* (2), **176** (2012) 1483-1544.

[BaLiu] *Hungarian Problem Book IV,* translated and edited by Robert Barrington Leigh and Andy Liu, The Mathematical Association of America, 2011.

[Bas] E.T. Baskoro, The Ramsey Number of Paths and Small Wheels, *Majalah Ilmiah Himpunan Matematika Indonesia,* MIHMI, **8** (2002) 13-16.

[BaHA] E.T. Baskoro, Hasmawati and H. Assiyatun, The Ramsey Numbers for Disjoint Unions of Trees, *Discrete Mathematics,* **306** (2006) 3297-3301.

[BaSu] E.T. Baskoro and Surahmat, The Ramsey Number of Paths with respect to Wheels, *Discrete Mathematics,* **294** (2005) 275-277.

[BaSNM] E.T. Baskoro, Surahmat, S.M. Nababan and M. Miller, On Ramsey Graph Numbers for Trees versus Wheels of Five or Six Vertices, *Graphs and Combinatorics,* **18** (2002) 717-721.

[-] E.T. Baskoro, see also [AliBB, AliBas, AliBT1, AliBT2, HafBa, HaABS, HaBA1, HaBA2, NoBa, SuAUB, SuBa1, SuBa2, SuBAU1, SuBAU2, SuBAU3, SuBB1, SuBB2, SuBB3, SuBB4, SuBT1, SuBT2, SuBTB, SuBUB].

[BatJA] M.S.A. Bataineh, M.M.M. Jaradat and L.M.N. Al-Zaleq, The Cycle-Complete Graph Ramsey Number $r(C_9, K_8)$, *International Scholarly Research Network - Algebra,* Article ID 926191, (2011), 10 pages.

[-] M.S.A. Bataineh, see also [BaJBJ, JaBVR].

[BenSk] F.S. Benevides and J. Skokan, The 3-Colored Ramsey Number of Even Cycles, *Journal of Combinatorial Theory,* Series B, **99** (2009) 690-708.

[-] S. Ben-Shimon, see [AlBK].

[Bev] D. Bevan, *personal communication* (2002).

[BePi] A. Beveridge and O. Pikhurko, On the Connectivity of Extremal Ramsey Graphs, *Australasian Journal of Combinatorics,* **41** (2008) 57-61.

[BiaS] A. Bialostocki and J. Schönheim, On Some Turán and Ramsey Numbers for C_4, in *Graph Theory and Combinatorics* (ed. B. Bollobás), Academic Press, London, (1984) 29-33.

[Biel1] H. Bielak, Ramsey and 2-local Ramsey Numbers for Disjoint Unions of Cycles, *Discrete Mathematics,* **307** (2007) 319-330.

[Biel2] H. Bielak, Ramsey Numbers for a Disjoint Union of Some Graphs, *Applied Mathematics Letters,* **22** (2009) 475-477.

[Biel3] H. Bielak, Multicolor Ramsey Numbers for Some Paths and Cycles, *Discussiones Mathematicae Graph Theory,* **29** (2009) 209-218.

[Biel4] H. Bielak, Ramsey Numbers for a Disjoint Union of Good Graphs, *Discrete Mathematics,* **310** (2010) 1501-1505.

[BieDa] H. Bielak and K. Dabrowska, The Ramsey Numbers for Some Subgraphs of Generalized Wheels versus Cycles and Paths, *Annales Universitatis Mariae Curie-Skłodowska Lublin-Polonia,* Sectio A, **LXIX** (2015) 1-7.

[-] H. Bielak, see also [LiBie, LiZBBH].

[Bier] J. Bierbrauer, Ramsey Numbers for the Path with Three Edges, *European Journal of Combinatorics,* **7** (1986) 205-206.

[BierB] J. Bierbrauer and A. Brandis, On Generalized Ramsey Numbers for Trees, *Combinatorica,* **5** (1985) 95-107.

[BiFJ] C. Biró, Z. Füredi and S. Jahanbekam, Large Chromatic Number and Ramsey Graphs, *Graphs and Combinatorics,* **29** (2013) 1183-1191.

Bl - Br

[BlLR]* K. Black, D. Leven and S.P. Radziszowski, New Bounds on Some Ramsey Numbers, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **78** (2011) 213-222.

[BlLi] T. Bloom and A. Liebenau, Ramsey Equivalence of K_n and $K_n + K_{n-1}$, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P3.4, **25**(3) (2018), 17 pages.

[Boh] T. Bohman, The Triangle-Free Process, *Advances in Mathematics,* **221** (2009) 1653-1677.

[BohK1] T. Bohman and P. Keevash, The Early Evolution of the H-Free Process, *Inventiones Mathematicae,* **181** (2010) 291-336.

[BohK2] T. Bohman and P. Keevash, Dynamic Concentration of the Triangle-Free Process, *Seventh European Conference on Combinatorics, Graph Theory and Applications,* 489-495, CRM Series, 16, Pisa, 2013.

[BohK3] T. Bohman and P. Keevash, Dynamic Concentration of the Triangle-Free Process, *preprint,* arXiv, http://arxiv.org/abs/1302.5963 (2013), 52 pages. Revised version (2019), 75 pages.

[BohZ] T. Bohman and Emily Zhu, On Multicolor Ramsey Numbers of Triple System Paths of Length 3, *preprint,* arXiv, http://arxiv.org/abs/1907.05236 (2020).

[BolJY+] B. Bollobás, C.J. Jayawardene, Yang Jian Sheng, Huang Yi Ru, C.C. Rousseau, and Zhang Ke Min, On a Conjecture Involving Cycle-Complete Graph Ramsey Numbers, *Australasian Journal of Combinatorics,* **22** (2000) 63-71.

[-] B. Bollobás, see also [JRB].

[BoH] R. Bolze and H. Harborth, The Ramsey Number $r(K_4-x, K_5)$, in *The Theory and Applications of Graphs,* (Kalamazoo, MI, 1980), John Wiley & Sons, New York, (1981) 109-116.

[BoEr] J.A. Bondy and P. Erdős, Ramsey Numbers for Cycles in Graphs, *Journal of Combinatorial Theory,* Series B, **14** (1973) 46-54.

[Boza1] L. Boza, Nuevas Cotas Superiores de Algunos Números de Ramsey del Tipo $r(K_m, K_n-e)$, in proceedings of the *VII Jornada de Matemática Discreta y Algoritmica,* JMDA 2010, Castro Urdiales, Spain, July 2010.

[Boza2] L. Boza, The Ramsey Number $r(K_5-P_3, K_5)$, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P90, **18**(1) (2011), 10 pages.

[Boza3]* L. Boza, Upper Bounds for Some Ramsey Numbers of K_n-e versus K_m, *manuscript* (2012).

[Boza4]* L. Boza, Números de Ramsey de Algunos Grafos de 4 Vértices y Todods los Grafos de 7 Vértices, in proceedings of the *VIII Jornada de Matemática Discreta y Algoritmica,* JMDA 2012, Almeria, Spain, July 2012.

[Boza5]* L. Boza, *personal communication* (2013).

[Boza6]* L. Boza, Sobre el Número de Ramsey $R(K_4, K_6-e)$, *VIII Encuentro Andaluz de Matemática Discreta,* Sevilla, Spain, October 2013.

[Boza7]* L. Boza, Sobre los Números de Ramsey $R(K_5-e, K_5)$ y $R(K_6-e, K_4)$, *IX Jornada de Matemática Discreta y Algoritmica,* JMDA 2014, Tarragona, Spain, July 2014.

[BoCGR] L. Boza, M. Cera, P. Garcia-Vázquez and M.P. Revuelta, On the Ramsey Numbers for Stars versus Complete Graphs, *European Journal of Combinatorics,* **31** (2010) 1680-1688.

[BoDD]* L. Boza, J. Dybizbański and T. Dzido, Three Color Ramsey Numbers for Graphs With at Most 4 Vertices, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P47, **19**(4) (2012), 16 pages.

[BoPo]* L. Boza and J.R. Portillo, Sobre el Número de Ramsey $R(K_4-e, K_7)$, in proceedings of the *VIII Jornada de Matemática Discreta y Algoritmica,* JMDA 2012, Almeria, Spain, July 2012.

[-] A. Brandis, see [BierB].

[Bra1] S. Brandt, Subtrees and Subforests in Graphs, *Journal of Combinatorial Theory,* Series B, **61** (1994) 63-70.

[Bra2] S. Brandt, Sufficient Conditions for Graphs to Contain All Subgraphs of a Given Type, *Ph.D. thesis,* Freie Universität Berlin, 1994.

[Bra3] S. Brandt, Expanding Graphs and Ramsey Numbers, *preprint No. A 96-24,* ftp://ftp.math.fu-berlin.de/pub/math/publ/pre/1996 (1996).

[BrBH1]** S. Brandt, G. Brinkmann and T. Harmuth, All Ramsey Numbers $r(K_3, G)$ for Connected Graphs of Order 9, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #R7, **5** (1998), 20 pages.

[BrBH2]** S. Brandt, G. Brinkmann and T. Harmuth, The Generation of Maximal Triangle-Free Graphs, *Graphs and Combinatorics,* **16** (2000) 149-157.

[Bren1] M. Brennan, Ramsey Numbers of Trees and Unicyclic Graphs versus Fans, *Discrete Mathematics,* **340** (2017) 969-983.

[Bren2] M. Brennan, Ramsey Numbers of Trees versus Odd Cycles, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P3.2, **23**(3) (2016), 12 pages.

[-] G. Brightwell. see [AllBS].

[Brin]** G. Brinkmann, All Ramsey Numbers $r(K_3, G)$ for Connected Graphs of Order 7 and 8, *Combinatorics, Probability and Computing,* **7** (1998) 129-140.

[BrCGM]* G. Brinkmann, K. Coolsaet, J. Goedgebeur and H. Mélot, House of Graphs: A database of interesting graphs, *Discrete Applied Mathematics,* **161** (2013) 311-314.

[BrGS]** G. Brinkmann, J. Goedgebeur and J.C. Schlage-Puchta, Ramsey Numbers $R(K_3, G)$ for Graphs of Order 10, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P36, **19**(4) (2012), 23 pages.

[-] G. Brinkmann, see also [BrBH1, BrBH2].

[-] H.J. Broersma, see [LiZBBH, LiZB, SaBr1, SaBr2, SaBr3, SaBr4, SuBB1, SuBB2, SuBB3, SuBB4, SuBTB, SuBUB, ZhaBC1, ZhaBC2, ZhaBC3, ZhaBC4, ZhaBC5].

[BroNN] S. Brooks, T. Nguyen and E. Nystrom, The Ramsey Number and Saturation of the Tristar, *manuscript* (2016).

[Brou] A.E. Brouwer, *Parameters of Strongly Regular Graphs,* status of feasible parameters up to 1300 vertices and of several SRG family parameters, https://www.win.tue.nl/~aeb/graphs/srg/srgtab.html.

[BrBH] M. Bruce, M. Budden and J. Hiller, Lexicographic Products of r-Uniform Hypergraphs and Some Applications to Hypergraph Ramsey Theory, *Australasian Journal of Combinatorics,* **70** (2018) 390-401.

Bu

[BudHLS] M. Budden, J. Hiller, J. Lambert and C. Sanford, The Lifting of Graphs to 3-Uniform Hypergraphs and Some Applications to Hypergraph Ramsey Theory, *Involve: A Journal of Mathematics,* **10** (2017) 65-76.

[BudHMP] M. Budden, J. Hiller, T. Meek and A. Penland, Algebraic Properties of a Hypergraph Lifting Map, *preprint,* arXiv, http://arxiv.org/abs/2012.13500 (2020).

[BudHP] M. Budden, J. Hiller and A. Penland, Constructive Methods in Gallai-Ramsey Theory for Hypergraphs, *Integers,* 20A (2020) A4, http://math.colgate.edu/~integers.

[BudHR1] M. Budden, J. Hiller and A. Rapp, Generalized Ramsey Theorems for r-Uniform Hypergraphs, *Australasian Journal of Combinatorics,* **63** (2015) 142-152.

[BudHR2] M. Budden, J. Hiller and A. Rapp, Hypergraph Ramsey Numbers Involving Paths, *Acta Universitatis Apulensis,* **48** (2016) 75-87.

[BudP] M. Budden and A. Penland, Trees and n-Good Hypergraphs, *Australasian Journal of Combinatorics,* **72**(2) (2018) 329-349. Corrigendum in **75**(1) (2019) 171-173.

[-] M. Budden, see also [BrBH].

[BurR]* J.P. Burling and S.W. Reyner, Some Lower Bounds of the Ramsey Numbers $n(k, k)$, *Journal of Combinatorial Theory,* Series B, **13** (1972) 168-169.

[Bu1] S.A. Burr, Generalized Ramsey Theory for Graphs - a Survey, in *Graphs and Combinatorics* (R. Bari and F. Harary eds.), Springer LNM **406**, Berlin, (1974) 52-75.

[Bu2] S.A. Burr, Ramsey Numbers Involving Graphs with Long Suspended Paths, *Journal of the London Mathematical Society* (2), **24** (1981) 405-413.

[Bu3] S.A. Burr, Multicolor Ramsey Numbers Involving Graphs with Long Suspended Path, *Discrete Mathematics,* **40** (1982) 11-20.

[Bu4] S.A. Burr, Diagonal Ramsey Numbers for Small Graphs, *Journal of Graph Theory,* **7** (1983) 57-69.

[Bu5] S.A. Burr, Ramsey Numbers Involving Powers of Sparse Graphs, *Ars Combinatoria,* **15** (1983) 163-168.

[Bu6] S.A. Burr, Determining Generalized Ramsey Numbers is NP-Hard, *Ars Combinatoria,* **17** (1984) 21-25.

[Bu7] S.A. Burr, What Can We Hope to Accomplish in Generalized Ramsey Theory?, *Discrete Mathematics,* **67** (1987) 215-225.

[Bu8] S.A. Burr, On the Ramsey Numbers $r(G, nH)$ and $r(nG, nH)$ When n Is Large, *Discrete Mathematics,* **65** (1987) 215-229.

[Bu9] S.A. Burr, On Ramsey Numbers for Large Disjoint Unions of Graphs, *Discrete Mathematics,* **70** (1988) 277-293.

[Bu10] S.A. Burr, On the Computational Complexity of Ramsey-type Problems, Mathematics of Ramsey Theory, *Algorithms and Combinatorics,* **5**, Springer, Berlin, 1990, 46-52.

[BuE1] S.A. Burr and P. Erdős, On the Magnitude of Generalized Ramsey Numbers for Graphs, in *Infinite and Finite Sets,* (A. Hajnal, R. Rado and V.T. Sós eds., Keszthely 1973) Vol. 1, 215-240, Colloq. Math. Soc. János Bolyai, Vol. 10, North-Holland, Amsterdam, 1975.

[BuE2] S.A. Burr and P. Erdős, Extremal Ramsey Theory for Graphs, *Utilitas Mathematica,* **9** (1976) 247-258.

[BuE3] S.A. Burr and P. Erdős, Generalizations of a Ramsey-Theoretic Result of Chvátal, *Journal of Graph Theory,* **7** (1983) 39-51.

[BEFRS1] S.A. Burr, P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, An Extremal Problem in Generalized Ramsey Theory, *Ars Combinatoria,* **10** (1980) 193-203.

[BEFRS2] S.A. Burr, P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Ramsey Numbers for the Pair Sparse Graph-Path or Cycle, *Transactions of the American Mathematical Society,* **269** (1982) 501-512.

[BEFRS3] S.A. Burr, P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, The Ramsey Number for the Pair Complete Bipartite Graph-Graph of Limited Degree, in *Graph Theory with Applications to Algorithms and Computer Science,* (Y. Alavi et al. eds.), John Wiley & Sons, New York, (1985) 163-174.

[BEFRS4] S.A. Burr, P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Some Complete Bipartite Graph-Tree Ramsey Numbers, *Annals of Discrete Mathematics,* **41** (1989) 79-89.

[BEFRSGJ] S.A. Burr, P. Erdős, R.J. Faudree, C.C. Rousseau, R.H. Schelp, R.J. Gould and M.S. Jacobson, Goodness of Trees for Generalized Books, *Graphs and Combinatorics,* **3** (1987) 1-6.

[BEFS] S.A. Burr, P. Erdős, R.J. Faudree and R.H. Schelp, On the Difference between Consecutive Ramsey Numbers, *Utilitas Mathematica,* **35** (1989) 115-118.

[BES] S.A. Burr, P. Erdős and J.H. Spencer, Ramsey Theorems for Multiple Copies of Graphs, *Transactions of the American Mathematical Society,* **209** (1975) 87-99.

[BuF] S.A. Burr and R.J. Faudree, On Graphs G for Which All Large Trees Are G-good, *Graphs and Combinatorics,* **9** (1993) 305-313.

[BFRS] S.A. Burr, R.J. Faudree, C.C. Rousseau and R.H. Schelp, On Ramsey Numbers Involving Starlike Multipartite Graphs, *Journal of Graph Theory,* **7** (1983) 395-409.

[BuG] S.A. Burr and J.W. Grossman, Ramsey Numbers of Graphs with Long Tails, *Discrete Mathematics,* **41** (1982) 223-227.

[BuRo1] S.A. Burr and J.A. Roberts, On Ramsey Numbers for Stars, *Utilitas Mathematica,* **4** (1973) 217-220.

[BuRo2] S.A. Burr and J.A. Roberts, On Ramsey Numbers for Linear Forests, *Discrete Mathematics,* **8** (1974) 245-250.

[BuRo3] S.A. Burr and V. Rosta, On the Ramsey Multiplicities of Graphs - Problems and Recent Results, *Journal of Graph Theory,* **4** (1980) 347-361.

[Bush] L.E. Bush, The William Lowell Putnam Mathematical Competition (question #2 in Part I asks for the proof of $R(3,3) \leq 6$), *American Mathematical Monthly,* **60** (1953) 539-542.

[-] S. Butler, see [GrBu].

Ca - Ch

[-] J.W. Cain, see [LinCa].

[CaET]* N.J. Calkin, P. Erdős and C.A. Tovey, New Ramsey Bounds from Cyclic Graphs of Prime Order, *SIAM Journal on Discrete Mathematics,* **10** (1997) 381-387.

[CalSR]* J.A. Calvert and M.J. Schuster and S.P. Radziszowski, Computing the Ramsey Number $R(K_5 - P_3, K_5)$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **82** (2012) 131-140.

[Car] D. Cariolaro, On the Ramsey Number $R(3, 6)$, *Australasian Journal of Combinatorics,* **37** (2007) 301-304.

[Caro] Y. Caro, Zero-Sum Problems - A Survey, *Discrete Mathematics,* **152** (1996) 93-113.

[CaLRZ] Y. Caro, Li Yusheng, C.C. Rousseau and Zhang Yuming, Asymptotic Bounds for Some Bipartite Graph - Complete Graph Ramsey Numbers, *Discrete Mathematics,* **220** (2000) 51-56.

[CaYZ] Y. Caro, R. Yuster and C. Zarb, Ramsey Numbers for Degree Monotone Paths, *Discrete Mathematics,* **340** (2017) 124-131.

[-] M. Cera, see [BoCGR].

[ChaMR] J. Chappelon, L.P. Montejano and J.L. Ramirez Alfonsin, On Ramsey Numbers of Complete Graphs with Dropped Stars, *Discrete Applied Mathematics,* **210** (2016) 200-206.

[-] J. Chappelon, see also [MonCR].

[ChGP] G. Chartrand, R.J. Gould and A.D. Polimeni, On Ramsey Numbers of Forests versus Nearly Complete Graphs, *Journal of Graph Theory,* **4** (1980) 233-239.

[ChRSPS] G. Chartrand, C.C. Rousseau, M.J. Stewart, A.D. Polimeni and J. Sheehan, On Star-Book Ramsey Numbers, in *Proceedings of the Fourth International Conference on the Theory and Applications of Graphs,* (Kalamazoo, MI 1980), John Wiley & Sons, (1981) 203-214.

[ChaS] G. Chartrand and S. Schuster, On the Existence of Specified Cycles in Complementary Graphs, *Bulletin of the American Mathematical Society,* **77** (1971) 995-998.

[Chen] Chen Guantao, A Result on C_4-Star Ramsey Numbers, *Discrete Mathematics,* **163** (1997) 243-246.

[ChenS] Chen Guantao and R.H. Schelp, Graphs with Linearly Bounded Ramsey Numbers, *Journal of Combinatorial Theory,* Series B, **57** (1993) 138-149.

[ChenYZ] Guantao Chen, Xiaowei Yu and Yi Zhao, Ramsey Number of Fans, *preprint,* arXiv, http://arxiv.org/abs/2007.00152 (2020).

[-] Chen Hong, see also [LiaWXCS, XWCS].

[ChenJ] Chen Jie, The Lower Bound of Some Ramsey Numbers (in Chinese), *Journal of Liaoning Normal University, Natural Science,* **25** (2002) 244-246.

[-] Kun Chen, see [ZhaCZ].

[-] Ming Chen, see [PeiCLY].

[-] Weiji Chen, see [LinCh].

[ChenCMN] Yaojun Chen, T.C. Edwin Cheng, Zhengke Miao and C.T. Ng, The Ramsey Numbers for Cycles versus Wheels of Odd Order, *Applied Mathematics Letters,* **22** (2009) 1875-1876.

[ChenCNZ] Yaojun Chen, T.C. Edwin Cheng, C.T. Ng and Yunqing Zhang, A Theorem on Cycle-Wheel Ramsey Number, *Discrete Mathematics,* **312** (2012) 1059-1061.

[ChenCX] Yaojun Chen, T.C. Edwin Cheng and Ran Xu, The Ramsey Number for a Cycle of Length Six versus a Clique of Order Eight, *Discrete Applied Mathematics,* **157** (2009) 8-12.

[ChenCZ1] Yaojun Chen, T.C. Edwin Cheng and Yunqing Zhang, The Ramsey Numbers $R(C_m, K_7)$ and $R(C_7, K_8)$, *European Journal of Combinatorics,* **29** (2008) 1337-1352.

[ChenZZ1] Chen Yaojun, Zhang Yunqing and Zhang Ke Min, The Ramsey Numbers of Paths versus Wheels, *Discrete Mathematics,* **290** (2005) 85-87.

[ChenZZ2] Chen Yaojun, Zhang Yunqing and Zhang Ke Min, The Ramsey Numbers of Stars versus Wheels, *European Journal of Combinatorics,* **25** (2004) 1067-1075.

[ChenZZ3] Chen Yaojun, Zhang Yunqing and Zhang Ke Min, The Ramsey Numbers $R(T_n, W_6)$ for $\Delta(T_n) \geq n-3$, *Applied Mathematics Letters,* **17** (2004) 281-285.

[ChenZZ4] Chen Yaojun, Zhang Yunqing and Zhang Ke Min, The Ramsey Numbers of Trees versus W_6 or W_7, *European Journal of Combinatorics,* **27** (2006) 558-564.

[ChenZZ5] Chen Yaojun, Zhang Yunqing and Zhang Ke Min, The Ramsey Numbers $R(T_n, W_6)$ for Small n, *Utilitas Mathematica,* **67** (2005) 269-284.

[ChenZZ6] Chen Yaojun, Zhang Yunqing and Zhang Ke Min, The Ramsey Numbers $R(T_n, W_6)$ for T_n without Certain Deletable Sets, *Journal of Systems Science and Complexity,* **18** (2005) 95-101.

[-] Chen Yaojun, see also [CheCZN, WaCh1, WaCh2, ZhZC, ZhaCh, ZhaBC1, ZhaBC2, ZhaBC3, ZhaBC4, ZhaBC5, ZhaCC1, ZhaCC2, ZhaCC3, ZhaCC4, ZhaCC5, ZhaCZ1, ZhaCZ2, ZhaZC].

[-] Chen Zhi, see [XuXC].

[Cheng] Cheng Ying, On Graphs Which Do Not Contain Certain Trees, *Ars Combinatoria,* **19** (1985) 119-151.

[CheCZN] T.C. Edwin Cheng, Yaojun Chen, Yunqing Zhang and C.T. Ng, The Ramsey Numbers for a Cycle of Length Six or Seven versus a Clique of Order Seven, *Discrete Mathematics,* **307** (2007) 1047-1053.

[-] T.C. Edwin Cheng, see also [ChenCMN, ChenCNZ, ChenCX, ChenCZ1, ZhaCC1, ZhaCC2, ZhaCC3, ZhaCC4, ZhaCC5].

[Chu1] F.R.K. Chung, On the Ramsey Numbers $N(3,3,...,3;2)$, *Discrete Mathematics,* **5** (1973) 317-321.

[Chu2] F.R.K. Chung, On Triangular and Cyclic Ramsey Numbers with k Colors, in *Graphs and Combinatorics* (R. Bari and F. Harary eds.), Springer LNM **406**, Berlin, (1974) 236-241.

[Chu3] F.R.K. Chung, A Note on Constructive Methods for Ramsey Numbers, *Journal of Graph Theory,* **5** (1981) 109-113.

[Chu4] F.R.K. Chung, Open problems of Paul Erdős in Graph Theory, *Journal of Graph Theory,* **25** (1997) 3-36.

[ChCD] F.R.K. Chung, R. Cleve and P. Dagum, A Note on Constructive Lower Bounds for the Ramsey Numbers $R(3, t)$, *Journal of Combinatorial Theory,* Series B, **57** (1993) 150-155.

[ChGra1] F.R.K. Chung and R.L. Graham, On Multicolor Ramsey Numbers for Complete Bipartite Graphs, *Journal of Combinatorial Theory,* Series B, **18** (1975) 164-169.

[ChGra2] F.R.K. Chung and R.L. Graham, *Erdős on Graphs, His Legacy of Unsolved Problems,* A K Peters, Wellesley, Massachusetts (1998).

[ChGri] F.R.K. Chung and C.M. Grinstead, A Survey of Bounds for Classical Ramsey Numbers, *Journal of Graph Theory,* **7** (1983) 25-37.

[-] F.R.K. Chung, see also [UCSD].

[Chv] V. Chvátal, Tree-Complete Graph Ramsey Numbers, *Journal of Graph Theory,* **1** (1977) 93.

[ChH1] V. Chvátal and F. Harary, Generalized Ramsey Theory for Graphs, II. Small Diagonal Numbers, *Proceedings of the American Mathematical Society,* **32** (1972) 389-394.

[ChH2] V. Chvátal and F. Harary, Generalized Ramsey Theory for Graphs, III. Small Off-Diagonal Numbers, *Pacific Journal of Mathematics,* **41** (1972) 335-345.

[ChH3] V. Chvátal and F. Harary, Generalized Ramsey Theory for Graphs, I. Diagonal Numbers, *Periodica Mathematica Hungarica,* **3** (1973) 115-124.

[ChRST] V. Chvátal, V. Rödl, E. Szemerédi and W.T. Trotter Jr., The Ramsey Number of a Graph with Bounded Maximum Degree, *Journal of Combinatorial Theory,* Series B, **34** (1983) 239-243.

[ChvS] V. Chvátal and A. Schwenk, On the Ramsey Number of the Five-Spoked Wheel, in *Graphs and Combinatorics* (R. Bari and F. Harary eds.), Springer LNM **406**, Berlin, (1974) 247-261.

Cl - Cs

[Clan] M. Clancy, Some Small Ramsey Numbers, *Journal of Graph Theory,* **1** (1977) 89-91.

[Clap] C. Clapham, The Ramsey Number $r(C_4, C_4, C_4)$, *Periodica Mathematica Hungarica,* **18** (1987) 317-318.

[ClEHMS] C. Clapham, G. Exoo, H. Harborth, I. Mengersen and J. Sheehan, The Ramsey Number of $K_5 - e$, *Journal of Graph Theory,* **13** (1989) 7-15.

[Clark] L. Clark, On Cycle-Star Graph Ramsey Numbers, *Congressus Numerantium,* **50** (1985) 187-192.

[-] L. Clark, see also [RanMCG].

[-] F.C. Clemen, see [BalCSW].

[CleDa] R. Cleve and P. Dagum, A Constructive $\Omega(t^{1.26})$ Lower Bound for the Ramsey Number $R(3,t)$, *International Computer Science Institute,* TR-89-009, Berkeley, CA, 1989.

[-] R. Cleve, see also [ChCD].

[Coc] E.J. Cockayne, Some Tree-Star Ramsey Numbers, *Journal of Combinatorial Theory,* Series B, **17** (1974) 183-187.

[CocL1] E.J. Cockayne and P.J. Lorimer, The Ramsey Number for Stripes, *Journal of the Australian Mathematical Society,* Series A, **19** (1975) 252-256.

[CocL2] E.J. Cockayne and P.J. Lorimer, On Ramsey Graph Numbers for Stars and Stripes, *Canadian Mathematical Bulletin,* **18** (1975) 31-34.

[CoPR] B. Codenotti, P. Pudlák and G. Resta, Some Structural Properties of Low-Rank Matrices Related to Computational Complexity, *Theoretical Computer Science,* **235** (2000) 89-107.

[CodFIM]* M. Codish, M. Frank, A. Itzhakov and A. Miller, Computing the Ramsey Number $R(4,3,3)$ Using Abstraction and Symmetry Breaking, *Constraints,* **21** (2016) 375-393.

[-] K. Coffey, see [AlmBCL].

[Coh] G. Cohen, Two-Source Dispersers for Polylogarithmic Entropy and Improved Ramsey Graphs, in *Proceedings of the 48-th Annual ACM Symposium on Theory of Computing,* STOC'16, Cambridge MA, 278-284. Extended version on arXiv, http://arxiv.org/abs/1506.04428 (2015).

[ColGJ] C. Collier-Cartaino, N. Graber and Tao Jiang, Linear Turán Numbers of Linear Cycles and Cycle-Complete Ramsey Numbers, *Combinatorics, Probability and Computing,* **27**(3) (2018) 358-386.

[CoPC] Special issue on Ramsey theory of *Combinatorics, Probability and Computing,* **12** (2003), Numbers 5 and 6.

[Con1] D. Conlon, A New Upper Bound for Diagonal Ramsey Numbers, *Annals of Mathematics,* **170** (2009) 941-960.

[Con2] D. Conlon, Hypergraph Packing and Sparse Bipartite Ramsey Numbers, *Combinatorics, Probability and Computing,* **18** (2009) 913-923.

[Con3] D. Conlon, The Ramsey Number of Dense Graphs, *Bulletin of the London Mathematical Society,* **45** (2013) 483-496.

[Con4] D. Conlon, The Ramsey Number of Books, *Advances in Combinatorics,* **3** (2019), 12 pages, https://doi.org/10.19086/aic.10808.

[ConFer] D. Conlon and A. Ferber, Lower Bounds for Multicolor Ramsey Numbers, *Advances in Mathematics,* **378** (2021), 5 pages, https://doi.org/10.1016/j.aim.2020.107528.

[ConFLS] D. Conlon, J. Fox, C. Lee and B. Sudakov, Ramsey Numbers of Cubes versus Cliques, *Combinatorica,* **36** (2016) 37-70.

[ConFR] D. Conlon, J. Fox and V. Rödl, Hedgehogs Are Not Color Blind, *Journal of Combinatorics,* **8** (2017) 475-485.

[ConFS1] D. Conlon, J. Fox and B. Sudakov, Ramsey Numbers of Sparse Hypergraphs, *Random Structures and Algorithms,* **35** (2009) 1-14.

[ConFS2] D. Conlon, J. Fox and B. Sudakov, Hypergraph Ramsey Numbers, *Journal of the American Mathematical Society,* **23** (2010) 247-266.

[ConFS3] D. Conlon, J. Fox and B. Sudakov, Large Almost Monochromatic Subsets in Hypergraphs, *Israel Journal of Mathematics,* **181** (2011) 423-432.

[ConFS4] D. Conlon, J. Fox and B. Sudakov, On Two Problems in Graph Ramsey Theory, *Combinatorica,* **32** (2012) 513-535.

[ConFS5] D. Conlon, J. Fox and B. Sudakov, Erdős-Hajnal-type Theorems in Hypergraphs, *Journal of Combinatorial Theory,* Series B, **102** (2012) 1142-1154.

[ConFS6] D. Conlon, J. Fox and B. Sudakov, An Improved Bound for the Stepping-Up Lemma, *Discrete Applied Mathematics,* **161** (2013) 1191-1196.

[ConFS7] D. Conlon, J. Fox and B. Sudakov, Recent Developments in Graph Ramsey Theory, *Surveys in Combinatorics,* London Mathematical Society Lecture Note Series, (2015) 49-118.

[ConFS8] D. Conlon, J. Fox and B. Sudakov, Short Proofs of Some Extremal Results II, *Journal of Combinatorial Theory,* Series B, **121** (2016) 173-196.

[ConFW] D. Conlon, J. Fox and Y. Wigderson, Ramsey Numbers of Books and Quasirandomness, *preprint,* arXiv, http://arxiv.org/abs/2001.00407 (2020).

[CooFKO1] O. Cooley, N. Fountoulakis, D. Kühn and D. Osthus, 3-Uniform Hypergraphs of Bounded Degree Have Linear Ramsey Numbers, *Journal of Combinatorial Theory,* Series B, **98** (2008) 484-505.

[CooFKO2] O. Cooley, N. Fountoulakis, D. Kühn and D. Osthus, Embeddings and Ramsey Numbers of Sparse k-uniform Hypergraphs, *Combinatorica,* **29** (2009) 263-297.

[-] O. Cooley, see also [KüCFO].

[-] K. Coolsaet, see [BrCGM].

[CsKo] R. Csákány and J. Komlós, The Smallest Ramsey Numbers, *Discrete Mathematics,* **199** (1999) 193-199.

D

[-] K. Dabrowska, see [BieDa].

[-] P. Dagum, see [ChCD, CleDa].

[DavJR] E. Davies, M. Jenssen and B. Roberts, Multicolour Ramsey Numbers of Paths and Even Cycles, *European Journal of Combinatorics,* **63** (2017) 124-133.

[DayJ] A.N. Day and J.R. Johnson, Multicolour Ramsey Numbers of Odd Cycles, *Journal of Combinatorial Theory,* Series B, **124** (2017) 56-63.

[DeBGS] L. DeBiasio, A. Gyárfás and G.N. Sárközy, Ramsey Numbers of Path-Matchings, Covering Designs and 1-Cores, *preprint,* arXiv, http://arxiv.org/abs/1909.01920 (2020).

[Den1] T. Denley, The Independence Number of Graphs with Large Odd Girth, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #R9, **1** (1994), 12 pages.

[Den2] T. Denley, The Ramsey Numbers for Disjoint Unions of Cycles, *Discrete Mathematics,* **149** (1996) 31-44.

[Dong] Dong Lin, A Note on a Lower Bound for $r(K_{m,n})$, *Journal of Tongji University (Natural Science),* **38** (2010) 776,778.

[DoLi] Lin Dong and Yusheng Li, A Construction for Ramsey Numbers for $K_{m,n}$, *European Journal of Combinatorics,* **31** (2010) 1667-1670.

[DoLL1] Lin Dong, Yusheng Li and Qizhong Lin, Ramsey Numbers Involving Graphs with Large Degrees, *Applied Mathematics Letters,* **22** (2009) 1577-1580.

[DoLL2] Dong Lin, Li Yusheng and Lin Qizhong, Ramsey Numbers of Cycles vs. Large Complete Graph, *Advances in Mathematics (China),* **39** (2010) 700-702.

[-] Dong Lin, see also [HeLD, LinLD].

[DuHu] Duan Chanlun and Huang Wenke, Lower Bound of Ramsey Number $r(3, 10)$ (in Chinese), *Acta Scientiarum Naturalium Universitatis Nei Mongol,* **31** (2000) 468-470.

[DuLR] D. Duffus, H. Lefmann and V. Rödl, Shift Graphs and Lower Bounds on Ramsey Numbers $r_k(l; r)$, *Discrete Mathematics,* **137** (1995) 177-187.

[Dyb1]* J. Dybizbański, On Some Ramsey Numbers of C_4 versus $K_{2,n}$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **87** (2013) 137-145.

[Dyb2]* J. Dybizbański, A Lower Bound on the Hypergraph Ramsey Number $R(4,5;3)$, *Contributions to Discrete Mathematics,* **13**(2) (2018) 112-115.

[Dyb3]* J. Dybizbański, University of Gdańsk, *personal communication* (2018). Addendum to [Dyb2] at https://inf.ug.edu.pl/ramsey.

[DyDz1]* J. Dybizbański and T. Dzido, On Some Ramsey Numbers for Quadrilaterals, *Electronic Journal of Combinatorics,* http:// www.combinatorics.org, #P154, **18**(1) (2011), 12 pages.

[DyDz2] J. Dybizbański and T. Dzido, On Some Ramsey Numbers for Quadrilaterals versus Wheels, *Graphs and Combinatorics,* **30** (2014) 573-579.

[DyDR] J. Dybizbański, T. Dzido and S.P. Radziszowski, On Some Three-Color Ramsey Numbers for Paths, *Discrete Applied Mathematics,* **204** (2016) 133-141.

[-] J. Dybizbański, see also [BoDD].

[Dzi1]* T. Dzido, Ramsey Numbers for Various Graph Classes (in Polish), *Ph.D. thesis,* University of Gdańsk, Poland, November 2005.

[Dzi2]* T. Dzido, Multicolor Ramsey Numbers for Paths and Cycles, *Discussiones Mathematicae Graph Theory,* **25** (2005) 57-65.

[DzFi1]* T. Dzido and R. Fidytek, The Number of Critical Colorings for Some Ramsey Numbers, *International Journal of Pure and Applied Mathematics,* ISSN 1311-8080, **38** (2007) 433-444.

[DzFi2]* T. Dzido and R. Fidytek, On Some Three Color Ramsey Numbers for Paths and Cycles, *Discrete Mathematics,* **309** (2009) 4955-4958.

[DzKP] T. Dzido, M. Kubale and K. Piwakowski, On Some Ramsey and Turán-type Numbers for Paths and Cycles, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #R55, **13** (2006), 9 pages.

[DzNS] T. Dzido, A. Nowik and P. Szuca, New Lower Bound for Multicolor Ramsey Numbers for Even Cycles, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #N13, **12** (2005), 5 pages.

[-] T. Dzido, see also [BoDD, DyDz1, DyDz2, DyDR, KhoDz].

E

[Ea1] Easy to obtain by simple combinatorics from other results, in particular by using graphs establishing lower bounds with smaller parameters.

[Ea2] Unique 2-(6,3,2) design gives lower bound 7, upper bound is easy.

[Ea3] Every edge $(3,3,3;2)$-coloring of K_{15} has 35 edges in each color [Hein], and since the number of triangles in K_{16} is not divisible by 3, hence no required triangle-coloring of K_{16} exists.

[Eaton] N. Eaton, Ramsey Numbers for Sparse Graphs, *Discrete Mathematics,* **185** (1998) 63-75.

[Eli] S. Eliahou, An Adaptive Upper Bound on the Ramsey Numbers $R(3, \ldots, 3)$, *Integers,* 20 (2020) A54, 7 pages, http://math.colgate.edu/~integers.

[Erd1] P. Erdős, Some Remarks on the Theory of Graphs, *Bulletin of the American Mathematical Society,* **53** (1947) 292-294.

[Erd2] P. Erdős, Some New Problems and Results in Graph Theory and Other Branches of Combinatorial Mathematics, *Combinatorics and Graph Theory* (Calcutta 1980), Berlin-NY Springer, LNM **885** (1981) 9-17.

[Erd3] P. Erdős, On the Combinatorial Problems Which I Would Most Like to See Solved, *Combinatorica,* **1** (1981) 25-42.

[Erd4] P. Erdős, On Some Problems in Graph Theory, Combinatorial Analysis and Combinatorial Number Theory, *Graph Theory and Combinatorics,* (Cambridge 1983), 1-17, Academic Press, London-New York, 1984.

[EFRS1] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Generalized Ramsey Theory for Multiple Colors, *Journal of Combinatorial Theory,* Series B, **20** (1976) 250-264.

[EFRS2] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, On Cycle-Complete Graph Ramsey Numbers, *Journal of Graph Theory,* **2** (1978) 53-64.

[EFRS3] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Ramsey Numbers for Brooms, *Congressus Numerantium,* **35** (1982) 283-293.

[EFRS4] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Multipartite Graph-Sparse Graph Ramsey Numbers, *Combinatorica,* **5** (1985) 311-318.

[EFRS5] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, A Ramsey Problem of Harary on Graphs with Prescribed Size, *Discrete Mathematics,* **67** (1987) 227-233.

[EFRS6] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Extremal Theory and Bipartite Graph-Tree Ramsey Numbers, *Discrete Mathematics,* **72** (1988) 103-112.

[EFRS7] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, The Book-Tree Ramsey Numbers, *Scientia,* Series A: Mathematical Sciences, Valparaíso, Chile, **1** (1988) 111-117.

[EFRS8] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Multipartite Graph-Tree Graph Ramsey Numbers, in *Graph Theory and Its Applications: East and West, Proceedings of the First China-USA International Graph Theory Conference,* Annals of the New York Academy of Sciences, **576** (1989) 146-154.

[EFRS9] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Ramsey Size Linear Graphs, *Combinatorics, Probability and Computing,* **2** (1993) 389-399.

[ErdG] P. Erdős and R.L. Graham, On Partition Theorems for Finite Sets, in *Infinite and Finite Sets,* (A. Hajnal, R. Rado and V.T. Sós eds.) Vol. 1, 515--527, Colloq. Math. Soc. János Bolyai, Vol. 10, North Holland, 1975.

[ErdH] P. Erdős and A. Hajnal, On Ramsey Like Theorems, Problems and Results, *Combinatorics,* Conference on Combinatorial Mathematics, Math. Institute, Oxford, (1972) 123-140.

[-] P. Erdős, see also [BoEr, BuE1, BuE2, BuE3, BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRSGJ, BEFS, BES, CaET].

[Ex1]* G. Exoo, Ramsey Numbers of Hypergraphs, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **2** (1987) 5-11.

[Ex2]* G. Exoo, Constructing Ramsey Graphs with a Computer, *Congressus Numerantium,* **59** (1987) 31-36.

[Ex3]* G. Exoo, Applying Optimization Algorithm to Ramsey Problems, in *Graph Theory, Combinatorics, Algorithms, and Applications* (Y. Alavi ed.), SIAM Philadelphia, (1989) 175-179.

[Ex4]* G. Exoo, A Lower Bound for $R(5, 5)$, *Journal of Graph Theory,* **13** (1989) 97-98.

[Ex5]* G. Exoo, On Two Classical Ramsey Numbers of the Form $R(3, n)$, *SIAM Journal on Discrete Mathematics,* **2** (1989) 488-490.

[Ex6]* G. Exoo, A Lower Bound for $r(K_5 - e, K_5)$, *Utilitas Mathematica,* **38** (1990) 187-188.

[Ex7]* G. Exoo, Three Color Ramsey Number of $K_4 - e$, *Discrete Mathematics,* **89** (1991) 301-305.

[Ex8]* G. Exoo, Indiana State University, *personal communication* (1992).

[Ex9]* G. Exoo, Announcement: On the Ramsey Numbers $R(4, 6)$, $R(5, 6)$ and $R(3, 12)$, *Ars Combinatoria,* **35** (1993) 85. The construction of a graph proving $R(4, 6) \geq 35$ is presented in detail at http://cs.indstate.edu/ge/RAMSEY (2001).

[Ex10]* G. Exoo, A Lower Bound for Schur Numbers and Multicolor Ramsey Numbers of K_3, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #R8, **1** (1994), 3 pages.

[Ex11]* G. Exoo, Indiana State University, *personal communication* (1997).

[Ex12]* G. Exoo, Some New Ramsey Colorings, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #R29, **5** (1998), 5 pages. The constructions are available electronically at http://cs.indstate.edu/ge/RAMSEY. The lower bounds presented in this paper have been improved.

[Ex14]* G. Exoo, Indiana State University, *New Lower Bounds for Table III,* (2000). Constructions available at http://cs.indstate.edu/ge/RAMSEY.

[Ex16]* G. Exoo, Indiana State University, *personal communication* (2005-2006). Constructions available at http://cs.indstate.edu/ge/RAMSEY.

[Ex17]* G. Exoo, Indiana State University, *personal communication* (2010-2011). Constructions available at http://cs.indstate.edu/ge/RAMSEY.

[Ex18]* G. Exoo, Indiana State University, *personal communication* (2012-2013). Constructions available at http://cs.indstate.edu/ge/RAMSEY.

[Ex19]* G. Exoo, On the Ramsey Number $R(4, 6)$, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P66, **19**(1) (2012), 5 pages.

[Ex20]* G. Exoo, On Some Small Classical Ramsey Numbers, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P68, **20**(1) (2013), 6 pages.

[Ex21]* G. Exoo, Ramsey Colorings from p-Groups, *in preparation,* (2013).

[Ex22]* G. Exoo, Indiana State University, *personal communication* (2015). Constructions available at http://cs.indstate.edu/ge/RAMSEY.

[Ex23]* G. Exoo, Indiana State University, *personal communication* (2017-2019). Constructions available at http://cs.indstate.edu/ge/RAMSEY, data on Paley graphs at http://cs.indstate.edu/ge/Paley.

[EHM1] G. Exoo, H. Harborth and I. Mengersen, The Ramsey Number of K_4 versus $K_5 - e$, *Ars Combinatoria,* **25A** (1988) 277-286.

[EHM2] G. Exoo, H. Harborth and I. Mengersen, On Ramsey Number of $K_{2,n}$, in *Graph Theory, Combinatorics, Algorithms, and Applications* (Y. Alavi, F.R.K. Chung, R.L. Graham and D.F. Hsu eds.), SIAM Philadelphia, (1989) 207-211.

[ExRe]* G. Exoo and D.F. Reynolds, Ramsey Numbers Based on C_5-Decompositions, *Discrete Mathematics,* **71** (1988) 119-127.

[ExT]* G. Exoo and M. Tatarevic, New Lower Bounds for 28 Classical Ramsey Numbers, *Electronic Journal of Combinatorics,* http:// www.combinatorics.org, #P3.11, **22**(3) (2015), 12 pages. Graphs available at the journal site and at http://cs.indstate.edu/ge/RAMSEY/ExTa.

[-] G. Exoo, see also [ClEHMS, XXER].

Fa - Fe

[FLPS] R.J. Faudree, S.L. Lawrence, T.D. Parsons and R.H. Schelp, Path-Cycle Ramsey Numbers, *Discrete Mathematics,* **10** (1974) 269-277.

[FM]** R.J. Faudree and B.D. McKay, A Conjecture of Erdős and the Ramsey Number $r(W_6)$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **13** (1993) 23-31.

[FRS1] R.J. Faudree, C.C. Rousseau and R.H. Schelp, All Triangle-Graph Ramsey Numbers for Connected Graphs of Order Six, *Journal of Graph Theory,* **4** (1980) 293-300.

[FRS2] R.J. Faudree, C.C. Rousseau and R.H. Schelp, Studies Related to the Ramsey Number $r(K_5-e)$, in *Graph Theory and Its Applications to Algorithms and Computer Science,* (Y. Alavi et al. eds.), John Wiley and Sons, New York, (1985) 251-271.

[FRS3] R.J. Faudree, C.C. Rousseau and R.H. Schelp, Generalizations of the Tree-Complete Graph Ramsey Number, in *Graphs and Applications,* (F. Harary and J.S. Maybee eds.), John Wiley and Sons, New York, (1985) 117-126.

[FRS4] R.J. Faudree, C.C. Rousseau and R.H. Schelp, Small Order Graph-Tree Ramsey Numbers, *Discrete Mathematics,* **72** (1988) 119-127.

[FRS5] R.J. Faudree, C.C. Rousseau and R.H. Schelp, A Good Idea in Ramsey Theory, in *Graph Theory, Combinatorics, Algorithms, and Applications* (San Francisco, CA 1989), SIAM Philadelphia, PA (1991) 180-189.

[FRS6] R.J. Faudree, C.C. Rousseau and R.H. Schelp, Problems in Graph Theory from Memphis, in *The Mathematics of Paul Erdős II,* R.L. Graham et al. (eds.), Springer, New York, (2013) 95-118.

[FRS7] R.J. Faudree, C.C. Rousseau and J. Sheehan, More from the Good Book, in *Proceedings of the Ninth Southeastern Conference on Combinatorics, Graph Theory, and Computing,* Utilitas Mathematica Publ., *Congressus Numerantium,* **XXI** (1978) 289-299.

[FRS8] R.J. Faudree, C.C. Rousseau and J. Sheehan, Strongly Regular Graphs and Finite Ramsey Theory, *Linear Algebra and its Applications,* **46** (1982) 221-241.

[FRS9] R.J. Faudree, C.C. Rousseau and J. Sheehan, Cycle-Book Ramsey Numbers, *Ars Combinatoria,* **31** (1991) 239-248.

[FS1] R.J. Faudree and R.H. Schelp, All Ramsey Numbers for Cycles in Graphs, *Discrete Mathematics,* **8** (1974) 313-329.

[FS2] R.J. Faudree and R.H. Schelp, Path Ramsey Numbers in Multicolorings, *Journal of Combinatorial Theory,* Series B, **19** (1975) 150-160.

[FS3] R.J. Faudree and R.H. Schelp, Ramsey Numbers for All Linear Forests, *Discrete Mathematics,* **16** (1976) 149-155.

[FS4] R.J. Faudree and R.H. Schelp, Some Problems in Ramsey Theory, in *Theory and Applications of Graphs,* (conference proceedings, Kalamazoo, MI 1976), Lecture Notes in Mathematics **642**, Springer, Berlin, (1978) 500-515.

[FSR] R.J. Faudree, R.H. Schelp and C.C. Rousseau, Generalizations of a Ramsey Result of Chvátal, in *Proceedings of the Fourth International Conference on the Theory and Applications of Graphs,* (Kalamazoo, MI 1980), John Wiley & Sons, (1981) 351-361.

[FSS1] R.J. Faudree, R.H. Schelp and J. Sheehan, Ramsey Numbers for Matchings, *Discrete Mathematics,* **32** (1980) 105-123.

[FSS2] R.J. Faudree, R.H. Schelp and M. Simonovits, On Some Ramsey Type Problems Connected with Paths, Cycles and Trees, *Ars Combinatoria,* **29A** (1990) 97-106.

[FSS3] R.J. Faudree, A. Schelten and I. Schiermeyer, The Ramsey Number $r(C_7, C_7, C_7)$, *Discussiones Mathematicae Graph Theory,* **23** (2003) 141-158.

[FaSi] R.J. Faudree and M. Simonovits, Ramsey Problems and Their Connection to Turán-Type Extremal Problems, *Journal of Graph Theory,* **16** (1992) 25-50.

[-] R.J. Faudree, see also [BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRSGJ, BEFS, BuF, BFRS, EFRS1, EFRS2, EFRS3, EFRS4, EFRS5, EFRS6, EFRS7, EFRS8, EFRS9].

[FeKR]** S. Fettes, R.L. Kramer and S.P. Radziszowski, An Upper Bound of 62 on the Classical Ramsey Number $R(3,3,3,3)$, *Ars Combinatoria,* **72** (2004) 41-63.

[-] A. Ferber, see [ConFer].

[Ferg] D.G. Ferguson, Topics in Graph Colouring and Graph Structures, *Ph.D. thesis,* Department of Mathematics, London School of Economics and Political Science, London, 2013. The problems on Ramsey theory are presented also in three arXiv preprints "The Ramsey Number of Mixed-Parity Cycles I, II and III", http://arxiv.org/abs/1508.07154, 1508.07171 and 1508.07176 (2015).

Fi - Fu

[Fid1]* R. Fidytek, Two- and Three-Color Ramsey Numbers for Paths and Cycles, *manuscript* (2010).

[Fid2]* R. Fidytek, *personal communication,* Ramsey Graphs $R(K_n, K_m - e)$, http://fidytek.inf.ug.edu.pl/ramsey (2010), available until 2014.

[-] R. Fidytek, see also [DzFi1, DzFi2].

[FiŁu1] A. Figaj and T. Łuczak, The Ramsey Number for a Triple of Long Even Cycles, *Journal of Combinatorial Theory,* Series B, **97** (2007) 584-596.

[FiŁu2] A. Figaj and T. Łuczak, The Ramsey Numbers for a Triple of Long Cycles, *Combinatorica,* **38**(4) (2018) 827-845.

[FizGM] G. Fiz Pontiveros, S. Griffiths and R. Morris, The Triangle-Free Process and $R(3, k)$, *Memoirs of the American Mathematical Society,* Vol. **263**, Number 1274 (2020), 125 pages, first version on arXiv, http://arxiv.org/abs/1302.6279 (2013).

[FizGMSS] G. Fiz Pontiveros, S. Griffiths, R. Morris, D. Saxton and J. Skokan, The Ramsey Number of the Clique and the Hypercube, *Journal of the London Mathematical Society,* **89** (2014) 680-702.

[-] G. Fiz Pontiveros, see also [GrMFSS].

[Fol] J. Folkman, Notes on the Ramsey Number $N(3,3,3,3)$, *Journal of Combinatorial Theory,* Series A, **16** (1974) 371-379.

[-] N. Fountoulakis, see [CooFKO1, CooFKO2, KüCFO].

[FoxLi] Jacob Fox and Ray Li, On Ramsey Numbers of Hedgehogs, *Combinatorics, Probability and Computing,* **29** (2020) 101-112.

[FoxPS1] J. Fox, J. Pach and A. Suk, Bounded VC-Dimension Implies the Schur-Erdős Conjecture, *36th International Symposium on Computational Geometry,* Dagstuhl Publishing, Germany, 46:1-46:8, (2020).

[FoxPS2] J. Fox, J. Pach and A. Suk, The Schur-Erdős Problem for Semi-Algebraic Colorings, *Israel Journal of Mathematics,* **239** (2020) 39-57.

[FoxSu1] J. Fox and B. Sudakov, Density Theorems for Bipartite Graphs and Related Ramsey-type Results, *Combinatorica,* **29** (2009) 153-196.

[FoxSu2] J. Fox and B. Sudakov, Two Remarks on the Burr-Erdős Conjecture, *European Journal of Combinatorics,* **30** (2009) 1630-1645.

[-] J. Fox, see also [ConFLS, ConFR, ConFS1, ConFS2, ConFS3, ConFS4, ConFS5, ConFS6, ConFS7, ConFS8, ConFW].

[-] M. Frank, see [CodFIM].

[FraWi] P. Frankl and R.M. Wilson, Intersection Theorems with Geometric Consequences, *Combinatorica,* **1** (1981) 357-368.

[Fra1] K. Fraughnaugh Jones, Independence in Graphs with Maximum Degree Four, *Journal of Combinatorial Theory,* Series B, **37** (1984) 254-269.

[Fra2] K. Fraughnaugh Jones, Size and Independence in Triangle-Free Graphs with Maximum Degree Three, *Journal of Graph Theory,* **14** (1990) 525-535.

[FrLo] K. Fraughnaugh and S.C. Locke, Finding Independent Sets in Triangle-Free Graphs, *SIAM Journal on Discrete Mathematics,* **9** (1996) 674-681.

[Fre] H. Fredricksen, Schur Numbers and the Ramsey Numbers $N(3,3,...,3;2)$, *Journal of Combinatorial Theory,* Series A, **27** (1979) 376-377.

[FreSw]* H. Fredricksen and M.M. Sweet, Symmetric Sum-Free Partitions and Lower Bounds for Schur Numbers, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #R32, **7** (2000), 9 pages.

[Fuj1]* H. Fujita, Ramsey Numbers and Ramsey Graphs, http://opal.inf.kyushu-u.ac.jp/~fujita/ramsey.html, 2014-2017.

[Fuj2] S. Fujita, Generalized Ramsey Numbers for Graphs with Three Disjoint Cycles versus a Complete Graph, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P14, **19**(2) (2012), 11 pages.

[-] Z. Füredi, see [AxFM, BiFJ].

Ga - Gr

[-] F. Gaitan, see [RanMCG].

[-] P. Garcia-Vázquez, see [BoCGR].

[Gas] W. Gasarch, Applications of Ramsey Theory to Computer Science, collection of pointers to papers, http://www.cs.umd.edu/~gasarch/TOPICS/ramsey/ramsey.html (2009, 2011, 2017).

[GauST] S. Gautam, A.K. Srivastava and A. Tripathi, On Multicolour Noncomplete Ramsey Graphs of Star Graphs, *Discrete Applied Mathematics,* **156** (2008) 2423-2428.

[Gerbi]* R. Gerbicz, New Lower Bounds for Two Color and Multicolor Ramsey Numbers, *preprint,* arXiv, http://arxiv.org/abs/1004.4374 (2010), pointed to in revision #14. Since 2015, better bounds in all cases were obtained by others.

[Gerbn] D. Gerbner, On Berge-Ramsey Problems, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P2.39, **27**(2) (2020), 8 pages.

[GerMOV] D. Gerbner, A. Methuku, G. Omidi and M. Vizer, Ramsey Problems for Berge Hypergraphs, *SIAM Journal on Discrete Mathematics,* **34** (2020) 351-369.

[GeGy] L. Gerencsér and A. Gyárfás, On Ramsey-Type Problems, *Annales Universitatis Scientiarum Budapestinensis, Eötvös Sect. Math.,* **10** (1967) 167-170.

[Gi1] G. Giraud, Une généralisation des nombres et de l'inégalité de Schur, *C.R. Acad. Sc. Paris,* Séries A-B, **266** (1968) A437-A440.

[Gi2] G. Giraud, Minoration de certains nombres de Ramsey binaires par les nombres de Schur généralisés, *C.R. Acad. Sc. Paris,* Séries A-B, **266** (1968) A481-A483.

[Gi3] G. Giraud, Nouvelles majorations des nombres de Ramsey binaires-bicolores, *C.R. Acad. Sc. Paris,* Séries A-B, **268** (1969) A5-A7.

[Gi4] G. Giraud, Majoration du nombre de Ramsey ternaire-bicolore en (4,4), *C.R. Acad. Sc. Paris,* Séries A-B, **269** (1969) A620-A622.

[Gi5] G. Giraud, Une minoration du nombre de quadrangles unicolores et son application à la majoration des nombres de Ramsey binaires-bicolores, *C.R. Acad. Sc. Paris,* Séries A-B, **276** (1973) A1173-A1175.

[Gi6] G. Giraud, Sur le problème de Goodman pour les quadrangles et la majoration des nombres de Ramsey, *Journal of Combinatorial Theory,* Series B, **27** (1979) 237-253.

[-] A.M. Gleason, see [GG].

[GoK] W. Goddard and D.J. Kleitman, An Upper Bound for the Ramsey Numbers $r(K_3, G)$, *Discrete Mathematics,* **125** (1994) 177-182.

[GoR1]** J. Goedgebeur and S.P. Radziszowski, New Computational Upper Bounds for Ramsey Numbers $R(3, k)$, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P30, **20**(1) (2013), 28 pages.

[GoR2]** J. Goedgebeur and S.P. Radziszowski, The Ramsey Number $R(3, K_{10}-e)$ and Computational Bounds for $R(3, G)$, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P19, **20**(4) (2013), 25 pages.

[-] J. Goedgebeur, see also [BrCGM, BrGS].

[GoMC] A. Gonçalves and E.L. Monte Carmelo, Some Geometric Structures and Bounds for Ramsey Numbers, *Discrete Mathematics,* **280** (2004) 29-38.

[GoJa1] R.J. Gould and M.S. Jacobson, Bounds for the Ramsey Number of a Disconnected Graph versus Any Graph, *Journal of Graph Theory,* **6** (1982) 413-417.

[GoJa2] R.J. Gould and M.S. Jacobson, On the Ramsey Number of Trees versus Graphs with Large Clique Number, *Journal of Graph Theory,* **7** (1983) 71-78.

[-] R.J. Gould, see also [BEFRSGJ, ChGP].

[-] N. Graber, see [ColGJ].

[GrBu] R.L. Graham and S. Butler, Rudiments of Ramsey Theory, second edition, *CBMS Regional Conference Series in Mathematics,* 123, American Mathematical Society 2015.

[GrNe] R.L. Graham and J. Nešetřil, Ramsey Theory and Paul Erdős (Recent Results from a Historical Perspective), *Bolyai Society Mathematical Studies,* **11**, Budapest (2002) 339-365.

[GrRö] R.L. Graham and V. Rödl, Numbers in Ramsey Theory, in *Surveys in Combinatorics,* (ed. C. Whitehead), Cambridge University Press, 1987, 111-153.

[GRR1] R.L. Graham, V. Rödl and A. Ruciński, On Graphs with Linear Ramsey Numbers, *Journal of Graph Theory,* **35** (2000) 176-192.

[GRR2] R.L. Graham, V. Rödl and A. Ruciński, On Bipartite Graphs with Linear Ramsey Numbers, Paul Erdős and his mathematics, *Combinatorica,* **21** (2001) 199-209.

[GRS] R.L. Graham, B.L. Rothschild and J.H. Spencer, *Ramsey Theory,* John Wiley & Sons, first edition 1980, second edition 1990, paperback of the second edition 2013.

[-] R.L. Graham, see also [ChGra1, ChGra2, ErdG].

[GrY] J.E. Graver and J. Yackel, Some Graph Theoretic Results Associated with Ramsey's Theorem, *Journal of Combinatorial Theory,* **4** (1968) 125-175.

[GG] R.E. Greenwood and A.M. Gleason, Combinatorial Relations and Chromatic Graphs, *Canadian Journal of Mathematics,* **7** (1955) 1-7.

[GrH] U. Grenda and H. Harborth, The Ramsey Number $r(K_3, K_7-e)$, *Journal of Combinatorics, Information & System Sciences,* **7** (1982) 166-169.

[GrMFSS] S. Griffiths, R. Morris, G. Fiz Pontiveros, D. Saxton and J. Skokan, On the Ramsey Number of the Triangle and the Cube, *Combinatorica,* **36** (2016) 71-89.

[-] S. Griffiths, see also [FizGM, FizGMSS].

[Gri] J.R. Griggs, An Upper Bound on the Ramsey Numbers $R(3,k)$, *Journal of Combinatorial Theory,* Series A, **35** (1983) 145-153.

[GR]** C. Grinstead and S. Roberts, On the Ramsey Numbers $R(3,8)$ and $R(3,9)$, *Journal of Combinatorial Theory,* Series B, **33** (1982) 27-51.

[-] C. Grinstead, see also [ChGri].

[Grol1] V. Grolmusz, Superpolynomial Size Set-Systems with Restricted Intersections mod 6 and Explicit Ramsey Graphs, *Combinatorica,* **20** (2000) 73-88.

[Grol2] V. Grolmusz, Low Rank Co-Diagonal Matrices and Ramsey Graphs, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #R15, **7** (2000), 7 pages.

[Grol3] V. Grolmusz, Set-Systems with Restricted Multiple Intersections, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #R8, **9** (2002), 10 pages.

[Gros1] J.W. Grossman, Some Ramsey Numbers of Unicyclic Graphs, *Ars Combinatoria,* **8** (1979) 59-63.

[Gros2] J.W. Grossman, The Ramsey Numbers of the Union of Two Stars, *Utilitas Mathematica,* **16** (1979) 271-279.

[GrHK] J.W. Grossman, F. Harary and M. Klawe, Generalized Ramsey Theory for Graphs, X: Double Stars, *Discrete Mathematics,* **28** (1979) 247-254.

[-] J.W. Grossman, see also [BuG].

Gu - Gy

[GuLi] Gu Hua and Li Yusheng, On Ramsey Number of $K_{2,t+1}$ vs $K_{1,n}$, *Journal of Nanjing University Mathematical Biquarterly,* **19** (2002) 150-153.

[GuSL] Gu Hua, Song Hongxue and Liu Xiangyang, Ramsey Numbers $r(K_{1,4}, G)$ for All Three-Partite Graphs G of Order Six, *Journal of Southeast University,* (English Edition), **20** (2004) 378-380.

[-] Gu Hua, see also [SonGQ].

[GuoV] Guo Yubao and L. Volkmann, Tree-Ramsey Numbers, *Australasian Journal of Combinatorics,* **11** (1995) 169-175.

[-] L. Gupta, see [GuGS].

[GuGS] S.K. Gupta, L. Gupta and A. Sudan, On Ramsey Numbers for Fan-Fan Graphs, *Journal of Combinatorics, Information & System Sciences,* **22** (1997) 85-93.

[GyLSS] A. Gyárfás, J. Lehel, G.N. Sárközy and R.H. Schelp, Monochromatic Hamiltonian Berge-Cycles in Colored Complete Uniform Hypergraphs, *Journal of Combinatorial Theory,* Series B, **98** (2008) 342-358.

[GyRa] A. Gyárfás and G. Raeisi, The Ramsey Number of Loose Triangles and Quadrangles in Hypergraphs, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P30, **19**(2) (2012), 9 pages.

[GyRSS] A. Gyárfás, M. Ruszinkó, G.N. Sárközy and E. Szemerédi, Three-color Ramsey Numbers for Paths, *Combinatorica,* **27** (2007) 35-69. *Corrigendum* in **28** (2008) 499-502.

[GySá1] A. Gyárfás and G.N. Sárközy, The 3-Colour Ramsey Number of a 3-Uniform Berge Cycle, *Combinatorics, Probability and Computing,* **20** (2011) 53-71.

[GySá2] A. Gyárfás and G.N. Sárközy, Star versus Two Stripes Ramsey Numbers and a Conjecture of Schelp, *Combinatorics, Probability and Computing,* **21** (2012) 179-186.

[GySá3] A. Gyárfás and G.N. Sárközy, Ramsey Number of a Connected Triangle Matching, *Journal of Graph Theory,* **83** (2016) 109-119.

[GySS1] A. Gyárfás, G.N. Sárközy and E. Szemerédi, The Ramsey Number of Diamond-Matchings and Loose Cycles in Hypergraphs, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #R126, **15** (2008), 14 pages.

[GySS2] A. Gyárfás, G.N. Sárközy and E. Szemerédi, Monochromatic Hamiltonian 3-Tight Berge Cycles in 2-Colored 4-Uniform Hypergraphs, *Journal of Graph Theory,* **63** (2010) 288-299.

[GySeT] A. Gyárfás, A. Sebő and N. Trotignon, The Chromatic Gap and Its Extremes, *Journal of Combinatorial Theory,* Series B, **102** (2012) 1155-1178.

[GyTu] A. Gyárfás and Z. Tuza, An Upper Bound on the Ramsey Number of Trees, *Discrete Mathematics,* **66** (1987) 309-310.

[-] A. Gyárfás, see also [AxGy, AxGLM, DeBGS, GeGy].

Ha

[HafBa] Y. Hafidh and E.T. Baskoro, The Ramsey Number for Tree Versus Wheel with Odd Order, *preprint,* arXiv, http://arxiv.org/abs/1912.05772 (2019).

[Häg] R. Häggkvist, On the Path-Complete Bipartite Ramsey Number, *Discrete Mathematics,* **75** (1989) 243-245.

[HagMa] Sh. Haghi and H.R. Maimani, A Note on the Ramsey Number of Even Wheels versus Stars, *Discussiones Mathematicae Graph Theory,* **38** (2018) 397-404.

[-] A. Hajnal, see [ErdH].

[HanPR] Jie Han, J. Polcyn and A. Ruciński, Turán and Ramsey Numbers for 3-Uniform Minimal Paths of Length 4, *preprint,* arXiv, http://arxiv.org/abs/2009.12593 (2020).

[Han]* D. Hanson, Sum-Free Sets and Ramsey Numbers, *Discrete Mathematics,* **14** (1976) 57-61.

[-] D. Hanson, see also [AbbH].

[Hans] M. Hansson, On Generalized Ramsey Numbers for Two Sets of Cycles, *Discrete Applied Mathematics,* **238** (2018) 86-94.

[HaoLin] Yiyuan Hao and Qizhong Lin, Ramsey Number of K_3 versus $F_{3,n}$, *Discrete Applied Mathematics,* **251** (2018) 345-348.

[Har1] F. Harary, Recent Results on Generalized Ramsey Theory for Graphs, in *Graph Theory and Applications,* (Y. Alavi et al. eds.) Springer, Berlin (1972) 125-138.

[Har2] F. Harary, Generalized Ramsey Theory I to XIII: Achievement and Avoidance Numbers, in *Proceedings of the Fourth International Conference on the Theory and Applications of Graphs,* (Kalamazoo, MI 1980), John Wiley & Sons, (1981) 373-390.

[-] F. Harary, see also [ChH1, ChH2, ChH3, GrHK].

[HaKr1]** H. Harborth and S. Krause, Ramsey Numbers for Circulant Colorings, *Congressus Numerantium,* **161** (2003) 139-150.

[HaKr2]** H. Harborth and S. Krause, Distance Ramsey Numbers, *Utilitas Mathematica,* **70** (2006) 197-200.

[HaMe1] H. Harborth and I. Mengersen, An Upper Bound for the Ramsey Number $r(K_5 - e)$, *Journal of Graph Theory,* **9** (1985) 483-485.

[HaMe2] H. Harborth and I. Mengersen, All Ramsey Numbers for Five Vertices and Seven or Eight Edges, *Discrete Mathematics,* **73** (1988/89) 91-98.

[HaMe3] H. Harborth and I. Mengersen, The Ramsey Number of $K_{3,3}$, in *Combinatorics, Graph Theory, and Applications,* Vol. **2** (Y. Alavi, G. Chartrand, O.R. Oellermann and J. Schwenk eds.), John Wiley & Sons, (1991) 639-644.

[-] H. Harborth, see also [BoH, ClEHMS, EHM1, EHM2, GrH].

[HaMe4] M. Harborth and I. Mengersen, Some Ramsey Numbers for Complete Bipartite Graphs, *Australasian Journal of Combinatorics,* **13** (1996) 119-128.

[-] T. Harmuth, see [BrBH1, BrBH2].

[Has] Hasmawati, The Ramsey Numbers for Disjoint Union of Stars, *Journal of the Indonesian Mathematical Society,* **16** (2010) 133-138.

[HaABS] Hasmawati, H. Assiyatun, E.T. Baskoro and A.N.M. Salman, Ramsey Numbers on a Union of Identical Stars versus a Small Cycle, in Computational Geometry and Graph Theory, Kyoto CGGT 2007, *LNCS* 4535, Springer, Berlin (2008) 85-89.

[HaBA1] Hasmawati, E.T. Baskoro and H. Assiyatun, Star-Wheel Ramsey Numbers, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **55** (2005) 123-128.

[HaBA2] Hasmawati, E.T. Baskoro and H. Assiyatun, The Ramsey Numbers for Disjoint Unions of Graphs, *Discrete Mathematics,* **308** (2008) 2046-2049.

[HaJu] Hasmawati and Jusmawati, Ramsey Number on a Union of Stars Versus a Small Cycle, *Jurnal Matematika Statistika dan Komputasi,* Universitas Hasanuddin, **13** (2017) 152-157.

[-] Hasmawati, see also [BaHA].

[HaŁP1+] P.E. Haxell, T. Łuczak, Y. Peng, V. Rödl, A. Ruciński, M. Simonovits and J. Skokan, The Ramsey Number for Hypergraph Cycles I, *Journal of Combinatorial Theory,* Series A, **113** (2006) 67-83.

[HaŁP2+] P.E. Haxell, T. Łuczak, Y. Peng, V. Rödl, A. Ruciński and J. Skokan, The Ramsey Number for 3-Uniform Tight Hypergraph Cycles, *Combinatorics, Probability and Computing,* **18** (2009) 165-203.

[HaŁT] P.E. Haxell, T. Łuczak and P.W. Tingley, Ramsey Numbers for Trees of Small Maximum Degree, *Combinatorica,* **22** (2002) 287-320.

He - Hu

[HeLD]* Changxiang He, Yusheng Li and Lin Dong, Three-Color Ramsey Numbers of K_n Dropping an Edge, *Graphs and Combinatorics,* **28** (2012) 663-669.

[HeWi] Xiaoyu He and Yuval Wigderson, Multicolor Ramsey Numbers via Pseudorandom Graphs, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P1.32, **27**(1) (2020), 8 pages.

[Hein] K. Heinrich, Proper Colourings of K_{15}, *Journal of the Australian Mathematical Society,* Series A, **24** (1977) 465-495.

[He1] G.R.T. Hendry, Diagonal Ramsey Numbers for Graphs with Seven Edges, *Utilitas Mathematica,* **32** (1987) 11-34.

[He2] G.R.T. Hendry, Ramsey Numbers for Graphs with Five Vertices, *Journal of Graph Theory,* **13** (1989) 245-248.

[He3] G.R.T. Hendry, The Ramsey Numbers $r(K_2+\overline{K}_3, K_4)$ and $r(K_1+C_4, K_4)$, *Utilitas Mathematica,* **35** (1989) 40-54, addendum in **36** (1989) 25-32.

[He4] G.R.T. Hendry, Critical Colorings for Clancy's Ramsey Numbers, *Utilitas Mathematica,* **41** (1992) 181-203.

[He5] G.R.T. Hendry, Small Ramsey Numbers II. Critical Colorings for $r(C_5+e, K_5)$, *Quaestiones Mathematica,* **17** (1994) 249-258.

[-] G.R.T. Hendry, see also [YH].

[HiIr]* R. Hill and R.W. Irving, On Group Partitions Associated with Lower Bounds for Symmetric Ramsey Numbers, *European Journal of Combinatorics,* **3** (1982) 35-50.

[-] J. Hiller, see [BrBH, BudHLS, BudHMP, BudHP, BudHR1, BudHR2].

[Hir] J. Hirschfeld, A Lower Bound for Ramsey's Theorem, *Discrete Mathematics,* **32** (1980) 89-91.

[Ho] Pak Tung Ho, On Ramsey Unsaturated and Saturated Graphs, *Australasian Journal of Combinatorics,* **46** (2010) 13-18.

[HoMe] M. Hoeth and I. Mengersen, Ramsey Numbers for Graphs of Order Four versus Connected Graphs of Order Six, *Utilitas Mathematica,* **57** (2000) 3-19.

[-] P. Holub, see [LiZBBH].

[-] N. Hommowun, see [AlmHS].

[Hook] J. Hook, Critical Graphs for $R(P_n, P_m)$ and the Star-Critical Ramsey Number for Paths, *Discussiones Mathematicae Graph Theory,* **35** (2015) 689-701.

[HoIs] J. Hook and G. Isaak, Star-Critical Ramsey Numbers, *Discrete Applied Mathematics,* **159** (2011) 328-334.

[HuSo] Huang Da Ming and Song En Min, Properties and Lower Bounds of the Third Order Ramsey Numbers (in Chinese), *Mathematica Applicata,* **9** (1996) 105-107.

[Hua1] Huang Guotai, Some Generalized Ramsey Numbers (in Chinese), *Mathematica Applicata,* **1** (1988) 97-101.

[Hua2] Huang Guotai, An Unsolved Problem of Gould and Jacobson (in Chinese), *Mathematica Applicata,* **9** (1996) 234-236.

[-] Huang Jian, see [HTHZ1, HTHZ2, HWSYZH].

[-] Huang Wenke, see [DuHu].

[HTHZ1] (also abbreviated as HT+) Yiru Huang, Fuping Tan, Jian Huang and Kemin Zhang, New Upper Bounds for Ramsey Number $R(K_m - e, K_n - e)$, *manuscript* (2016).

[HTHZ2] Huang Yiru, Tan Fuping, Huang Jian and Zhang Chaohui, On the Upper Bounds Formulas of Multicolored Ramsey Number (in Chinese), *Journal of Jishou University,* Natural Science Edition, **38** (2017) 1-6.

[HWSYZH] (also abbreviated as HW+) Huang Yi Ru, Wang Yuandi, Sheng Wancheng, Yang Jiansheng, Zhang Ke Min and Huang Jian, New Upper Bound Formulas with Parameters for Ramsey Numbers, *Discrete Mathematics,* **307** (2007) 760-763.

[HZ1] Huang Yi Ru and Zhang Ke Min, A New Upper Bound Formula on Ramsey Numbers, *Journal of Shanghai University, Natural Science,* **7** (1993) 1-3.

[HZ2] Huang Yi Ru and Zhang Ke Min, An New Upper Bound Formula for Two Color Classical Ramsey Numbers, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **28** (1998) 347-350.

[HZ3] Huang Yi Ru and Zhang Ke Min, New Upper Bounds for Ramsey Numbers, *European Journal of Combinatorics,* **19** (1998) 391-394.

[-] Huang Yi Ru, see also [BolJY+, YHZ1, YHZ2].

I

[Ihr]* F. Ihringer, Two Small Improvements on Ramsey Numbers, *unpublished report,* http://math.ihringer.org/publications.php (2020).

[Ir] R.W. Irving, Generalised Ramsey Numbers for Small Graphs, *Discrete Mathematics,* **9** (1974) 251-264.

[-] R.W. Irving, see also [HiIr].

[-] G. Isaak, see [HoIs].

[Isb1] J.R. Isbell, $N(4,4;3) \geq 13$, *Journal of Combinatorial Theory,* **6** (1969) 210.

[Isb2] J.R. Isbell, $N(5,4;3) \geq 24$, *Journal of Combinatorial Theory,* Series A, **34** (1983) 379-380.

[Ishi] Y. Ishigami, Linear Ramsey Numbers for Bounded-Degree Hypergraphs, *Electronic Notes in Discrete Mathematics,* **29** (2007) 47-51.

[-] A. Itzhakov, see [CodFIM].

J

[Jack] E. Jackowska, The 3-Color Ramsey Number for a 3-Uniform Loose Path of Length 3, *Australasian Journal of Combinatorics,* **63** (2015) 314-320.

[JacPR] E. Jackowska, J. Polcyn and A. Ruciński, Multicolor Ramsey Numbers and Restricted Turán Numbers for the Loose 3-Uniform Path of Length Three, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P3.5, **24** (2017), 21 pages.

[Jaco] M.S. Jacobson, On the Ramsey Number for Stars and a Complete Graph, *Ars Combinatoria,* **17** (1984) 167-172.

[-] M.S. Jacobson, see also [BEFRSGJ, GoJa1, GoJa2].

[-] S. Jahanbekam, see [BiFJ].

[-] A.M.M. Jaradat, see [BaJBJ].

[JaAl] M.M.M. Jaradat and B.M.N. Alzaleq, The Cycle-Complete Graph Ramsey Number $r(C_8, K_8)$, *SUT Journal of Mathematics,* **43** (2007) 85-98.

[JaBa] M.M.M. Jaradat and A.M.M. Baniabedalruhman, The Cycle-Complete Graph Ramsey Number $r(C_8, K_7)$, *International Journal of Pure and Applied Mathematics,* **41** (2007) 667-677.

[JaBVR] M.M.M. Jaradat, M.S. Bataineh, T. Vetrik and A.M.M. Rabaiah, A Note on the Ramsey Numbers for Theta Graphs versus the Wheel of Order 5, *AKCE International Journal of Graphs and Combinatorics,* **15** (2018) 187-189.

[-] M.M.M. Jaradat, see also [BaJBJ, BatJA].

[-] I. Javaid, see [AliTJ].

[JaNR]* C.J. Jayawardene, D. Narváez and S. Radziszowski, Star-Critical Ramsey Numbers for Cycles versus K_4, *Discussiones Mathematicae Graph Theory,* **41** (2021) 381-390.

[JR1] C.J. Jayawardene and C.C. Rousseau, An Upper Bound for the Ramsey Number of a Quadrilateral versus a Complete Graph on Seven Vertices, *Congressus Numerantium,* **130** (1998) 175-188.

[JR2] C.J. Jayawardene and C.C. Rousseau, Ramsey Numbers $r(C_6, G)$ for All Graphs G of Order Less than Six, *Congressus Numerantium,* **136** (1999) 147-159.

[JR3] C.J. Jayawardene and C.C. Rousseau, The Ramsey Numbers for a Quadrilateral vs. All Graphs on Six Vertices, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **35** (2000) 71-87. Erratum in **51** (2004) 221. Erratum by L. Boza in **89** (2014) 155-156.

[JR4] C.J. Jayawardene and C.C. Rousseau, Ramsey Numbers $r(C_5, G)$ for All Graphs G of Order Six, *Ars Combinatoria,* **57** (2000) 163-173.

[JR5] C.J. Jayawardene and C.C. Rousseau, The Ramsey Number for a Cycle of Length Five vs. a Complete Graph of Order Six, *Journal of Graph Theory,* **35** (2000) 99-108.

[JRB} C. Jayawardene, C.C. Rousseau and B. Bollobás, How Ramsey Theory Can Be Used to Solve Harary's Problem for $K_{2,k}$, *preprint,* arXiv, http://arxiv.org/abs/1901.01552 (2019).

[-] C.J. Jayawardene, see also [BolJY+, RoJa1, RoJa2].

[JenSk] M. Jenssen and J. Skokan, Exact Ramsey Numbers of Odd Cycles via Nonlinear Optimisation, *Advances in Mathematics,* **376** (2021), online 107444.

[-] M. Jenssen, see also [DavJR].

[-] Jiang Baoqi, see [SunYJLS].

[JiSa] Tao Jiang and M. Salerno, Ramsey Numbers of Some Bipartite Graphs versus Complete Graphs, *Graphs and Combinatorics,* **27** (2011) 121-128.

[-] Tao Jiang, see also [ColGJ].

[JiLSX] Yu Jiang, Meilian Liang, Yongqi Sun and Xiaodong Xu, On Ramsey Numbers $R(K_4 - e, K_t)$, *preprint,* (2019). To appear in *Graphs and Combinatorics* (2021).

[JiLTX1]* Yu Jiang, Meilian Liang, Yanmei Teng and Xiaodong Xu, The Cyclic Triangle-Free Process, *Symmetry,* **11** (2019) 955.

[JiLTX2]* Yu Jiang, Meilian Liang, Yanmei Teng and Xiaodong Xu, Random Cyclic Triangle-Free Graphs with Prime Orders, *preprint,* (2020).

[Jin]** Jin Xia, Ramsey Numbers Involving a Triangle: Theory & Applications, *Technical Report RIT-TR-93-019,* MS thesis, Department of Computer Science, Rochester Institute of Technology, 1993.

[-] Jin Xia, see also [RaJi].

[-] J.R. Johnson, see [DayJ].

[JoPe] K. Johst and Y. Person, On the Multicolor Ramsey Number of a Graph with m Edges, *Discrete Mathematics,* **339** (2016) 2857-2860.

[-] Jusmawati, see [HaJu].

[JGT] *Journal of Graph Theory,* special volume on Ramsey theory, **7**, Number 1, (1983).

K

[KaOS] S. Kadota, T. Onozuka and Y. Suzuki, The Graph Ramsey Number $R(F_l, K_6)$, *Discrete Mathematics,* **342** (2019) 1028-1037.

[Ka1] J.G. Kalbfleisch, Construction of Special Edge-Chromatic Graphs, *Canadian Mathematical Bulletin,* **8** (1965) 575-584.

[Ka2]* J.G. Kalbfleisch, Chromatic Graphs and Ramsey's Theorem, *Ph.D. thesis,* University of Waterloo, January 1966.

[Ka3] J.G. Kalbfleisch, On Robillard's Bounds for Ramsey Numbers, *Canadian Mathematical Bulletin,* **14** (1971) 437-440.

[KaSt] J.G. Kalbfleisch and R.G. Stanton, On the Maximal Triangle-Free Edge-Chromatic Graphs in Three Colors, *Journal of Combinatorial Theory,* **5** (1968) 9-20.

[KamRa] A. Kamranian and G. Raeisi, On the Star-Critical Ramsey Number of a Forest versus Complete Graphs, *preprint,* http://arxiv.org/abs/1912.00703 (2019).

[-] A. Kamranian, see also [RaeK].

[KáRos] G. Károlyi and V. Rosta, Generalized and Geometric Ramsey Numbers for Cycles, *Theoretical Computer Science,* **263** (2001) 87-98.

[KeeLS] P. Keevash, E. Long and J. Skokan, Cycle-Complete Ramsey Numbers, *International Mathematics Research Notices,* rnz119 (2019) 1-26.

[-] P. Keevash, see also [BohK1, BohK2, BohK3].

[KerRo] M. Kerber and C. Rowan, CommonLisp program for computing upper bounds on classical Ramsey numbers, http://www.cs.bham.ac.uk/~mmk/demos/ramsey-upper-limit.lisp (2009).

[Kéry] G. Kéry, On a Theorem of Ramsey (in Hungarian), *Matematikai Lapok,* **15** (1964) 204-224.

[KhoDz] F. Khoeini and T. Dzido, On Some Three Color Ramsey Numbers for Paths, Cycles, Stripes and Stars, *Graphs and Combinatorics,* **35** (2019) 559-567.

[Kim] J.H. Kim, The Ramsey Number $R(3,t)$ Has Order of Magnitude $t^2/\log t$, *Random Structures and Algorithms,* **7** (1995) 173-207.

[KlaM1] K. Klamroth and I. Mengersen, Ramsey Numbers of K_3 versus (p,q)-Graphs, *Ars Combinatoria,* **43** (1996) 107-120.

[KlaM2] K. Klamroth and I. Mengersen, The Ramsey Number of $r(K_{1,3}, C_4, K_4)$, *Utilitas Mathematica,* **52** (1997) 65-81.

[-] K. Klamroth, see also [ArKM].

[-] M. Klawe, see [GrHK].

[-] D.J. Kleitman, see [GoK].

[KniSu] C. Knierim and P. Su, Improved Bounds on the Multicolor Ramsey Numbers of Paths and Even Cycles, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P1.26, **26**(1) (2019), 17 pages.

[KoSS1] Y. Kohayakawa, M. Simonovits and J. Skokan, The 3-colored Ramsey Number of Odd Cycles, *Electronic Notes in Discrete Mathematics,* **19** (2005) 397-402.

[KoSS2] Y. Kohayakawa, M. Simonovits and J. Skokan, The 3-colored Ramsey Number of Odd Cycles, to appear in *Journal of Combinatorial Theory,* Series B (2013).

[Köh] W. Köhler, On a Conjecture by Grossman, *Ars Combinatoria,* **23** (1987) 103-106.

[-] J. Komlós, see [CsKo, AjKS, AjKSS].

[Kol1]* M. Kolodyazhny, Novye Nizhnie Granitsy Chisel Ramseya $R(3, 12)$ i $R(3, 13)$ (in Russian), *Matematicheskoye i Informacionnoe Modelirovanie,* Tyumen, **14** (2015) 126-130.

[Kol2]* M. Kolodyazhny, graphs available at http://aluarium.net/forum/wiki-article-17.html, *personal communication* (2016).

[Kor] A. Korolova, Ramsey Numbers of Stars versus Wheels of Similar Sizes, *Discrete Mathematics,* **292** (2005) 107-117.

[KosMV1] A. Kostochka, D. Mubayi and J. Verstraëte, On Independent Sets in Hypergraphs, *Random Structures and Algorithms,* **44** (2014) 224-239.

[KosMV2] A. Kostochka, D. Mubayi and J. Verstraëte, Hypergraph Ramsey Numbers: Triangles versus Cliques, *Journal of Combinatorial Theory,* Series A, **120** (2013) 1491-1507.

[KosPR] A. Kostochka, P. Pudlák and V. Rödl, Some Constructive Bounds on Ramsey Numbers, *Journal of Combinatorial Theory,* Series B, **100** (2010) 439-445.

[KoRö1] A.V. Kostochka and V. Rödl, On Graphs with Small Ramsey Numbers, *Journal of Graph Theory,* **37** (2001) 198-204.

[KoRö2] A.V. Kostochka and V. Rödl, On Graphs with Small Ramsey Numbers, II, *Combinatorica,* **24** (2004) 389-401.

[KoRö3] A.V. Kostochka and V. Rödl, On Ramsey Numbers of Uniform Hypergraphs with Given Maximum Degree, *Journal of Combinatorial Theory,* Series A, **113** (2006) 1555-1564.

[KoSu] A.V. Kostochka and B. Sudakov, On Ramsey Numbers of Sparse Graphs, *Combinatorics, Probability and Computing,* **12** (2003) 627-641.

[-] R.L. Kramer, see [FeKR].

[KrRod] I. Krasikov and Y. Roditty, On Some Ramsey Numbers of Unicyclic Graphs, *Bulletin of the Institute of Combinatorics and its Applications,* **33** (2001) 29-34.

[-] S. Krause, see [HaKr1, HaKr2].

[KrLR]* D.L. Kreher, Wei Li and S. Radziszowski, Lower Bounds for Multicolor Ramsey Numbers from Group Orbits, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **4** (1988) 87-95.

[-] D.L. Kreher, see also [RaK1, RaK2, RaK3, RaK4].

[Kriv] M. Krivelevich, Bounding Ramsey Numbers through Large Deviation Inequalities, *Random Structures and Algorithms,* **7** (1995) 145-155.

[-] M. Krivelevich, see also [AlBK, AlKS].

[KroMe] M. Krone and I. Mengersen, The Ramsey Numbers $r(K_5-2K_2, 2K_3)$, $r(K_5-e, 2K_3)$ and $r(K_5, 2K_3)$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **81** (2012) 257-260.

[Krü] O. Krüger, An Invariant for Minimum Triangle-Free Graphs, *Australasian Journal of Combinatorics,* **74** (2019) 371-388.

[-] M. Kubale, see [DzKP].

[KüCFO] D. Kühn, O. Cooley, N. Fountoulakis and D. Osthus, Ramsey Numbers of Sparse Hypergraphs, *Electronic Notes in Discrete Mathematics,* **29** (2007) 29-33.

[-] D. Kühn, see also [CooFKO1, CooFKO2].

[Kuz]* E. Kuznetsov, Computational Lower Limits on Small Ramsey Numbers, *preprint,* arXiv, http://arxiv.org/abs/1505.07186 (2016).

La - Le

[-] P.C.B. Lam, see [ShiuLL].

[-] J. Lambert, see [BudHLS].

[LaRo] B. Landman and A. Robertson, *Ramsey Theory on the Integers,* Student Mathematical Library, American Mathematical Society, first edition **24** (2004), second edition **73** (2014).

[LaLR]** A. Lange, I. Livinsky and S.P. Radziszowski, Computation of the Ramsey Numbers $R(C_4, K_9)$ and $R(C_4, K_{10})$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **97** (2016) 139-154.

[-] C. Langhoff, see [AlmBCL].

[La1] S.L. Lawrence, Cycle-Star Ramsey Numbers, *Notices of the American Mathematical Society,* **20** (1973) Abstract A-420.

[La2] S.L. Lawrence, Bipartite Ramsey Theory, *Notices of the American Mathematical Society,* **20** (1973) Abstract A-562.

[-] S.L. Lawrence, see also [FLPS].

[LayMa] C. Laywine and J.P. Mayberry, A Simple Construction Giving the Two Non-isomorphic Triangle-Free 3-Colored K_{16}'s, *Journal of Combinatorial Theory,* Series B, **45** (1988) 120-124.

[LaMu] F. Lazebnik and D. Mubayi, New Lower Bounds for Ramsey Numbers of Graphs and Hypergraphs, *Advances in Applied Mathematics,* **28** (2002) 544-559.

[LaWo1] F. Lazebnik and A. Woldar, New Lower Bounds on the Multicolor Ramsey Numbers $r_k(C_4)$, *Journal of Combinatorial Theory,* Series B, **79** (2000) 172-176.

[LaWo2] F. Lazebnik and A. Woldar, General Properties of Some Families of Graphs Defined by Systems of Equations, *Journal of Graph Theory,* **38** (2001) 65-86.

[Lee] Choongbum Lee, Ramsey Numbers of Degenerate Graphs, *Annals of Mathematics,* **185** (2017) 791-829.

[-] Choongbum Lee, see also [ConFLS].

[Lef] H. Lefmann, Ramsey Numbers for Monotone Paths and Cycles, *Ars Combinatoria,* **35** (1993) 271-279.

[-] H. Lefmann, see also [DuLR].

[-] J. Lehel, see [BaLS, GyLSS].

[LeMu] J. Lenz and D. Mubayi, Multicolor Ramsey Numbers for Complete Bipartite versus Complete Graphs, *Journal of Graph Theory,* **77** (2014) 19-38.

[Les]* A. Lesser, Theoretical and Computational Aspects of Ramsey Theory, *Examensarbeten i Matematik,* Matematiska Institutionen, Stockholms Universitet, **3**, http://www2.math.su.se/gemensamt/grund/exjobb/matte/2001 (2001).

[-] D. Leven, see [BlLR].

Li

[-] Li Bingxi, see [SunYWLX, SunYXL].

[LiBie] Binlong Li and H. Bielak, On the Ramsey-Goodness of Paths, *Graphs and Combinatorics,* **32** (2016) 2541-2549.

[LiNing1] Binlong Li and Bo Ning, On Path-Quasar Ramsey Numbers, *Annales Universitatis Mariae Curie-Skłodowska Lublin-Polonia,* Sectio A, **LXVIII** (2014) 11-17.

[LiNing2] Binlong Li and Bo Ning, The Ramsey Numbers of Paths versus Wheels: a Complete Solution, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P4.41, **21**(4) (2014), 30 pages.

[LiSch] Binlong Li and I. Schiermeyer, On Star-Wheel Ramsey Numbers, *Graphs and Combinatorics,* **32** (2016) 733-739.

[LiZBBH] Binlong Li, Yanbo Zhang, H. Bielak, H. Broersma and P. Holub, Closing the Gap on Path-Kipas Ramsey Numbers, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P3.21, **22**(3) (2015), 7 pages.

[LiZB] Binlong Li, Yanbo Zhang and H. Broersma, An Exact Formula for All Star-Kipas Ramsey Numbers, *Graphs and Combinatorics,* **33** (2017) 141-148.

[LiWa1] Li Da Yong and Wang Zhi Jian, The Ramsey Number $r(mC_4, nC_4)$ (in Chinese), *Journal of Shanghai Tiedao University,* **20** (1999) 66-70, 83.

[LiWa2] Li Da Yong and Wang Zhi Jian, The Ramsey Numbers $r(mC_4, nC_5)$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **45** (2003) 245-252.

[-] Dongxin Li, see [ZhuZL].

[-] Li Guiqing, see [SLLL, SLZL].

[LiLi]* Li Mingbo and Li Yusheng, Ramsey Numbers and Triangle-Free Cayley Graphs (in Chinese), *Journal of Tongji University (Natural Science),* Shanghai, **43**(11) (2015) 1750-1752.

[-] Li Jinwen, see [ZLLS].

[LiSLW]* Li Qiao, Su Wenlong, Luo Haipeng and Wu Kang, Lower Bounds for Some Two-Color Ramsey Numbers, *manuscript* (2011). By 2016, the results there have been improved by others.

[-] Li Qiao, see also [SuLL, SLLL].

[-] Ray Li, see [FoxLi].

[-] Wei Li, see [KrLR].

[Li1] Li Yusheng, Some Ramsey Numbers of Graphs with Bridge, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **25** (1997) 225-229.

[Li2] Li Yusheng, The Shannon Capacity of a Communication Channel, Graph Ramsey Number and a Conjecture of Erdős, *Chinese Science Bulletin,* **46** (2001) 2025-2028.

[Li3] Yusheng Li, Ramsey Numbers of a Cycle, *Taiwanese Journal of Mathematics,* **12** (2008) 1007-1013.

[Li4] Yusheng Li, The Multi-Color Ramsey Number of an Odd Cycle, *Journal of Graph Theory,* **62** (2009) 324-328.

[LiLih] Yusheng Li and Ko-Wei Lih, Multi-Color Ramsey Numbers of Even Cycles, *European Journal of Combinatorics,* **30** (2009) 114-118.

[LiR1] Li Yusheng and C.C. Rousseau, On Book-Complete Graph Ramsey Numbers, *Journal of Combinatorial Theory,* Series B, **68** (1996) 36-44.

[LiR2] Li Yusheng and C.C. Rousseau, Fan-Complete Graph Ramsey Numbers, *Journal of Graph Theory,* **23** (1996) 413-420.

[LiR3] Li Yusheng and C.C. Rousseau, On the Ramsey Number $r(H+\overline{K}_n, K_n)$, *Discrete Mathematics,* **170** (1997) 265-267.

[LiR4] Li Yusheng and C.C. Rousseau, A Ramsey Goodness Result for Graphs with Many Pendant Edges, *Ars Combinatoria,* **49** (1998) 315-318.

[LiRS] Li Yusheng, C.C. Rousseau and L. Soltés, Ramsey Linear Families and Generalized Subdivided Graphs, *Discrete Mathematics,* **170** (1997) 269-275.

[LiRZ1] Li Yusheng, C.C. Rousseau and Zang Wenan, Asymptotic Upper Bounds for Ramsey Functions, *Graphs and Combinatorics,* **17** (2001) 123-128.

[LiRZ2] Li Yusheng, C.C. Rousseau and Zang Wenan, An Upper Bound on Ramsey Numbers, *Applied Mathematics Letters,* **17** (2004) 663-665.

[LiShen] Yusheng Li and Jian Shen, Bounds for Ramsey Numbers of Complete Graphs Dropping an Edge, *European Journal of Combinatorics,* **29** (2008) 88-94.

[LiTZ] Li Yusheng, Tang Xueqing and Zang Wenan, Ramsey Functions Involving $K_{m,n}$ with n Large, *Discrete Mathematics,* **300** (2005) 120-128.

[LiZa1] Li Yusheng and Zang Wenan, Ramsey Numbers Involving Large Dense Graphs and Bipartite Turán Numbers, *Journal of Combinatorial Theory,* Series B, **87** (2003) 280-288.

[LiZa2] Li Yusheng and Zang Wenan, The Independence Number of Graphs with a Forbidden Cycle and Ramsey Numbers, *Journal of Combinatorial Optimization,* **7** (2003) 353-359.

[-] Li Yusheng, see also [BaiLi, BaLX, CaLRZ, Doli, DoLL1, DoLL2, GuLi, HeLD, LiLi, LinLi1, LinLi2, LinLD, LinLS, LiuLi1, LiuLi2, PeiLi, PeiCLY, ShiuLL, SonLi, SunLi, WaLi, YuLi].

[-] Li Zhenchong, see [LuSL, LuLL].

Lia - Liv

[LiaRX] Meilian Liang, S. Radziszowski and Xiaodong Xu, On a Diagonal Conjecture for Classical Ramsey Numbers, *Discrete Applied Mathematics,* **267** (2019) 195-200.

[-] Meilian Liang, see also [JiLSX, JiLTX1, JiLTX2, LuLL, XuLL].

[LiaWXS]* Wenzhong Liang, Kang Wu, Xiaodong Xu and Wenlong Su, New Lower Bounds for Seven Classical Ramsey Numbers, *in preparation,* (2011).

[LiaWXCS]* Liang Wenzhong, Wu Kang, Xu Chengzhang, Chen Hong and Su Wenlong, Using the Two Stage Automorphism of Paley to Calculate the Lower Bound of Ramsey, *Journal of Inner Mongolia Normal University,* **41** (2012) 591-596.

[LidP]** B. Lidický and F. Pfender, Semidefinite Programming and Ramsey Numbers, *preprint,* arXiv, http://arxiv.org/abs/1704.03592 (2017). Revised version (2020).

[-] A. Liebenau, see [BlLi].

[-] Ko-Wei Lih, see [LiLih].

[LinCh] Qizhong Lin and Weiji Chen, New Upper Bound for Multicolor Ramsey Number of Odd Cycles, *Discrete Mathematics,* **342** (2019) 217-220.

[LinLi1] Qizhong Lin and Yusheng Li, On Ramsey Numbers of Fans, *Discrete Applied Mathematics,* **157** (2009) 191-194.

[LinLi2] Qizhong Lin and Yusheng Li, Ramsey Numbers of K_3 and $K_{n,n}$, *Applied Mathematics Letters,* **25** (2012) 380-384.

[LinLD] Qizhong Lin, Yusheng Li and Lin Dong, Ramsey Goodness and Generalized Stars, *European Journal of Combinatorics,* **31** (2010) 1228-1234.

[LinLS]* Qizhong Lin, Yusheng Li and Jian Shen, Lower Bounds for $r_2(K_1+G)$ and $r_3(K_1+G)$ from Paley Graph and Generalization, *European Journal of Combinatorics,* **40** (2014) 65-72.

[LinLiu] Qizhong Lin and Xiudi Liu, Ramsey Numbers Involving Large Books, *SIAM Journal on Discrete Mathematics,* **35** (2021) 23-34.

[LinP] Qizhong Lin and Xing Peng, Large Book-Cycle Ramsey Numbers, *preprint,* arXiv, http://arxiv.org/abs/1909.13533 (2019).

[-] Qizhong Lin, see also [DoLL1, DoLL2, HaoLin].

[-] Lin Xiaohui, see [SunYJLS, SunYLZ1, SunYLZ2].

[LinCa]* M. Lindsay and J.W. Cain, Improved Lower Bounds on the Classical Ramsey Numbers $R(4, 22)$ and $R(4, 25)$, *preprint,* arXiv, http://arxiv.org/abs/1510.06102 (2015).

[Lind] B. Lindström, Undecided Ramsey-Numbers for Paths, *Discrete Mathematics,* **43** (1983) 111-112.

[Ling] A.C.H. Ling, Some Applications of Combinatorial Designs to Extremal Graph Theory, *Ars Combinatoria,* **67** (2003) 221-229.

[-] Andy Liu, see [AbbL, BaLiu].

[-] Hong Liu, see [AxGLM].

[-] Liu Linzhong, see [ZLLS].

[LiuLi1] Meng Liu and Yusheng Li, Ramsey Numbers of a Fixed Odd-Cycle and Generalized Books and Fans, *Discrete Mathematics,* **339** (2016) 2481-2489.

[LiuLi2] Meng Liu and Yusheng Li, Ramsey Numbers and Bipartite Ramsey Numbers via Quasi-Random Graphs, *Discrete Mathematics,* **344** (2021), https://doi.org/10.1016/j.disc.2020.112162.

[-] Liu Shu Yan, see [SonBL].

[Liu]* Sixue Cliff Liu, Lower Bounds for Small Ramsey Numbers on Hypergraphs, in *Proceedings of COCOON 2019,* Xi'an, China, LNCS 11653, Springer, (2019) 412-424.

[-] Liu Xiangyang, see [GuSL].

[-] Xiudi Liu, see [LinLiu].

[-] Liu Yanwu, see [SonYL].

[-] Zhiguo Liu, see [WuSL].

[-] I. Livinsky, see [LaLR].

Lo - Lu

[Loc] S.C. Locke, Bipartite Density and the Independence Ratio, *Journal of Graph Theory,* **10** (1986) 47-53.

[-] S.C. Locke, see also [FrLo].

[LocMc]* M. Locus Dawsey and D. McCarthy, Generalized Paley Graphs and Their Complete Subgraphs of Orders Three and Four, *preprint,* arXiv, http://arxiv.org/abs/2006.14716 (2020).

[-] E. Long, see [KeeLS].

[Lor] P.J. Lorimer, The Ramsey Numbers for Stripes and One Complete Graph, *Journal of Graph Theory,* **8** (1984) 177-184.

[LorMu] P.J. Lorimer and P.R. Mullins, Ramsey Numbers for Quadrangles and Triangles, *Journal of Combinatorial Theory,* Series B, **23** (1977) 262-265.

[LorSe] P.J. Lorimer and R.J. Segedin, Ramsey Numbers for Multiple Copies of Complete Graphs, *Journal of Graph Theory,* **2** (1978) 89-91.

[LorSo] P.J. Lorimer and W. Solomon, The Ramsey Numbers for Stripes and Complete Graphs 1, *Discrete Mathematics,* **104** (1992) 91-97. Corrigendum in *Discrete Mathematics,* **131** (1994) 395.

[-] P.J. Lorimer, see also [CocL1, CocL2].

[Lortz] R. Lortz, A Note on the Ramsey Number of $K_{2,2}$ versus $K_{3,n}$, *Discrete Mathematics,* **306** (2006) 2976-2982.

[LoM1] R. Lortz and I. Mengersen, On the Ramsey Numbers $r(K_{2,n-1}, K_{2,n})$ and $r(K_{2,n}, K_{2,n})$, *Utilitas Mathematica,* **61** (2002) 87-95.

[LoM2] R. Lortz and I. Mengersen, Bounds on Ramsey Numbers of Certain Complete Bipartite Graphs, *Results in Mathematics,* **41** (2002) 140-149.

[LoM3]* R. Lortz and I. Mengersen, Off-Diagonal and Asymptotic Results on the Ramsey Number $r(K_{2,m}, K_{2,n})$, *Journal of Graph Theory,* **43** (2003) 252-268.

[LoM4]* R. Lortz and I. Mengersen, Further Ramsey Numbers for Small Complete Bipartite Graphs, *Ars Combinatoria,* **79** (2006) 195-203.

[LoM5] R. Lortz and I. Mengersen, Ramsey Numbers for Small Graphs versus Small Disconnected Graphs, *Australasian Journal of Combinatorics,* **51** (2011) 89-108.

[LoM6] R. Lortz and I. Mengersen, On the Ramsey Numbers of Certain Graphs of Order Five versus All Connected Graphs of Order Six, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **90** (2014) 197-222.

[LoM7] R. Lortz and I. Mengersen, On the Ramsey Numbers for Stars versus Connected Graphs of Order Six, *Australasian Journal of Combinatorics,* **73** (2019) 1-24.

[LoM8] R. Lortz and I. Mengersen, On the Ramsey Numbers of Non-Star Trees versus Connected Graphs of Order Six, to appear in *Discussiones Mathematicae Graph Theory,* https://doi.org/10.7151/dmgt.2370.

[-] V. Lozin, see [AtLZ].

[Łuc] T. Łuczak, $R(C_n, C_n, C_n) \leq (4+o(1))n$, *Journal of Combinatorial Theory,* Series B, **75** (1999) 174-187.

[ŁuPo1] T. Łuczak and J. Polcyn, On the Multicolor Ramsey Number for 3-Paths of Length Three, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P1.27, **24** (2017), 4 pages.

[ŁuPo2] T. Łuczak and J. Polcyn, The Multipartite Ramsey Number for the 3-Path of Length Three, *Discrete Mathematics,* **341** (2018) 1270-1274.

[ŁuPR] T. Łuczak, J. Polcyn and A. Ruciński, On Multicolor Ramsey Numbers for Loose k-Paths of Length Three, *European Journal of Combinatorics,* **71** (2018) 43-50.

[ŁucSS] T. Łuczak, M. Simonovits and J. Skokan, On the Multi-Colored Ramsey Numbers of Cycles, *Journal of Graph Theory,* **69** (2012) 169-175.

[-] T. Łuczak, see also [AllŁPZ, FiŁu1, FiŁu2, HaŁP1+, HaŁP2+, HaŁT].

[LuSL]* Luo Haipeng, Su Wenlong and Li Zhenchong, The Properties of Self-Complementary Graphs and New Lower Bounds for Diagonal Ramsey Numbers, *Australasian Journal of Combinatorics,* **25** (2002) 103-116.

[LuSS1]* Luo Haipeng, Su Wenlong and Shen Yun-Qiu, New Lower Bounds of Ten Classical Ramsey Numbers, *Australasian Journal of Combinatorics,* **24** (2001) 81-90.

[LuSS2]* Luo Haipeng, Su Wenlong and Shen Yun-Qiu, New Lower Bounds for Two Multicolor Classical Ramsey Numbers, *Radovi Matematički,* **13** (2004) 15-21, pointed to in past revisions. Since 2015, better bounds were obtained by others.

[-] Luo Haipeng, see also [LiSLW, SuL, SuLL, SLLL, SLZL, WSLX1, WSLX2, XuLL].

[LuLL]* Liang Luo, Meilian Liang and Zhenchong Li, Computation of Ramsey Numbers $R(C_m, W_n)$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **81** (2012) 145-149.

M

[Mac]* J. Mackey, Combinatorial Remedies, *Ph.D. thesis,* Department of Mathematics, University of Hawaii, 1994.

[-] W. Macready, see [RanMCG].

[Mad] P. Madarasi, The Ramsey Number of a Long Cycle and Complete Graphs, *preprint,* arXiv, http://arxiv.org/abs/2003.12691 (2020).

[-] C. Magnant, see [MaoWMS].

[MaOm1] L. Maherani and G.R. Omidi, Around a Conjecture of Erdős on Graph Ramsey Numbers, *preprint,* arXiv, http://arxiv.org/abs/1211.6287 (2012).

[MaOm2] L. Maherani and G.R. Omidi, Monochromatic Hamiltonian Berge-Cycles in Colored Hypergraphs, *Discrete Mathematics,* **340** (2017) 2043-2052.

[MaORS1] L. Maherani, G.R. Omidi, G. Raeisi and M. Shahsiah, The Ramsey Number of Loose Paths in 3-Uniform Hypergraphs, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P12, **20**(1) (2013), 8 pages.

[MaORS2] L. Maherani, G.R. Omidi, G. Raeisi and M. Shahsiah, On Three-Color Ramsey Number of Paths, *Graphs and Combinatorics,* **31** (2015) 2299-2308.

[-] H.R. Maimani, see [HagMa].

[MaoWMS] Yaping Mao, Zhao Wang, Colton Magnant and Ingo Schiermeyer, Ramsey and Gallai-Ramsey Number for Wheels, *preprint,* arXiv, http://arxiv.org/abs/1905.12414 (2019).

[Mat]* R. Mathon, Lower Bounds for Ramsey Numbers and Association Schemes, *Journal of Combinatorial Theory,* Series B, **42** (1987) 122-127.

[-] J.P. Mayberry, see [LayMa].

[-] D. McCarthy, see [LocMc].

[McS] C. McDiarmid and A. Steger, Tidier Examples for Lower Bounds on Diagonal Ramsey Numbers, *Journal of Combinatorial Theory,* Series A, **74** (1996) 147-152.

[McK1]** B.D. McKay, Australian National University, *personal communication* (2003+). Graphs available at http://cs.anu.edu.au/people/bdm/data/ramsey.html.

[McK2]** B.D. McKay, A Class of Ramsey-Extremal Hypergraphs, *Transactions on Combinatorics,* **6**(3) (2017) 37-43.

[McK3]** B.D. McKay, Australian National University, *personal communication* (2016).

[MPR]** B.D. McKay, K. Piwakowski and S.P. Radziszowski, Ramsey Numbers for Triangles versus Almost-Complete Graphs, *Ars Combinatoria,* **73** (2004) 205-214.

[MR1]** B.D. McKay and S.P. Radziszowski, The First Classical Ramsey Number for Hypergraphs is Computed, *Proceedings of the Second Annual ACM-SIAM Symposium on Discrete Algorithms,* SODA'91, San Francisco, (1991) 304-308.

[MR2]* B.D. McKay and S.P. Radziszowski, A New Upper Bound for the Ramsey Number $R(5, 5)$, *Australasian Journal of Combinatorics,* **5** (1992) 13-20.

[MR3]** B.D. McKay and S.P. Radziszowski, Linear Programming in Some Ramsey Problems, *Journal of Combinatorial Theory,* Series B, **61** (1994) 125-132.

[MR4]** B.D. McKay and S.P. Radziszowski, $R(4, 5) = 25$, *Journal of Graph Theory,* **19** (1995) 309-322.

[MR5]** B.D. McKay and S.P. Radziszowski, Subgraph Counting Identities and Ramsey Numbers, *Journal of Combinatorial Theory,* Series B, **69** (1997) 193-209.

[McZ]** B.D. McKay and Zhang Ke Min, The Value of the Ramsey Number $R(3,8)$, *Journal of Graph Theory,* **16** (1992) 99-105.

[-] B.D. McKay, see also [AnM1, AnM2, FM].

[McN]** J. McNamara, SUNY Brockport, *personal communication* (1995).

[McR]** J. McNamara and S.P. Radziszowski, The Ramsey Numbers $R(K_4-e, K_6-e)$ and $R(K_4-e, K_7-e)$, *Congressus Numerantium,* **81** (1991) 89-96.

[-] T. Meek, see [BudHMP].

[-] H. Mélot, see [BrCGM].

[MengZZ] Yanjun Meng, Yanbo Zhang and Yunqing Zhang, The Ramsey Number of Wheels versus a Fan of Order Five, *Ars Combinatoria,* **137** (2018) 365-372.

[MeO] I. Mengersen and J. Oeckermann, Matching-Star Ramsey Sets, *Discrete Applied Mathematics,* **95** (1999) 417-424.

[-] I. Mengersen, see also [ArKM, ClEHMS, EHM1, EHM2, HoMe, HaMe1, HaMe2, HaMe3, HaMe4, KlaM1, KlaM2, KroMe, LoM1, LoM2, LoM3, LoM4, LoM5, LoM6, LoM7, LoM8].

[Mér] A. Méroueh, The Ramsey Number of Loose Cycles versus Cliques, *Journal of Graph Theory,* **90** (2019) 172-188.

[-] A. Methuku, see [GerMOV].

[-] Zhengke Miao, see [ChenCMN].

[-] A. Miller, see [CodFIM].

[-] M. Miller, see [BaSNM].

[MiPal] T.K. Mishra and S.P. Pal, Lower Bounds for Ramsey Numbers for Complete Bipartite and 3-Uniform Tripartite Subgraphs, WALCOM 2013, *LNCS* 7748, Springer, Berlin (2013) 257-264.

[MiSa] H. Mizuno and I. Sato, Ramsey Numbers for Unions of Some Cycles, *Discrete Mathematics,* **69** (1988) 283-294.

[MoCa] E.L. Monte Carmelo, Configurations in Projective Planes and Quadrilateral-Star Ramsey Numbers, *Discrete Mathematics,* **308** (2008) 3986-3991.

[-] E.L. Monte Carmelo, see also [GoMC].

[MonCR] L.P. Montejano, J. Chappelon and J.L. Ramirez Alfonsin, Ramsey for Complete Graphs with a Dropped Edge or a Triangle, *Electronic Notes in Discrete Mathematics,* **62** (2017) 21-25.

[-] L.P. Montejano, see also [ChaMR].

[-] R. Morris, see [FizGM, FizGMSS, GrMFSS].

[MoSST] G.O. Mota, G.N. Sárközy, M. Schacht and A. Taraz, Ramsey Numbers for Bipartite Graphs with Small Bandwidth, *European Journal of Combinatorics,* **48** (2015) 165-176.

[Mub1] D. Mubayi, Improved Bounds for the Ramsey Number of Tight Cycles versus Cliques, *Combinatorics, Probability and Computing,* **25** (2016) 791-796.

[Mub2] D. Mubayi, Variants of the Erdős-Szekeres and Erdős-Hajnal Ramsey Problems, *European Journal of Combinatorics,* **62** (2017) 197-205.

[MuR] D. Mubayi and V. Rödl, Hypergraph Ramsey Numbers: Tight Cycles versus Cliques, *Bulletin of the London Mathematical Society,* **48** (2016) 127-134.

[MuSuk1] D. Mubayi and A. Suk, Off-Diagonal Hypergraph Ramsey Numbers, *Journal of Combinatorial Theory,* Series B, **125** (2017) 168-177.

[MuSuk2] D. Mubayi and A. Suk, Constructions in Ramsey Theory, *Journal of the London Mathematical Society,* **97** (2018) 247-257.

[MuSuk3] D. Mubayi and A. Suk, New Lower Bounds for Hypergraph Ramsey Numbers, *Bulletin of the London Mathematical Society,* **50** (2018) 189-201.

[MuSuk4] D. Mubayi and A. Suk, A Survey of Hypergraph Ramsey Problems, in *Discrete Mathematics and Applications,* Springer Optimization and Its Applications, Cham, vol. 165 (2020).

[-] D. Mubayi, see also [AxFM, AxGLM, KosMV1, KosMV2, LaMu, LeMu].

[-] P.R. Mullins, see [LorMu].

[-] S. Musdalifah, see [SuAM, SuAAM].

N

[-] S.M. Nababan, see [BaSNM].

[NaORS] B. Nagle, S. Olsen, V. Rödl and M. Schacht, On the Ramsey Number of Sparse 3-Graphs, *Graphs and Combinatorics,* **24** (2008) 205-228.

[Nar]* D. Narváez, Some Multicolor Ramsey Numbers Involving Cycles, *MS thesis,* Department of Computer Science, Rochester Institute of Technology, 2015.

[-] D. Narváez, see also [JaNR].

[Neš] J. Nešetřil, Ramsey Theory, chapter 25 in *Handbook of Combinatorics,* ed. R.L. Graham, M. Grötschel and L. Lovász, The MIT-Press, Vol. II, 1996, 1331-1403.

[NeOs] J. Nešetřil and P. Ossona de Mendez, Fraternal Augmentations, Arrangeability and Linear Ramsey Numbers, *European Journal of Combinatorics,* **30** (2009) 1696-1703.

[-] J. Nešetřil, see also [GrNe].

[-] C.T. Ng, see [ChenCMN, ChenCNZ, CheCZN].

[-] T. Nguyen, see [BroNN].

[Nik] V. Nikiforov, The Cycle-Complete Graph Ramsey Numbers, *Combinatorics, Probability and Computing,* **14** (2005) 349-370.

[NiRo1] V. Nikiforov and C.C. Rousseau, Large Generalized Books Are p-Good, *Journal of Combinatorial Theory,* Series B, **92** (2004) 85-97.

[NiRo2] V. Nikiforov and C.C. Rousseau, Book Ramsey Numbers I, *Random Structures and Algorithms,* **27** (2005) 379-400.

[NiRo3] V. Nikiforov and C.C. Rousseau, A Note on Ramsey Numbers for Books, *Journal of Graph Theory,* **49** (2005) 168-176.

[NiRo4] V. Nikiforov and C.C. Rousseau, Ramsey Goodness and Beyond, *Combinatorica,* **29** (2009) 227-262.

[NiRS] V. Nikiforov, C.C. Rousseau and R.H. Schelp, Book Ramsey Numbers and Quasi-Randomness, *Combinatorics, Probability and Computing,* **14** (2005) 851-860.

[-] Bo Ning, see [LiNing1, LiNing2].

[NoSZ] S. Norin, Yue Ru Sun and Yi Zhao, Asymptotics of Ramsey Numbers of Double Stars, *preprint,* arXiv, http://arxiv.org/abs/1605.03612 (2016).

[NoBa]* E. Noviani and E.T. Baskoro, On the Ramsey Number of 4-Cycle versus Wheel, *Indonesian Journal of Combinatorics,* **1** (2016) 9-21.

[-] A. Nowik, see [DzNS].

[-] E. Nystrom, see [BroNN].

O

[-] J. Oeckermann, see [MeO].

[-] S. Olsen, see [NaORS].

[OmRa1] G.R. Omidi and G. Raeisi, On Multicolor Ramsey Number of Paths versus Cycles, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P24, **18**(1) (2011), 16 pages.

[OmRa2] G.R. Omidi and G. Raeisi, A Note on the Ramsey Number of Stars - Complete Graphs, European Journal of Combinatorics, **32** (2011) 598-599.

[OmRa3] G.R. Omidi and G. Raeisi, Ramsey Numbers for Multiple Copies of Hypergraphs, *preprint,* arXiv, http://arxiv.org/abs/1303.0474 (2013).

[OmRR] G.R. Omidi, G. Raeisi and Z. Rahimi, Star versus Stripes Ramsey Numbers, *European Journal of Combinatorics,* **67** (2018) 268-274.

[OmSh1] G.R. Omidi and M. Shahsiah, Ramsey Numbers of 3-Uniform Loose Paths and Loose Cycles, *Journal of Combinatorial Theory,* Series A, **121** (2014) 64-73.

[OmSh2] G.R. Omidi and M. Shahsiah, Diagonal Ramsey Numbers of Loose Cycles in Uniform Hypergraphs, *SIAM Journal on Discrete Mathematics,* **31** (2017) 1634-1669.

[OmSh3] G.R. Omidi and M. Shahsiah, Ramsey Numbers of 4-Uniform Loose Cycles, *Discrete Applied Mathematics,* **230** (2017) 112-120.

[OmSh4] G.R. Omidi and M. Shahsiah, Ramsey Numbers of Uniform Loose Paths and Cycles, *Discrete Mathematics,* **340** (2017) 1426-1434.

[-] G.R. Omidi, see also [GerMOV, MaOm1, MaOm2, MaORS1, MaORS2].

[-] T. Onozuka, see [KaOS].

[-] P. Ossona de Mendez, see [NeOs].

[-] D. Osthus, see [CooFKO1, CooFKO2, KüCFO].

P

[-] J. Pach, see [FoxPS1, FoxPS2].

[-] S.P. Pal, see [MiPal].

[-] Linqiang Pan, see [ShaXBP, ShaXSP].

[Par1] T.D. Parsons, The Ramsey Numbers $r(P_m, K_n)$, *Discrete Mathematics,* **6** (1973) 159-162.

[Par2] T.D. Parsons, Path-Star Ramsey Numbers, *Journal of Combinatorial Theory,* Series B, **17** (1974) 51-58.

[Par3] T.D. Parsons, Ramsey Graphs and Block Designs, I, *Transactions of the American Mathematical Society,* **209** (1975) 33-44.

[Par4] T.D. Parsons, Ramsey Graphs and Block Designs, *Journal of Combinatorial Theory,* Series A, **20** (1976) 12-19.

[Par5] T.D. Parsons, Graphs from Projective Planes, *Aequationes Mathematica,* **14** (1976) 167-189.

[Par6] T.D. Parsons, Ramsey Graph Theory, in *Selected Topics in Graph Theory,* (L.W. Beineke and R.J. Wilson eds.), Academic Press, (1978) 361-384.

[-] T.D. Parsons, see also [FLPS].

[PeiLi] Chaoping Pei and Yusheng Li, Ramsey Numbers Involving a Long Path, *Discrete Mathematics,* **339** (2016) 564-570.

[PeiCLY] Chaoping Pei, Ming Chen, Yusheng Li and Pei Yu, Ramsey Good Graphs with Long Suspended Paths, *Graphs and Combinatorics,* **34** (2018) 759-767.

[Peng] Xing Peng, The Ramsey Number of Generalized Loose Paths in Hypergraphs, *Discrete Mathematics,* **339** (2016) 539-546.

[-] Xing Peng, see also [LinP].

[-] Yuejian Peng, see [HaŁP1+, HaŁP2+].

[-] A. Penland, see [BudHMP, BudHP, BudP].

[-] Y. Person, see [JoPe].

[-] F. Pfender, see [LidP].

[-] O. Pikhurko, see [BePi].

[Piw1]* K. Piwakowski, Applying Tabu Search to Determine New Ramsey Graphs, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #R6, **3**(1) (1996), 4 pages. The lower bounds presented in this paper have been improved.

[Piw2]** K. Piwakowski, A New Upper Bound for $R_3(K_4 - e)$, *Congressus Numerantium,* **128** (1997) 135-141.

[PR1]** K. Piwakowski and S.P. Radziszowski, $30 \le R(3,3,4) \le 31$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **27** (1998) 135-141.

[PR2]** K. Piwakowski and S.P. Radziszowski, Towards the Exact Value of the Ramsey Number $R(3,3,4)$, *Congressus Numerantium,* **148** (2001) 161-167.

[-] K. Piwakowski, see also [MPR, DzKP].

[Pokr] A. Pokrovskiy, Calculating Ramsey Numbers by Partitioning Coloured Graphs, *Journal of Graph Theory,* **84** (2017) 477-500.

[PoSu1] A. Pokrovskiy and B. Sudakov, Ramsey Goodness of Paths, *Journal of Combinatorial Theory,* Series B, **122** (2017) 384-390.

[PoSu2] A. Pokrovskiy and B. Sudakov, Ramsey Goodness of Cycles, *SIAM Journal on Discrete Mathematics,* **34** (2020) 1884-1908.

[-] A. Pokrovskiy, see also [BalPS].

[Pol] J. Polcyn, One More Turán Number and Ramsey Number for the Loose 3-Uniform Path of Length Three, *Discussiones Mathematicae Graph Theory,* **37** (2017) 443-464.

[PoRRS] J. Polcyn, V. Rödl, A. Ruciński and E. Szemerédi, Short Paths in Quasi-Random Triple Systems with Sparse Underlying Graphs, *Journal of Combinatorial Theory,* Series B, **96** (2006) 584-607.

[PoRu] J. Polcyn and A. Ruciński, Refined Turán Numbers and Ramsey Numbers for the Loose 3-Uniform Path of Length Three, *Discrete Mathematics,* **340** (2017) 107-118.

[-] J. Polcyn, see also [AllŁPZ, HanPR, JacPR, ŁuPo1, ŁuPo2, ŁuPR].

[-] A.D. Polimeni, see [ChGP, ChRSPS].

[-] J.R. Portillo, see [BoPo].

[-] L.M. Pretorius, see [SwPr].

[-] P. Pudlák, see [AlPu, CoPR, KosPR].

Q

[-] Qian Xinjin, see [SonGQ].

Ra - Re

[-] A.M.M. Rabaiah, see [JaBVR].

[Ra1]** S.P. Radziszowski, The Ramsey Numbers $R(K_3, K_8-e)$ and $R(K_3, K_9-e)$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **8** (1990) 137-145.

[Ra2] S.P. Radziszowski, Small Ramsey Numbers, *Technical Report RIT-TR-93-009,* Department of Computer Science, Rochester Institute of Technology, 1993.

[Ra3]** S.P. Radziszowski, On the Ramsey Number $R(K_5-e, K_5-e)$, *Ars Combinatoria,* **36** (1993) 225-232.

[Ra4] S.P. Radziszowski, Ramsey Numbers Involving Cycles, in *Ramsey Theory: Yesterday, Today and Tomorrow* (ed. A. Soifer), Progress in Mathematics 285, Springer-Birkhauser 2011, 41-62.

[RaJi] S.P. Radziszowski and Jin Xia, Paths, Cycles and Wheels in Graphs without Antitriangles, *Australasian Journal of Combinatorics,* **9** (1994) 221-232.

[RaK1]* S.P. Radziszowski and D.L. Kreher, Search Algorithm for Ramsey Graphs by Union of Group Orbits, *Journal of Graph Theory,* **12** (1988) 59-72.

[RaK2]** S.P. Radziszowski and D.L. Kreher, On $R(3, k)$ Ramsey Graphs: Theoretical and Computational Results, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **4** (1988) 37-52.

[RaK3]** S.P. Radziszowski and D.L. Kreher, Upper Bounds for Some Ramsey Numbers $R(3, k)$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **4** (1988) 207-212.

[RaK4] S.P. Radziszowski and D.L. Kreher, Minimum Triangle-Free Graphs, *Ars Combinatoria,* **31** (1991) 65-92.

[RaT]* S.P. Radziszowski and Kung-Kuen Tse, A Computational Approach for the Ramsey Numbers $R(C_4, K_n)$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **42** (2002) 195-207.

[RaST]* S.P. Radziszowski, J. Stinehour and Kung-Kuen Tse, Computation of the Ramsey Number $R(W_5, K_5)$, *Bulletin of the Institute of Combinatorics and its Applications,* **47** (2006) 53-57.

[-] S.P. Radziszowski, see also [BaRT, BlLR, CalSR, DyDR, FeKR, GoR1, GoR2, JaNR, KrLR, LaLR, LiaRX, MPR, MR1, MR2, MR3, MR4, MR5, McR, PR1, PR2, ShWR, WuSR, WuSZR, XuR1, XuR2, XuR3, XuR4, XSR1, XSR2, XXER, XuXR, ZhuXR].

[RaeK] G. Raeisi and A. Kamranian, Ramsey Number of Disjoint Union of Good Hypergraphs, *The 50th Annual Iranian Mathematics Conference,* Shiraz University, 26-29 August 2019, 750-753.

[RaeZ] G. Raeisi and A. Zaghian, Ramsey Number of Wheels versus Cycles and Trees, *Canadian Mathematical Bulletin,* **59** (2016) 190-196.

[-] G. Raeisi, see also [GyRa, KamRa, MaORS1, MaORS2, OmRa1, OmRa2, OmRa3, OmRR].

[-] Z. Rahimi, see [OmRR].

[-] J.L. Ramirez Alfonsin, see [ChaMR, MonCR].

[Ram] F.P. Ramsey, On a Problem of Formal Logic, *Proceedings of the London Mathematical Society,* **30** (1930) 264-286.

[RanMCG]* M. Ranjbar, W. Macready, L. Clark and F. Gaitan, Generalized Ramsey Numbers through Adiabatic Quantum Optimization, *Quantum Information Processing,* **15** (2016) 3519-3542.

[-] A. Rao, see [BarRSW].

[Rao]* S. Rao, Applying a Genetic Algorithm to Improve the Lower Bounds of Multi-Color Ramsey Numbers, *MS thesis,* Department of Computer Science, Rochester Institute of Technology, 1997.

[-] A. Rapp, see [BudHR1, BudHR2].

[ReWi] R.C. Read and R.J. Wilson, *An Atlas of Graphs,* Clarendon Press, Oxford, 1998.

[-] G. Resta, see [CoPR].

[-] M.P. Revuelta, see [BoCGR].

[-] S.W. Reyner, see [BurR].

[-] D.F. Reynolds, see [ExRe].

Ro - Ru

[Rob] B. Roberts, Ramsey Numbers of Connected Clique Matchings, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P1.36, **24** (2017), 7 pages.

[-] B. Roberts, see also [DavJR].

[Rob1] F.S. Roberts, *Applied Combinatorics,* Prentice-Hall, Englewood Cliffs, 1984.

[-] J.A. Roberts, see [BuRo1, BuRo2].

[-] S. Roberts, see [GR].

[Rob2]* A. Robertson, New Lower Bounds for Some Multicolored Ramsey Numbers, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #R12, **6** (1999), 6 pages.

[Rob3]* A. Robertson, Difference Ramsey Numbers and Issai Numbers, *Advances in Applied Mathematics,* **25** (2000) 153-162.

[Rob4] A. Robertson, New Lower Bounds Formulas for Multicolored Ramsey Numbers, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #R13, **9** (2002), 6 pages.

[-] A. Robertson, see also [LaRo].

[-] Y. Roditty, see [KrRod].

[RöTh] V. Rödl and R. Thomas, Arrangeability and Clique Subdivisions, in *The Mathematics of Paul Erdős II,* 236-239, Algorithms and Combinatorics **14**, Springer, Berlin, 1997.

[-] V. Rödl, see also [AlRö, ChRST, ConFR, DuLR, GrRö, GRR1, GRR2, HaŁP1+, HaŁP2+, KosPR, KoRö1, KoRö2, KoRö3, MuR, NaORS, PoRRS].

[-] L. Rónyai, see [AlRóS].

[Ros1] V. Rosta, On a Ramsey Type Problem of J.A. Bondy and P. Erdős, I & II, *Journal of Combinatorial Theory,* Series B, **15** (1973) 94-120.

[Ros2] V. Rosta, Ramsey Theory Applications, Dynamic Survey in *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #DS13, (2004), 43 pages.

[-] V. Rosta, see also [BuRo3, KáRos].

[-] B.L. Rothschild, see [GRS].

[Rou] C.C. Rousseau, *personal communication* (2006).

[RoJa1] C.C. Rousseau and C.J. Jayawardene, The Ramsey Number for a Quadrilateral vs. a Complete Graph on Six Vertices, *Congressus Numerantium,* **123** (1997) 97-108.

[RoJa2] C.C. Rousseau and C.J. Jayawardene, Harary's Problem for $K_{2,k}$, *manuscript* (1999).

[RoS1] C.C. Rousseau and J. Sheehan, On Ramsey Numbers for Books, *Journal of Graph Theory,* **2** (1978) 77-87.

[RoS2] C.C. Rousseau and J. Sheehan, A Class of Ramsey Problems Involving Trees, *Journal of the London Mathematical Society (2),* **18** (1978) 392-396.

[-] C.C. Rousseau, see also [BolJY+, BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRSG, BFRSJ, CaLRZ, ChRSPS, EFRS1, EFRS2, EFRS3, EFRS4, EFRS5, EFRS6, EFRS7, EFRS8, EFRS9, FRS1, FRS2, FRS3, FRS4, FRS5, FRS6, FRS7, FRS8, FRS9, FSR, JR1, JR2, JR3, JR4, JR5, JRB, LiR1, LiR2, LiR3, LiR4, LiRS, LiRZ1, LiRZ2, NiRo1, NiRo2, NiRo3, NiRo4, NiRS].

[-] C. Rowan, see [KerRo].

[Row1]* F. Rowley, Constructive Lower Bounds for Ramsey Numbers from Linear Graphs, *Australasian Journal of Combinatorics,* **68** (2017) 385-395.

[Row2]* F. Rowley, Some Further Results in Ramsey Graph Construction, *Australasian Journal of Combinatorics,* **78** (2020) 1-10.

[Row3]* F. Rowley, A Generalised Linear Ramsey Graph Construction, *preprint,* arXiv, http://arxiv.org/abs/1912.01164 (2020).

[-] P. Rowlinson, see [YR1, YR2, YR3].

[Rub] M. Rubey, Technische Universität Wien, an electronic resource for values of small Ramsey numbers, http://www.findstat.org/StatisticsDatabase/St000479, 2016.

[-] A. Ruciński, see [HanPR, JacPR, GRR1, GRR2, HaŁP1+, HaŁP2+, ŁuPR, PoRRS, PoRu].

[-] M. Ruszinkó, see [GyRSS].

Sa - Se

[Sah] Ashwin Sah, Diagonal Ramsey via Effective Quasirandomness, *preprint,* arXiv, http://arxiv.org/abs/2005.09251 (2020).

[-] M. Salerno, see [JiSa].

[SaTWZ] N. Salia, C. Tompkins, Zhiyu Wang and O. Zamora, Ramsey Numbers of Berge-Hypergraphs and Related Structures, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P4.40, **26**(4) (2019), 25 pages.

[SaBr1] A.N.M. Salman and H.J. Broersma, The Ramsey Numbers of Paths versus Kipases, *Electronic Notes in Discrete Mathematics,* **17** (2004) 251-255.

[SaBr2] A.N.M. Salman and H.J. Broersma, Paths-Fan Ramsey Numbers, *Discrete Applied Mathematics,* **154** (2006) 1429-1436.

[SaBr3] A.N.M. Salman and H.J. Broersma, The Ramsey Numbers for Paths versus Wheels, *Discrete Mathematics,* **307** (2007) 975-982.

[SaBr4] A.N.M. Salman and H.J. Broersma, Path-Kipas Ramsey Numbers, *Discrete Applied Mathematics,* **155** (2007) 1878-1884.

[-] A.N.M. Salman, see also [HaABS].

[Sán] A. Sánchez-Flores, An Improved Bound for Ramsey Number $N(3,3,3,3;2)$, *Discrete Mathematics,* **140** (1995) 281-286.

[-] C. Sanford, see [BudHLS].

[Sanh] N. Sanhueza-Matamala, Stability and Ramsey Numbers for Cycles and Wheels, *Discrete Mathematics,* **339** (2016) 1557-1565.

[Sár1] G.N. Sárközy, Monochromatic Cycle Partitions of Edge-Colored Graphs, *Journal of Graph Theory,* **66** (2011) 57-64.

[Sár2] G.N. Sárközy, On the Multi-Colored Ramsey Numbers of Paths and Even Cycles, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P3.53, **23**(3) (2016), 9 pages.

[-] G.N. Sárközy, see also [DeBGS, GyLSS, GyRSS, GySá1, GySá2, GySá3, GySS1, GySS2, MoSST].

[-] I. Sato, see [MiSa].

[-] D. Saxton, see [FizGMSS, GrMFSS].

[-] M. Schacht, see [MoSST, NaORS].

[Scha] M. Schaefer, Graph Ramsey Theory and the Polynomial Hierarchy, *Journal of Computer and System Sciences,* **62** (2001) 290-322.

[-] R.H. Schelp, see [BaLS, BaSS, BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRSGJ, BEFS, BFRS, ChenS, EFRS1, EFRS2, EFRS3, EFRS4, EFRS5, EFRS6, EFRS7, EFRS8, EFRS9, FLPS, FRS1,

FRS2, FRS3, FRS4, FRS5, FRS6, FS1, FS2, FS3, FS4, FSR, FSS1, FSS2, GyLSS, NiRS].

[SchSch1]* A. Schelten and I. Schiermeyer, Ramsey Numbers $r(K_3, G)$ for Connected Graphs G of Order Seven, *Discrete Applied Mathematics,* **79** (1997) 189-200.

[SchSch2] A. Schelten and I. Schiermeyer, Ramsey Numbers $r(K_3, G)$ for $G \cong K_7 - 2P_2$ and $G \cong K_7 - 3P_2$, *Discrete Mathematics,* **191** (1998) 191-196.

[-] A. Schelten, see also [FSS3].

[Schi1] I. Schiermeyer, All Cycle-Complete Graph Ramsey Numbers $r(C_m, K_6)$, *Journal of Graph Theory,* **44** (2003) 251-260.

[Schi2] I. Schiermeyer, The Cycle-Complete Graph Ramsey Number $r(C_5, K_7)$, *Discussiones Mathematicae Graph Theory,* **25** (2005) 129-139.

[-] I. Schiermeyer, see also [FSS3, LiSch, MaoWMS, SchSch1, SchSch2].

[-] J.C. Schlage-Puchta, see [BrGS].

[-] A. Schneider, see [AlmHS].

[-] J. Schönheim, see [BiaS].

[Schu] C.-U. Schulte, Ramsey-Zahlen für Bäume und Kreise, *Ph.D. thesis,* Heinrich-Heine-Universität Düsseldorf, (1992).

[-] M.J. Schuster, see [CalSR].

[-] S. Schuster, see [ChaS].

[-] A. Schwenk, see [ChvS].

[Scob] M.W. Scobee, On the Ramsey Number $R(m_1P_3, m_2P_3, m_3P_3)$ and Related Results, ..., *MA thesis,* University of Louisville (1993).

[-] A. Sebő, see [GySeT].

[-] R.J. Segedin, see [LorSe].

Sh

[Shah] M. Shahsiah, Ramsey Numbers of 5-Uniform Loose Cycles, *preprint,* arXiv, http://arxiv.org/abs/1806.07720 (2018).

[-] M. Shahsiah, see also [MaORS1, MaORS2, OmSh1, OmSh2, OmSh3, OmSh4].

[-] R. Shaltiel, see [BarRSW].

[Shao]* Zehui Shao, *personal communication* (2008).

[ShaoWX]* Shao Zehui, Wang Zicheng and Xiao Jianhua, Lower Bounds for Ramsey Numbers Based on Simulated Annealing Algorithm (in Chinese), *Computer Engineering and Applications,* **45** (2009) 70-71.

[ShaXBP]* Zehui Shao, Jin Xu, Qiquan Bao and Linqiang Pan, Computation of Some Generalized Ramsey Numbers, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **75** (2010) 217-228.

[ShaXB]* Zehui Shao, Xiaodong Xu and Qiquan Bao, On the Ramsey Numbers $R(C_m, B_n)$, *Ars Combinatoria,* **94** (2010) 265-271.

[ShaXSP]* Zehui Shao, Xiaodong Xu, Xiaolong Shi and Linqiang Pan, Some Three-Color Ramsey Numbers $R(P_4, P_5, C_k)$ and $R(P_4, P_6, C_k)$, *European Journal of Combinatorics,* **30** (2009) 396-403.

[-] Zehui Shao, see also [SunSh, XSR1, XSR2].

[Shas] A. Shastri, Lower Bounds for Bi-Colored Quaternary Ramsey Numbers, *Discrete Mathematics,* **84** (1990) 213-216.

[She1] J.B. Shearer, A Note on the Independence Number of Triangle-Free Graphs, *Discrete Mathematics,* **46** (1983) 83-87.

[She2]* J.B. Shearer, Lower Bounds for Small Diagonal Ramsey Numbers, *Journal of Combinatorial Theory,* Series A, **42** (1986) 302-304.

[She3] J.B. Shearer, A Note on the Independence Number of Triangle-Free Graphs II, *Journal of Combinatorial Theory,* Series B, **53** (1991) 300-307.

[She4]* J.B. Shearer, Independence Numbers of Paley Graphs (data for primes 1 mod 4 up to 7000), http://www.research.ibm.com/people/s/shearer/indpal.html (1996).

[-] J. Sheehan, see [ChRSPS, ClEHMS, FRS7, FRS8, FRS9, FSS1, RoS1, RoS2].

[-] Jian Shen, see [LiShen, LinLS].

[-] Shen Yun-Qiu, see [LuSS1, LuSS2].

[-] Sheng Wancheng, see [HWSYZH].

[ShWR]* D. Shetler, M. Wurtz and S.P. Radziszowski, On Some Multicolor Ramsey Numbers Involving K_3+e and K_4-e, *SIAM Journal on Discrete Mathematics,* **26** (2012) 1256-1264.

[-] Shi Lei, see [SunYJLS].

[Shi1] Lingsheng Shi, Cube Ramsey Numbers Are Polynomial, *Random Structures and Algorithms,* **19** (2001) 99--101.

[Shi2] Lingsheng Shi, Upper Bounds for Ramsey Numbers, *Discrete Mathematics,* **270** (2003) 251-265.

[Shi3] Lingsheng Shi, Linear Ramsey Numbers of Sparse Graphs, *Journal of Graph Theory,* **50** (2005) 175-185.

[Shi4] Lingsheng Shi, The Tail Is Cut for Ramsey Numbers of Cubes, *Discrete Mathematics,* **307** (2007) 290-292.

[Shi5] Lingsheng Shi, Ramsey Numbers of Long Cycles versus Books or Wheels, *European Journal of Combinatorics,* **31** (2010) 828-838.

[ShZ1] Shi Ling Sheng and Zhang Ke Min, An Upper Bound Formula for Ramsey Numbers, *manuscript* (2001).

[ShZ2] Shi Ling Sheng and Zhang Ke Min, A Sequence of Formulas for Ramsey Numbers, *manuscript* (2001).

[-] Xiaolong Shi, see [ShaXSP].

[ShiuLL] Shiu Wai Chee, Peter Che Bor Lam and Li Yusheng, On Some Three-Color Ramsey Numbers, *Graphs and Combinatorics,* **19** (2003) 249-258.

Si - St

[Sid1] A.F. Sidorenko, On Turán Numbers $T(n, 5, 4)$ and Number of Monochromatic 4-cliques in 2-colored 3-graphs (in Russian), *Voprosy Kibernetiki,* **64** (1980) 117-124.

[Sid2] A.F. Sidorenko, An Upper Bound on the Ramsey Number $R(K_3, G)$ Depending Only on the Size of the Graph G, *Journal of Graph Theory,* **15** (1991) 15-17.

[Sid3] A.F. Sidorenko, The Ramsey Number of an N-Edge Graph versus Triangle Is at Most $2N+1$, *Journal of Combinatorial Theory,* Series B, **58** (1993) 185-196.

[-] M. Simonovits, see [AjKSS, BaSS, FSS2, FaSi, HaŁP1+, KoSS1, KoSS2, ŁucSS].

[-] J. Skokan, see [AllBS, BalCSW, BenSk, FizGMSS, GrMFSS, HaŁP1+, HaŁP2+, JenSk, KeeLS, KoSS1, KoSS2, ŁucSS].

[-] M.J. Smuga-Otto, see [AbbS].

[Sob] A. Sobczyk, Euclidian Simplices and the Ramsey Number $R(4,4;3)$, *Technical Report #10, Clemson University* (1967).

[Soi1] A. Soifer, *The Mathematical Coloring Book, Mathematics of coloring and the colorful life of its creators,* Springer 2009.

[Soi2] A. Soifer, *Ramsey Theory: Yesterday, Today and Tomorrow,* Progress in Mathematics 285, Springer-Birkhauser 2011.

[-] W. Solomon, see [LorSo].

[-] L. Soltés, see [LiRS].

[Song1] Song En Min, Study of Some Ramsey Numbers (in Chinese), a note (announcement of results without proofs), *Mathematica Applicata,* **4**(2) (1991) 6.

[Song2] Song En Min, New Lower Bound Formulas for the Ramsey Numbers $N(k,k,...,k;2)$ (in Chinese), *Mathematica Applicata,* **6** (1993) suppl., 113-116.

[Song3] Song En Min, An Investigation of Properties of Ramsey Numbers (in Chinese), *Mathematica Applicata,* **7** (1994) 216-221.

[Song4] Song En Min, Properties and New Lower Bounds of the Ramsey Numbers $R(p, q;4)$ (in Chinese), *Journal of Huazhong University of Science and Technology,* **23** (1995) suppl. II, 1-4.

[SonYL] Song En Min, Ye Weiguo and Liu Yanwu, New Lower Bounds for Ramsey Number $R(p, q;4)$, *Discrete Mathematics,* **145** (1995) 343-346.

[-] Song En Min, see also [HuSo, ZLLS].

[Song5] Song Hongxue, Asymptotic Upper Bounds for Wheel-Complete Graph Ramsey Numbers, *Journal of Southeast University* (English Edition), ISSN 1003-7985, **20** (2004) 126-129.

[Song6] Song Hongxue, A Ramsey Goodness Result for Graphs with Large Pendent Trees, *Journal of Mathematical Study (China),* **42** (2009) 36-39.

[Song7] Song Hong-xue, Asymptotic Upper Bounds for $K_2 + T_m$: Complete Graph Ramsey Numbers, *Journal of Mathematics (China),* **30** (2010) 797-802.

[Song8] Song Hongxue, Asymptotic Lower Bounds of Ramsey Numbers for r-Uniform Hypergraphs, *Advances in Mathematics, Shuxue Jinzhan,* **40** (2011) 179-186.

[Song9] Hongxue Song, Asymptotic Upper Bounds for $K_{1,m,k}$: Complete Graph Ramsey Numbers, *Ars Combinatoria,* **111** (2013) 137-144.

[SonBL] Song Hong Xue, Bai Lu Feng and Liu Shu Yan, Asymptotic Upper Bounds for the Wheel-Complete Graph Ramsey Numbers (in Chinese), *Acta Mathematica Scientia, Series A,* ISSN 1003-3998, **26** (2006) 741-746.

[SonGQ] Song Hongxue, Gu Hua and Qian Xinjin, On the Ramsey Number of K_3 versus K_2+T_n (in Chinese), *Journal of Liaoning Normal University, Natural Science Edition,* ISSN 1000-1735, **27** (2004) 142-145.

[SonLi] Song Hongxue and Li Yusheng, Asymptotic Lower Bounds of Ramsey Numbers for 4-Uniform Hypergraphs (in Chinese), *Journal of Nanjing University Mathematical Biquarterly,* **26** (2009) 216-224.

[-] Song Hongxue, see also [GuSL].

[Spe1] J.H. Spencer, Ramsey's Theorem - A New Lower Bound, *Journal of Combinatorial Theory,* Series A, **18** (1975) 108-115.

[Spe2] J.H. Spencer, Asymptotic Lower Bounds for Ramsey Functions, *Discrete Mathematics,* **20** (1977) 69-76.

[Spe3] J.H. Spencer, Eighty Years of Ramsey $R(3, k)$ ... and Counting! in *Ramsey Theory: Yesterday, Today and Tomorrow* (ed. A. Soifer), Progress in Mathematics 285, Springer-Birkhauser 2011, 27-39.

[-] J.H. Spencer, see also [BES, GRS].

[-] T.S. Spencer, see [BahS].

[Spe4]* T. Spencer, University of Nebraska at Omaha, *personal communication* (1993), and, Upper Bounds for Ramsey Numbers via Linear Programming, *manuscript* (1994).

[-] A.K. Srivastava, see [GauST].

[Stahl] S. Stahl, On the Ramsey Number $R(F, K_m)$ where F is a Forest, *Canadian Journal of Mathematics,* **27** (1975) 585-589.

[-] R.G. Stanton, see [KaSt].

[Stat] W. Staton, Some Ramsey-type Numbers and the Independence Ratio, *Transactions of the American Mathematical Society,* **256** (1979) 353-370.

[-] A. Steger, see [McS].

[-] J. Stinehour, see [RaST].

[Stev] S. Stevens, Ramsey Numbers for Stars versus Complete Multipartite Graphs, *Congressus Numerantium,* **73** (1990) 63-71.

[-] M.J. Stewart, see [ChRSPS].

[Stone] J.C. Stone, Utilizing a Cancellation Algorithm to Improve the Bounds of $R(5, 5)$, (1996). This paper claimed incorrectly that $R(5,5) = 50$.

Su - Suk

[-] Pascal Su, see [KniSu].

[SuL]* Su Wenlong and Luo Haipeng, Prime Order Cyclic Graphs and New Lower Bounds for Three Classical Ramsey Numbers $R(4, n)$ (in Chinese), *Journal of Mathematical Study,* **31**, 4 (1998) 442-446.

[SuLL]* Su Wenlong, Luo Haipeng and Li Qiao, New Lower Bounds of Classical Ramsey Numbers $R(4,12)$, $R(5,11)$ and $R(5,12)$, *Chinese Science Bulletin,* **43**, 6 (1998) 528.

[SLLL]* Su Wenlong, Luo Haipeng, Li Guiqing and Li Qiao, Lower Bounds of Ramsey Numbers Based on Cubic Residues, *Discrete Mathematics,* **250** (2002) 197-209.

[SLZL]* Su Wenlong, Luo Haipeng, Zhang Zhengyou and Li Guiqing, New Lower Bounds of Fifteen Classical Ramsey Numbers, *Australasian Journal of Combinatorics,* **19** (1999) 91-99.

[-] Su Wenlong, see also [LiaWXCS, LiaWXS, LuSL, LiSLW, LuSS1, LuSS2, WSLX1, WSLX2, XWCS].

[Sud1] B. Sudakov, A Note on Odd Cycle-Complete Graph Ramsey Numbers, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #N1, **9** (2002), 4 pages.

[Sud2] B. Sudakov, Large K_r-Free Subgraphs in K_s-Free Graphs and Some Other Ramsey-Type Problems, *Random Structures and Algorithms,* **26** (2005) 253-265.

[Sud3] B. Sudakov, Ramsey Numbers and the Size of Graphs, *SIAM Journal on Discrete Mathematics,* **21** (2007) 980-986.

[Sud4] B. Sudakov, A Conjecture of Erdős on Graph Ramsey Numbers, *Advances in Mathematics,* **227** (2011) 601-609

[-] B. Sudakov, see also [AlKS, BalPS, ConFLS, ConFS1, ConFS2, ConFS3, ConFS4, ConFS5, ConFS6, ConFS7, ConFS8, FoxSu1, FoxSu2, KoSu, PoSu1, PoSu2].

[-] A. Sudan, see [GuGS].

[Sudar1] I.W. Sudarsana, The Goodness of Long Path with Respect to Multiple Copies of Small Wheel, *Far East Journal of Mathematical Sciences,* **59** (2011) 47-55.

[Sudar2] I.W. Sudarsana, The Goodness of Long Path with Respect to Multiple Copies of Complete Graphs, *Journal of the Indonesian Mathematical Society,* **20** (2014) 31-35.

[Sudar3] I.W. Sudarsana, The Goodness of Path or Cycle with Respect to Multiple Copies of Complete Graphs of Order Three, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **126** (2016) 359-367.

[SuAM] I.W. Sudarsana, Adiwijaya and S. Musdalifah, The Ramsey Number for a Linear Forest versus Two Identical Copies of Complete Graphs, COCOON 2010, *LNCS* 6196, Springer, Berlin (2010) 209-215.

[SuAAM] I.W. Sudarsana, H. Assiyatun, Adiwijaya and S. Musdalifah, The Ramsey Number for a Linear Forest versus Two Identical Copies of Complete Graph, *Discrete Mathematics, Algorithms and Applications,* **2** (2010) 437-444.

[SuAUB] I.W. Sudarsana, H. Assiyatun, S. Uttunggadewa and E.T. Baskoro, On the Ramsey Numbers $R(S_{2,m}, K_{2,q})$ and $R(sK_2, K_s+C_n)$, *Ars Combinatoria,* **119** (2015) 235-246.

[SuBAU1] I.W. Sudarsana, E.T. Baskoro, H. Assiyatun and S. Uttunggadewa, The Ramsey Number of a Certain Forest with Respect to a Small Wheel, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **71** (2009) 257-264.

[SuBAU2] I.W. Sudarsana, E.T. Baskoro, H. Assiyatun and S. Uttunggadewa, The Ramsey Numbers of Linear Forest versus $3K_3 \bigcup 2K_4$, *Journal of the Indonesian Mathematical Society,* **15** (2009) 61-67.

[SuBAU3] I.W. Sudarsana, E.T. Baskoro, H. Assiyatun and S. Uttunggadewa, The Ramsey Numbers for the Union Graph with H-Good Components, *Far East Journal of Mathematical Sciences,* **39** (2010) 29-40.

[-] A. Suk, see [FoxPS1, FoxPS2, MuSuk1, MuSuk2, MuSuk3, MuSuk4].

Sun - Sz

[SunSh]** Minhong Sun and Zehui Shao, Exact Values of Some Generalized Ramsey Numbers, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **107** (2018) 277-283.

[Sun]* Sun Yongqi, Research on Ramsey Numbers of Some Graphs (in Chinese), *Ph.D. thesis,* Dalian University of Technology, China, July 2006.

[SunY]* Sun Yongqi and Yang Yuansheng, Study of the Three Color Ramsey Number $R_3(C_8)$ (in Chinese), *Journal of Beijing Jiaotong University,* **35** (2011) 14-17.

[SunYJLS] Sun Yongqi, Yang Yuansheng, Jiang Baoqi, Lin Xiaohui and Shi Lei, On Multicolor Ramsey Numbers for Even Cycles in Graphs, *Ars Combinatoria,* **84** (2007) 333-343.

[SunYLZ1]* Sun Yongqi, Yang Yuansheng, Lin Xiaohui and Zheng Wenping, The Value of the Ramsey Number $R_4(C_4)$, *Utilitas Mathematica,* **73** (2007) 33-44.

[SunYLZ2]* Sun Yongqi, Yang Yuansheng, Lin Xiaohui and Zheng Wenping, On the Three Color Ramsey Numbers $R(C_m, C_4, C_4)$, *Ars Combinatoria,* **84** (2007) 3-11.

[SunYW]* Sun Yongqi, Yang Yuansheng and Wang Zhihai, The Value of the Ramsey Number $R_5(C_6)$, *Utilitas Mathematica,* **76** (2008) 25-31.

[SunYWLX]* Sun Yongqi, Yang Yuansheng, Wang Wei, Li Bingxi and Xu Feng, Study of Three Color Ramsey numbers $R(C_{m_1}, C_{m_2}, C_{m_3})$ (in Chinese), *Journal of Dalian University of Technology,* ISSN 1000-8608, **46** (2006) 428-433.

[SunYXL] Sun Yongqi, Yang Yuansheng, Xu Feng and Li Bingxi, New Lower Bounds on the Multicolor Ramsey Numbers $R_r(C_{2m})$, *Graphs and Combinatorics,* **22** (2006) 283-288.

[-] Sun Yongqi, see also [JiLSX, WuSL, WuSR, WuSZR, ZhaSW, ZhuSWZ].

[-] Yue Ru Sun, see [NoSZ].

[SunLi] Sun Yuqin and Li Yusheng, On an Upper Bound of Ramsey Number $r_k(K_{m,n})$ with Large n, *Heilongjiang Daxue Ziran Kexue Xuebao,* ISSN 1001-7011, **23** (2006) 668-670.

[SunZ1] Zhi-Hong Sun, Ramsey Numbers for Trees, *Bulletin of the Australian Mathematical Society,* **86** (2012) 164-176.

[SunZ2] Zhi-Hong Sun, Ramsey Numbers for Trees II, *preprint,* arXiv, http://arxiv.org/abs/1410.7637 (2014). Revised version (2016).

[SunW] Zhi-Hong Sun and Lin-Lin Wang, Turán's Problem for Trees, *Journal of Combinatorics and Number Theory,* **3** (2011) 51-69.

[SunWW] Zhi-Hong Sun, Lin-Lin Wang and Yi-Li Wu, Turán's Problem and Ramsey Numbers for Trees, *Colloquium Mathematicum,* **139** (2015) 273-298.

[Sur] Surahmat, Cycle-Wheel Ramsey Numbers. Some results, open problems and conjectures. *Math Track,* ISSN 1817-3462, 1818-5495, **2** (2006) 56-64.

[SuBa1] Surahmat and E.T. Baskoro, On the Ramsey Number of a Path or a Star versus W_4 or W_5, *Proceedings of the 12-th Australasian Workshop on Combinatorial Algorithms,* Bandung, Indonesia, July 14-17 (2001) 174-179.

[SuBa2] Surahmat and E.T. Baskoro, The Ramsey Number of Linear Forest versus Wheel, paper presented at the *13-th Australasian Workshop on Combinatorial Algorithms,* Fraser Island, Queensland, Australia, July 7-10, 2002.

[SuBB1] Surahmat, E.T. Baskoro and H.J. Broersma, The Ramsey Numbers of Large Star-like Trees versus Large Odd Wheels, *Technical Report* #1621, Faculty of Mathematical Sciences, University of Twente, The Netherlands, (2002).

[SuBB2] Surahmat, E.T. Baskoro and H.J. Broersma, The Ramsey Numbers of Large Cycles versus Small Wheels, *Integers: Electronic Journal of Combinatorial Number Theory,* http://www.integers-ejcnt.org/vol4.html, #A10, **4** (2004), 9 pages.

[SuBB3] Surahmat, E.T. Baskoro and H.J. Broersma, The Ramsey Numbers of Fans versus K_4, *Bulletin of the Institute of Combinatorics and its Applications,* **43** (2005) 96-102.

[SuBB4] Surahmat, E.T. Baskoro and H.J. Broersma, The Ramsey Numbers of Large Star and Large Star-Like Trees versus Odd Wheels, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **65** (2008) 153-162.

[SuBT1] Surahmat, E.T. Baskoro and I. Tomescu, The Ramsey Numbers of Large Cycles versus Wheels, *Discrete Mathematics,* **306** (2006), 3334-3337.

[SuBT2] Surahmat, E.T. Baskoro and I. Tomescu, The Ramsey Numbers of Large Cycles versus Odd Wheels, *Graphs and Combinatorics,* **24** (2008), 53-58.

[SuBTB] Surahmat, E.T. Baskoro, I. Tomescu and H.J. Broersma, On Ramsey Numbers of Cycles with Respect to Generalized Even Wheels, *manuscript* (2006).

[SuBUB] Surahmat, E.T. Baskoro, S. Uttunggadewa and H.J. Broersma, An Upper Bound for the Ramsey Number of a Cycle of Length Four versus Wheels, in *LNCS* 3330, Springer, Berlin (2005) 181-184.

[SuTo] Surahmat and I. Tomescu, On Path-Jahangir Ramsey Numbers, *Applied Mathematical Sciences,* **8**(99) (2014) 4899-4904.

[-] Surahmat, see also [AliSur, BaSu, BaSNM].

[-] Y. Suzuki, see [KaOS].

[SwPr] C.J. Swanepoel and L.M. Pretorius, Upper Bounds for a Ramsey Theorem for Trees, *Graphs and Combinatorics,* **10** (1994) 377-382.

[-] M.M. Sweet, see [FreSw].

[-] T. Szabó, see [AlRóS].

[Szem] E. Szemerédi, Regular Partitions of Graphs, Problèmes Combinatoires et Théorie des Graphes (Orsay, 1976), Colloques Internationaux du Centre National de la Recherche Scientifique, CNRS Paris, **260** (1978) 399--401.

[-] E. Szemerédi, see also [AjKS, AjKSS, ChRST, GyRSS, GySS1, GySS2, PoRRS].

[-] P. Szuca, see [DzNS].

T

[-] Fuping Tan, see [HTHZ1, HTHZ2].

[-] Tang Xueqing, see [LiTZ].

[-] A. Taraz, see [MoSST].

[Tat]* M. Tatarevic, *personal communication,* graph constructions for lower bounds on Ramsey numbers at http://github.com/milostatarevic/ramsey-numbers/tree/master/graphs (2020).

[-] M. Tatarevic, see also [ExT].

[-] Yanmei Teng, see [JiLTX1, JiLTX2].

[-] R. Thomas, see [RöTh].

[Tho] A. Thomason, An Upper Bound for Some Ramsey Numbers, *Journal of Graph Theory,* **12** (1988) 509-517.

[-] P.W. Tingley, see [HaŁT].

[-] I. Tomescu, see [AliBT1, AliBT2, AliTJ, SuBT1, SuBT2, SuBTB, SuTo].

[-] C. Tompkins, see [SaTWZ].

[-] C.A. Tovey, see [CaET].

[Tri] A. Tripathi, An Introduction to Ramsey's Theorem, *Proceedings of Telangana Academy of Sciences,* **1** (2020) 202-213.

[-] A. Tripathi, see also [GauST].

[Tr] Trivial results.

[-] N. Trotignon, see [GySeT].

[-] W.T. Trotter Jr., see [ChRST].

[Tse1]* Kung-Kuen Tse, On the Ramsey Number of the Quadrilateral versus the Book and the Wheel, *Australasian Journal of Combinatorics,* **27** (2003) 163-167.

[Tse2]* Kung-Kuen Tse, A Note on the Ramsey Numbers $R(C_4, B_n)$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **58** (2006) 97-100.

[Tse3]* Kung-Kuen Tse, A Note on Some Ramsey Numbers $R(C_p, C_q, C_r)$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **62** (2007) 189-192.

[-] Kung-Kuen Tse, see also [BaRT, RaST, RaT].

[-] Z. Tuza, see [GyTu].

U

[UCSD] University of California San Diego, Fan Chung and a team of students, *Erdős' Problems on Graphs, Ramsey Theory,* http://www.math.ucsd.edu/~erdosproblems/RamseyTheory.html (2010-2012).

[-] S. Uttunggadewa, see [SuAUB, SuBAU1, SuBAU2, SuBAU3, SuBUB].

V

[VO]** Steven Van Overberghe, Algorithms for Computing Ramsey Numbers, *MS Thesis in Mathematics,* Ghent University, Belgium, 2020. Constructions at https://github.com/Steven-VO/circulant-Ramsey.

[-] J. Verstraëte, see [KosMV1, KosMV2].

[-] T. Vetrik, see [JaBVR].

[-] M. Vizer, see [GerMOV].

[-] L. Volkmann, see [GuoV].

W

[-] A.Z. Wagner, see [BalCSW].

[Walk] K. Walker, Dichromatic Graphs and Ramsey Numbers, *Journal of Combinatorial Theory,* **5** (1968) 238-243.

[Wall] W.D. Wallis, On a Ramsey Number for Paths, *Journal of Combinatorics, Information & System Sciences,* **6** (1981) 295-296.

[Wan] Wan Honghui, Upper Bounds for Ramsey Numbers $R(3, 3, \cdots, 3)$ and Schur Numbers, *Journal of Graph Theory,* **26** (1997) 119-122.

[-] Wang Gongben, see [WW, WWY1, WWY2].

[-] Lin-Lin Wang, see [SunW, SunWW].

[Wang] Longqin Wang, Some Multi-Color Ramsey Numbers on Stars versus Path, Cycle or Wheel, *Graphs and Combinatorics,* **36** (2020) 515-524.

[WaCh1] Longqin Wang and Yaojun Chen, The Ramsey Numbers of Trees versus Generalized Wheels, *Graphs and Combinatorics,* **35** (2019) 189-193.

[WaCh2] Longqin Wang and Yaojun Chen, The Ramsey Numbers of Two Sets of Cycles, *Journal of Graph Theory,* **96** (2021) 129-136, https://doi.org/10.1002/jgt.22564 (2020).

[WW]* Wang Qingxian and Wang Gongben, New Lower Bounds of Ramsey Numbers $r(3, q)$ (in Chinese), *Acta Scientiarum Naturalium, Universitatis Pekinensis,* **25** (1989) 117-121. The lower bounds presented in this paper have been improved.

[WWY1]* Wang Qingxian, Wang Gongben and Yan Shuda, A Search Algorithm And New Lower Bounds for Ramsey Numbers $r(3, q)$, *manuscript* (1994).

[WWY2]* Wang Qingxian, Wang Gongben and Yan Shuda, The Ramsey Numbers $R(K_3, K_q - e)$ (in Chinese), *Beijing Daxue Xuebao Ziran Kexue Ban,* **34** (1998) 15-20.

[-] Wang Wei, see [SunYWLX, SunYXL].

[WaLi] Ye Wang and Yusheng Li, Lower Bounds for Ramsey Numbers of K_n with a Small Subgraph Removed, *Discrete Applied Mathematics,* **160** (2012) 2063-2068.

[-] Zhao Wang, see [MaoWMS].

[-] Wang Yuandi, see [HWSYZH].

[-] Wang Zhihai, see [SunYW].

[-] Wang Zhi Jian, see [LiWa1, LiWa2].

[-] Zhiyu Wang, see [SaTWZ].

[-] Wang Zicheng, see [ShaoWX].

[West] D. West, *Introduction to Graph Theory,* second edition, Prentice Hall, 2001.

[Wh] E.G. Whitehead, The Ramsey Number $N(3,3,3,3;2)$, *Discrete Mathematics,* **4** (1973) 389-396.

[-] A. Wigderson, see [BarRSW].

[Wig] Y. Wigderson, An Improved Lower Bound on Multicolor Ramsey Numbers, *preprint,* arXiv, http://arxiv.org/abs/2009.12020 (2020).

[-] Y. Wigderson, see also [HeWi, ConFW].

[-] E.R. Williams, see [AbbW].

[-] R.J. Wilson, see [ReWi].

[-] R.M. Wilson, see [FraWi].

[-] A. Woldar, see [LaWo1, LaWo2].

[WSLX1]* Kang Wu, Wenlong Su, Haipeng Luo and Xiaodong Xu, New Lower Bound for Seven Classical Ramsey Numbers $R(3, q)$, *Applied Mathematics Letters,* **22** (2009) 365-368.

[WSLX2]* Kang Wu, Wenlong Su, Haipeng Luo and Xiaodong Xu, A Generalization of Generalized Paley Graphs and New Lower Bounds for $R(3, q)$, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #N25, **17** (2010), 10 pages.

[-] Wu Kang, see also [LiaWXCS, LiaWXS, LiSLW, XWCS].

[WuSL] Yali Wu, Yongqi Sun and Zhiguo Liu, The Ramsey Numbers $R(C_{\leq n}, K_m)$, *Ars Combinatoria,* **131** (2017) 227-237.

[WuSR] Wu Yali, Sun Yongqi and S.P. Radziszowski, Wheel and Star-Critical Ramsey Numbers for Quadrilateral, *Discrete Applied Mathematics,* **186** (2015) 260-271.

[WuSZR] Wu Yali, Sun Yongqi, Zhang Rui and S.P. Radziszowski, Ramsey Numbers of C_4 versus Wheels and Stars, *Graphs and Combinatorics,* **31** (2015) 2437-2446.

[-] Wu Yali, see also [ZhaSW, ZhuSWZ].

[-] Yi-Li Wu, see [SunWW].

[-] M. Wurtz, see [ShWR].

X

[-] Xiao Jianhua, see [ShaoWX].

[XieZ]* Xie Jiguo and Zhang Xiaoxian, A New Lower Bound for Ramsey Number $r(3, 13)$ (in Chinese), *Journal of Lanzhou Railway Institute,* **12** (1993) 87-89.

[-] Xie Zheng, see [XuX1, XuX2, XuXC, XXER, XuXR].

[XWCS]* Chengzhang Xu, Kang Wu, Hong Chen and Wenlong Su, New Lower Bounds for Some Ramsey Numbers Based on Cyclic Graphs, *in preparation,* (2011).

[-] Xu Chengzhang, see also [LiaWXCS].

[XuYZ] Chuandong Xu, Hongna Yang and Shenggui Zhang, On Characterizing the Critical Graphs for Matching Ramsey Numbers, *Discrete Applied Mathematics,* **287** (2020) 15-20.

[-] Jin Xu, see [ShaXBP].

[-] Xu Feng, see [SunYWLX, SunYXL].

[-] Ran Xu, see [ChenCX].

[XuLL] Xiaodong Xu, Meilian Liang and Haipeng Luo, *Ramsey Theory. Unsolved Problems and Results.* De Gruyter, Berlin; University of Science and Technology of China Press, 2018.

[XuR1] Xiaodong Xu and S.P. Radziszowski, An Improvement to Mathon's Cyclotomic Ramsey Colorings, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #N1, **16**(1) (2009), 5 pages.

[XuR2] Xiaodong Xu and S.P. Radziszowski, $28 \le R(C_4, C_4, C_3, C_3) \le 36$, *Utilitas Mathematica,* **79** (2009) 253-257.

[XuR3] Xiaodong Xu and S.P. Radziszowski, Bounds on Shannon Capacity and Ramsey Numbers from Product of Graphs, *IEEE Transactions on Information Theory,* **59** (2013) 4767-4770.

[XuR4] Xiaodong Xu and S.P. Radziszowski, On Some Open Questions for Ramsey and Folkman Numbers, in *Graph Theory, Favorite Conjectures and Open Problems,* Vol. 1, edited by R. Gera, S. Hedetniemi and C. Larson, Problem Books in Mathematics, Springer 2016, 43-62.

[XSR1]* Xiaodong Xu, Zehui Shao and S.P. Radziszowski, Bounds on Some Ramsey Numbers Involving Quadrilateral, *Ars Combinatoria,* **90** (2009) 337-344.

[XSR2]* Xiaodong Xu, Zehui Shao and S.P. Radziszowski, More Constructive Lower Bounds on Classical Ramsey Numbers, *SIAM Journal on Discrete Mathematics,* **25** (2011) 394-400.

[XuX1]* Xu Xiaodong and Xie Zheng, A Constructive Approach for the Lower Bounds on the Ramsey Numbers $r(k, l)$, *manuscript* (2002).

[XuX2] Xu Xiaodong and Xie Zheng, A Constructive Approach for the Lower Bounds on Multicolor Ramsey Numbers, *manuscript* (2002).

[XuXC] Xu Xiaodong, Xie Zheng and Chen Zhi, Upper Bounds for Ramsey Numbers $R_n(3)$ and Schur Numbers (in Chinese), *Mathematics in Economics,* **19**(1) (2002) 81-84.

[XXER]* Xu Xiaodong, Xie Zheng, G. Exoo and S.P. Radziszowski, Constructive Lower Bounds on Classical Multicolor Ramsey Numbers, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #R35, **11**(1) (2004), 24 pages.

[XuXR] Xu Xiaodong, Xie Zheng and S.P. Radziszowski, A Constructive Approach for the Lower Bounds on the Ramsey Numbers $R(s, t)$, *Journal of Graph Theory,* **47** (2004) 231-239.

[-] Xu Xiaodong, see also [JiLSX, JiLTX1, JiLTX2, LiaRX, LiaWXS, ShaXB, ShaXSP, WSLX1, WSLX2, ZhuXR].

[-] Xu Zhiqiang, see [BaLX].

Y

[-] J. Yackel, see [GrY].

[-] Yan Shuda, see [WWY1, WWY2].

[-] Hongna Yang, see [XuYZ].

[YHZ1] Yang Jian Sheng, Huang Yi Ru and Zhang Ke Min, The Value of the Ramsey Number $R(C_n, K_4)$ is $3(n-1)+1$ $(n \geq 4)$, *Australasian Journal of Combinatorics,* **20** (1999) 205-206.

[YHZ2] Yang Jian Sheng, Huang Yi Ru and Zhang Ke Min, $R(C_6, K_5) = 21$ and $R(C_7, K_5) = 25$, *European Journal of Combinatorics,* **22** (2001) 561-567.

[-] Yang Jian Sheng, see also [BolJY+, HWSYZH].

[YY]** Yang Yuansheng, On the Third Ramsey Numbers of Graphs with Six Edges, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **17** (1995) 199-208.

[YH]* Yang Yuansheng and G.R.T. Hendry, The Ramsey Number $r(K_1+C_4, K_5-e)$, *Journal of Graph Theory,* **19** (1995) 13-15.

[YR1]** Yang Yuansheng and P. Rowlinson, On the Third Ramsey Numbers of Graphs with Five Edges, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **11** (1992) 213-222.

[YR2]* Yang Yuansheng and P. Rowlinson, On Graphs without 6-Cycles and Related Ramsey Numbers, *Utilitas Mathematica,* **44** (1993) 192-196.

[YR3]* Yang Yuansheng and P. Rowlinson, The Third Ramsey Numbers for Graphs with at Most Four Edges, *Discrete Mathematics,* **125** (1994) 399-406.

[-] Yang Yuansheng, see also [SunY, SunYJLS, SunYLZ1, SunYLZ2, SunYW, SunYWLX, SunYXL].

[-] Ye Weiguo, see [SonYL].

[YuLi] Pei Yu and Yusheng Li, All Ramsey Numbers for Brooms in Graphs, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P3.29, **23**(3) (2016), 8 pages.

[-] Pei Yu, see also [PeiCLY].

[Yu1]* Yu Song Nian, A Computer Assisted Number Theoretical Construction of $(3, k)$-Ramsey Graphs, *Annales Universitatis Scientiarum Budapestinensis, Sect. Comput.,* **10** (1989) 35-44.

[Yu2]* Yu Song Nian, Maximal Triangle-Free Circulant Graphs and the Function $K(c)$ (in Chinese), *Journal of Shanghai University, Natural Science,* **2** (1996) 678-682.

[-] Xiaowei Yu, see [ChenYZ].

[-] R. Yuster, see [CaYZ].

Z

[-] A. Zaghian, see [RaeZ].

[-] V. Zamaraev, see [AtLZ].

[-] O. Zamora, see [SaTWZ].

[-] Zang Wenan, see [LiRZ1, LiRZ2, LiTZ, LiZa1, LiZa2].

[-] C. Zarb, see [CaYZ].

[Zeng] Zeng Wei Bin, Ramsey Numbers for Triangles and Graphs of Order Four with No Isolated Vertex, *Journal of Mathematical Research & Exposition,* **6** (1986) 27-32.

[-] Zhang Chaohui, see [HTHZ2].

[ZhZC] Fangfang Zhang, Yunqing Zhang and Yaojun Chen, On Three-color Ramsey Numbers $R(C_4, K_{1,m}, P_n)$, *Finite Fields and Their Applications,* **55** (2019) 97-108.

[-] Hanshuo Zhang, see [ZhuSWZ].

[ZhZ1] Zhang Ke Min and Zhang Shu Sheng, Some Tree-Stars Ramsey Numbers, *Proceedings of the Second Asian Mathematical Conference* 1995, 287-291, World Sci. Publishing, River Edge, NJ, 1998.

[ZhZ2] Zhang Ke Min and Zhang Shu Sheng, The Ramsey Numbers for Stars and Stripes, *Acta Mathematica Scientia,* **25A** (2005) 1067-1072.

[-] Zhang Ke Min, see also [BolJY+, ChenZZ1, ChenZZ2, ChenZZ3, ChenZZ4, ChenZZ5, ChenZZ6, HTHZ1, HWSYZH, HZ1, HZ2, HZ3, McZ, ShZ1, ShZ2, YHZ1, YHZ2, ZhaCZ1, ZhaCZ2, ZZ3].

[ZhaCC1] Lianmin Zhang, Yaojun Chen and T.C. Edwin Cheng, The Ramsey Numbers for Cycles versus Wheels of Even Order, *European Journal of Combinatorics,* **31** (2010) 254-259.

[ZhaCZ] Lianmin Zhang, Kun Chen and Dongmei Zhu, Some Tree-Book Ramsey Numbers, *Ars Combinatoria,* **130** (2017) 97-102.

[-] Lianmin Zhang, see also [ZhuZL].

[ZhaSW]* Zhang Rui, Sun Yongqi and Wu Yali, On the Four Color Ramsey Numbers for Hexagons, *Ars Combinatoria,* **111** (2013) 515-522.

[-] Zhang Rui, see also [WuSZR].

[-] Shenggui Zhang, see [XuYZ].

[-] Zhang Shu Sheng, see [ZhZ1, ZhZ2].

[-] Zhang Xiaoxian, see [XieZ].

[ZhaCC2] Xuemei Zhang, Yaojun Chen and T.C. Edwin Cheng, Some Values of Ramsey Numbers for C_4 versus Stars, *Finite Fields and Their Applications,* **45** (2017) 73-85.

[ZhaCC3] Xuemei Zhang, Yaojun Chen and T.C. Edwin Cheng, Polarity Graphs and Ramsey Numbers for C_4 versus Stars, *Discrete Mathematics,* **340** (2017) 655-660.

[ZhaCC4] Xuemei Zhang, Yaojun Chen and T.C. Edwin Cheng, On Three Color Ramsey Numbers for $R(C_4, C_4, K_{1,n})$, *Discrete Mathematics,* **342** (2019) 285-291.

[ZhaCh] Yanbo Zhang and Yaojun Chen, The Ramsey Numbers of Fans versus a Complete Graph of Order Five, *Electronic Journal of Graph Theory and Applications,* **2** (2014) 66-69.

[ZhaBC1] Yanbo Zhang, Hajo Broersma and Yaojun Chen, A Remark on Star-C_4 and Wheel-C_4 Ramsey Numbers, *Electronic Journal of Graph Theory and Applications,* **2** (2014) 110-114.

[ZhaBC2] Yanbo Zhang, Hajo Broersma and Yaojun Chen, Three Results on Cycle-Wheel Ramsey Numbers, *Graphs and Combinatorics,* **31** (2015) 2467-2479.

[ZhaBC3] Yanbo Zhang, Hajo Broersma and Yaojun Chen, Ramsey Numbers of Trees versus Fans, *Discrete Mathematics,* **338** (2015) 994-999.

[ZhaBC4] Yanbo Zhang, Hajo Broersma and Yaojun Chen, On Fan-Wheel and Tree-Wheel Ramsey Numbers, *Discrete Mathematics,* **339** (2016) 2284-2287.

[ZhaBC5] Yanbo Zhang, Hajo Broersma and Yaojun Chen, Narrowing Down the Gap on Cycle-Star Ramsey Numbers, *Journal of Combinatorics,* **7** (2016) 481-493.

[ZhaZC] Yanbo Zhang, Yunqing Zhang and Yaojun Chen, The Ramsey Numbers of Wheels versus Odd Cycles, *Discrete Mathematics,* **323** (2014) 76-80.

[ZhaZZ] Zhang Yanbo, Zhu Shiping and Zhang Yunqing, Ramsey Numbers for 7-Cycle versus Wheels with Small Order (in Chinese), *Journal of Nanjing University, Mathematical Biquarterly,* **30** (2013) 48-55.

[-] Yanbo Zhang, see also [AllŁPZ, LiZBBH, LiZB, MengZZ].

[-] Zhang Yuming, see [CaLRZ].

[Zhang1] Zhang Yunqing, On Ramsey Numbers of Short Paths versus Large Wheels, *Ars Combinatoria,* **89** (2008) 11-20.

[Zhang2] Zhang Yunqing, The Ramsey Numbers for Stars of Odd Small Order versus a Wheel of Order Nine, *Nanjing Daxue Xuebao Shuxue Bannian Kan,* ISSN 0469-5097, **25** (2008) 35-40.

[ZhaCC5] Yunqing Zhang, T.C. Edwin Cheng and Yaojun Chen, The Ramsey Numbers for Stars of Odd Order versus a Wheel of Order Nine, *Discrete Mathematics, Algorithms and Applications,* **1** (2009) 413-436.

[ZhaCZ1] Yunqing Zhang, Yaojun Chen and Kemin Zhang, The Ramsey Numbers for Stars of Even Order versus a Wheel of Order Nine, *European Journal of Combinatorics,* **29** (2008) 1744-1754.

[ZhaCZ2] Yunqing Zhang, Yaojun Chen and Kemin Zhang, The Ramsey Numbers for Trees of High Degree versus a Wheel of Order Nine, *manuscript* (2009).

[ZZ3] Yunqing Zhang and Ke Min Zhang, The Ramsey Number $R(C_8, K_8)$, *Discrete Mathematics,* **309** (2009) 1084-1090.

[-] Zhang Yunqing, see also [ChenCNZ, ChenCZ1, ChenZZ1, ChenZZ2, ChenZZ3, ChenZZ4, ChenZZ5, ChenZZ6, CheCZN, MengZZ, ZhZC, ZhaZC, ZhaZZ].

[-] Zhang Zhengyou, see [SLZL].

[ZLLS] Zhang Zhongfu, Liu Linzhong, Li Jinwen and Song En Min, Some Properties of Ramsey Numbers, *Applied Mathematics Letters,* **16** (2003) 1187-1193.

[Zhao] Yi Zhao, Proof of the $(n/2-n/2-n/2)$ Conjecture for Large n, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P27, **18**(1) (2011), 61 pages.

[-] Yi Zhao, see also [ChenYZ, NoSZ].

[-] Zheng Wenping, see [SunYLZ1, SunYLZ2].

[Zhou1] Zhou Huai Lu, Some Ramsey Numbers for Graphs with Cycles (in Chinese), *Mathematica Applicata,* **6** (1993) 218.

[Zhou2] Zhou Huai Lu, The Ramsey Number of an Odd Cycle with Respect to a Wheel (in Chinese), *Journal of Mathematics, Shuxue Zazhi* (Wuhan), **15** (1995) 119-120.

[Zhou3] Zhou Huai Lu, On Book-Wheel Ramsey Number, *Discrete Mathematics,* **224** (2000) 239-249.

[ZhuZL] Dongmei Zhu, Lianmin Zhang and Dongxin Li, The Ramsey Numbers of Large Trees versus Wheels, *Bulletin of the Iranian Mathematical Society,* **42**(4) (2016) 879-880.

[-] Dongmei Zhu, see also [ZhaCZ].

[-] Emily Zhu, see [BohZ].

[ZhuXR] Rujie Zhu and Xiaodong Xu and S.P. Radziszowski, A Small Step Forwards on the Erdős-Sós Problem Concerning the Ramsey Numbers $R(3, k)$, *Discrete Applied Mathematics,* **214** (2016) 216-221.

[-] Zhu Shiping, see [ZhaZZ].

[ZhuSWZ] Weiguo Zhu, Yongqi Sun, Yali Wu and Hanshuo Zhang, Exact Values of Multicolor Ramsey Numbers $R_l(C_{\leq l+1})$, *Graphs and Combinatorics,* **36** (2020) 839-852.