Small Ramsey Numbers

Stanisław P. Radziszowski
Department of Computer Science
Rochester Institute of Technology
Rochester, NY 14623, spr@cs.rit.edu
http://www.cs.rit.edu/~spr

Submitted: June 11, 1994; Revision #14: January 12, 2014
http://www.combinatorics.org

ABSTRACT: We present data which, to the best of our knowledge, includes all known nontrivial values and bounds for specific graph, hypergraph and multicolor Ramsey numbers, where the avoided graphs are complete or complete without one edge. Many results pertaining to other more studied cases are also presented. We give references to all cited bounds and values, as well as to previous similar compilations. We do not attempt complete coverage of asymptotic behavior of Ramsey numbers, but rather we concentrate on their specific values.

Mathematical Reviews Subject Number 05C55

Revisions

1993, February	preliminary version, RIT-TR-93-009 [Ra2]
1994, July 3	first posted on the web at the *ElJC*
1994, November 7	*ElJC* revision #1
1995, August 28	*ElJC* revision #2
1996, March 25	*ElJC* revision #3
1997, July 11	*ElJC* revision #4
1998, July 9	*ElJC* revision #5
1999, July 5	*ElJC* revision #6
2000, July 25	*ElJC* revision #7
2001, July 12	*ElJC* revision #8
2002, July 15	*ElJC* revision #9
2004, July 4	*ElJC* revision #10
2006, August 1	*ElJC* revision #11
2009, August 4	*ElJC* revision #12
2011, August 22	*ElJC* revision #13
2014, January 12	*ElJC* revision #14

Table of Contents **2**

1. Scope and Notation **3**

2. Classical Two-Color Ramsey Numbers **4**

2.1 Values and bounds for $R(k,l)$, $k \le 10$, $l \le 15$ 4
2.2 Bounds for $R(k,l)$, higher parameters 6
2.3 General results on $R(k,l)$ 8

3. Two Colors: $K_n - e$, K_3, $K_{m,n}$ **11**

3.1 Dropping one edge from complete graph 11
3.2 Triangle versus other graphs 13
3.3 Complete bipartite graphs 14

4. Two Colors: Numbers Involving Cycles **18**

4.1 Cycles, cycles versus paths and stars 18
4.2 Cycles versus complete graphs 19
4.3 Cycles versus wheels 21
4.4 Cycles versus books 22
4.5 Cycles versus other graphs 23

5. General Graph Numbers in Two Colors **24**

5.1 Paths 24
5.2 Wheels 24
5.3 Books 25
5.4 Trees and forests 25
5.5 Stars, stars versus other graphs 26
5.6 Paths versus other graphs 27
5.7 Fans, fans versus other graphs 28
5.8 Wheels versus other graphs 28
5.9 Books versus other graphs 29
5.10 Trees and forests versus other graphs 29
5.11 Cases for $n(G)$, $n(H) \le 5$ 30
5.12 Mixed cases 31
5.13 Multiple copies of graphs, disconnected graphs 31
5.14 General results for special graphs 32
5.15 General results for sparse graphs 32
5.16 General results 34

6. Multicolor Ramsey Numbers **36**

6.1 Bounds for classical numbers 36
6.2 General results for complete graphs 38
6.3 Cycles 39
6.4 Paths, paths versus other graphs 43
6.5 Special cases 45
6.6 General results for special graphs 46
6.7 General results 47

7. Hypergraph Numbers **49**

7.1 Values and bounds for numbers 49
7.2 Cycles and paths 49
7.3 General results for 3-uniform hypergraphs 51
7.4 General results 51

8. Cumulative Data and Surveys **52**

8.1 Cumulative data for two colors 52
8.2 Cumulative data for three colors 53
8.3 Electronic resources 54
8.4 Surveys 54

9. Concluding Remarks **56**

References **57-94**

1. Scope and Notation

There is vast literature on Ramsey type problems starting in 1930 with the original paper of Ramsey [Ram]. Graham, Rothschild and Spencer in their book [GRS] present an exciting development of Ramsey Theory. The subject has grown amazingly, in particular with regard to asymptotic bounds for various types of Ramsey numbers (see the survey papers [GrRö, Neš, ChGra2, Ros2]), but the progress on evaluating the basic numbers themselves has been unsatisfactory for a long time. In the last three decades, however, considerable progress has been obtained in this area, mostly by employing computer algorithms. The few known exact values and several bounds for different numbers are scattered among many technical papers. This compilation is a fast source of references for the best results known for specific numbers. It is not supposed to serve as a source of definitions or theorems, but these can be easily accessed via the references gathered here.

Ramsey Theory studies conditions when a combinatorial object contains necessarily some smaller given objects. The role of Ramsey numbers is to quantify some of the general existential theorems in Ramsey Theory.

Let $G_1, G_2, \ldots, G_m$ be graphs or s-uniform hypergraphs (s is the number of vertices in each edge). $R(G_1, G_2, \ldots, G_m; s)$ denotes the m-color **Ramsey number** for s-uniform graphs/hypergraphs, avoiding G_i in color i for $1 \le i \le m$. It is defined as the least integer n such that, in any coloring with m colors of the s-subsets of a set of n elements, for some i the s-subsets of color i contain a sub-(hyper)graph isomorphic to G_i (not necessarily induced). The value of $R(G_1, G_2, \ldots, G_m; s)$ is fixed under permutations of the first m arguments. If $s = 2$ (standard graphs) then s can be omitted. If G_i is a complete graph K_k, then we may write k instead of G_i, and if $G_i = G$ for all i we may use the abbreviation $R_m(G; s)$ or $R_m(G)$. For $s = 2$, $K_k - e$ denotes a K_k without one edge, and for $s = 3$, $K_k - t$ denotes a K_k without one triangle (hyperedge).

The graph nG is formed by n disjoint copies of G, $G \cup H$ stands for vertex disjoint union of graphs, and the **join** $G + H$ is obtained by adding all of the edges between vertices of G and H to $G \cup H$. P_i is a **path** on i vertices, C_i is a **cycle** of length i, and W_i is a **wheel** with $i-1$ spokes, i.e. a graph formed by some vertex x, connected to all vertices of the cycle C_{i-1} (thus $W_i = K_1 + C_{i-1}$). $K_{n,m}$ is a complete n by m bipartite graph, in particular $K_{1,n}$ is a **star** graph. The **book** graph $B_i = K_2 + \overline{K}_i = K_1 + K_{1,i}$ has $i+2$ vertices, and can be seen as i triangular pages attached to a single edge. The **fan** graph F_n is defined by $F_n = K_1 + nK_2$. For a graph G, $n(G)$ and $e(G)$ denote the number of vertices and edges, respectively, and $\delta(G)$ and $\Delta(G)$ minimum and maximum degree of G. Finally, $\chi(G)$ denotes the chromatic number of G. In general, we follow the notation used by West [West].

Section 2 contains the data for the classical two color Ramsey numbers $R(k, l)$ for complete graphs, section 3 for the much studied two color cases of $K_n - e$, K_3, $K_{m,n}$, and section 4 for numbers involving cycles. Section 5 lists other often studied two color cases for general graphs. The multicolor and hypergraph cases are gathered in sections 6 and 7, respectively. Finally, section 8 gives pointers to cumulative data and to other surveys.

2. Classical Two-Color Ramsey Numbers

2.1. Values and bounds for $R(k,l)$, $k \le 10$, $l \le 15$

k \ l	3	4	5	6	7	8	9	10	11	12	13	14	15
3	6	9	14	18	23	28	36	40 42	47 50	52 59	59 68	66 77	73 87
4		18	25	36 41	49 61	58 84	73 115	92 149	98 191	128 238	133 291	141 349	153 417
5			43 49	58 87	80 143	101 216	126 316	144 442	171 633	191 848	213 1138	239 1461	265 1878
6				102 165	113 298	132 495	169 780	179 1171	253 1804	263 2566	317 3703	5033	401 6911
7					205 540	217 1031	241 1713	289 2826	405 4553	417 6954	511 10578	15263	22112
8						282 1870	317 3583	6090	10630	16944	817 27485	41525	861 63609
9							565 6588	581 12677	22325	38832	64864		
10								798 23556	45881	81123			1265

Table I. Known nontrivial values and bounds for two color Ramsey numbers $R(k,l) = R(k,l;2)$.

k \ l	4	5	6	7	8	9	10	11	12	13	14	15
3	GG	GG	Kéry	Ka2 GrY	GR MZ	Ka2 GR	Ex5 GoR1	Ex20 GoR1	Ex12 Les	Piw1 GoR1	Ex8 GoR1	WW GoR1
4	GG	Ka1 MR4	Ex19 MR5	Ex3 Mac	Ex20 Mac	Ex16 Mac	HaKr1 Mac	Ex17 Spe4	SLL Spe4	2.3.e Spe4	XXR Spe4	XXR Spe4
5		Ex4 MR5	Ex9 HZ1	CaET Spe4	HaKr1 Spe4	Ex17 Mac	Ex17 Mac	Gerb HW+	Gerb HW+	Gerb HW+	Gerb HW+	Ex16 HW+
6			Ka1 Mac	Ex16 Mac	XSR2 Mac	XXER Mac	Ex16 Mac	XXR HW+	XSR2 HW+	XXER HW+	HW+	2.3.h HW+
7				She2 Mac	XSR2 Mac	XSR2 HZ1	2.3.h Mac	XXER HW+	XSR2 HW+	XXR HW+	HW+	HW+
8					BR Mac	XXER Ea1	HZ1	HW+	HW+	XXER HW+	HW+	2.3.h HW+
9						She2 ShZ1	XSR2 Ea1	HW+	HW+	HW+		
10							She2 Shi2	HW+	HW+			2.3.h

References for Table I;
HW+ abbreviates HWSYZH, as enhanced by Boza [Boza5], see 2.1.m.

We split the data into the table of values and a table with corresponding references. In Table I, known exact values appear as centered entries, lower bounds as top entries, and upper bounds as bottom entries. For some of the exact values two references are given when the lower and upper bound credits are different.

(a) The task of proving $R(3,3) \le 6$ was the second problem in Part I of the William Lowell Putnam Mathematical Competition held in March 1953 [Bush].

(b) Greenwood and Gleason [GG] established the initial values $R(3,4) = 9$, $R(3,5) = 14$ and $R(4,4) = 18$ in 1955.

(c) Kéry [Kéry] proved that $R(3,6) = 18$ in 1964, but only in 2007 an elementary and self-contained proof of this result appeared in English [Car].

(d) All of the critical graphs for the numbers $R(k,l)$ (graphs on $R(k,l)-1$ vertices without K_k and without K_l in the complement) are known for $k=3$ and $l=3$, 4, 5 [Kéry], 6 [Ka2], 7 [RK2, MZ], 8 [BrGS] and 9 [GoR1], and there are 1, 3, 1, 7, 191, 477142, and 1 of them, respectively. All $(3,k)$-graphs, for $k \le 6$, were enumerated in [RK2], and all (4,4)-graphs in [MR2]. There exists a unique critical graph for $R(4,4)$ [Ka2]. There are 350904 known critical graphs for $R(4,5)$ [MR4], but there might be more of them.

(e) In [MR5], strong evidence is given for the conjecture that $R(5,5) = 43$ and that there exist exactly 656 critical graphs on 42 vertices.

(f) The graphs constructed by Exoo in [Ex9, Ex12-Ex20], and some others, are available electronically from http://ginger.indstate.edu/ge/RAMSEY. Fujita [Fuj1] maintains a website with some lower bound constructions; in particular, it presents the bound $R(4,8) \ge 58$ obtained independently from Exoo.

(g) *Cyclic* (or *circular*) graphs are often used for Ramsey graph constructions. Several cyclic graphs establishing lower bounds were given in the Ph.D. dissertation by J.G. Kalbfleisch in 1966, and many others were published in the next few decades (see [RK1]). Harborth and Krause [HaKr1] presented all best lower bounds up to 102 from cyclic graphs avoiding complete graphs. In particular, no lower bound in Table I can be improved with a cyclic graph on less than 102 vertices, except possibly for $R(3,k)$ for $k \ge 13$. See also item 2.3.k and section 5.16 [HaKr1]. Several best lower bounds from *distance colorings*, a slightly more general concept than circular graphs, are presented in [HaKr2].

(h) The claim that $R(5,5) = 50$ posted on the web [Stone] is in error, and despite being shown to be incorrect more than once, this value is still being cited by some authors. The bound $R(3,13) \ge 60$ [XieZ] cited in the 1995 version of this survey was shown to be incorrect in [Piw1]. Another incorrect construction for $R(3,10) \ge 41$ was described in [DuHu].

(i) There are really only two general upper bound inequalities useful for small parameters, namely 2.3.a and 2.3.b. Stronger upper bounds for specific parameters were difficult to obtain, and they often involved massive computations, like those for the cases of (3,8) [MZ], (3,10) [GoR1], (4,5) [MR4], (4,6) and (5,5) [MR5]. The bound $R(6,6) \le 166$, only 1 more than the best known [Mac], is an easy consequence of a theorem in [Walk]

(2.3.b) and $R(4,6) \le 41$.

(j) T. Spencer [Spe4], Mackey [Mac], and Huang and Zhang [HZ1], using the bounds for minimum and maximum number of edges in (4,5) Ramsey graphs listed in [MR3, MR5], were able to establish new upper bounds for several higher Ramsey numbers, improving on all of the previous longstanding best results by Giraud [Gi3, Gi5, Gi6].

(k) Only some of the higher bounds implied by 2.3.* are shown, and more similar bounds could be derived. In general, we show bounds beyond the contiguous small values if they improve on results previously reported in this survey or published elsewhere. Some easy upper bounds implied by 2.3.a are marked as [Ea1].

(l) In 2009, we have recomputed the upper bounds in Table I marked [HZ1] using the method from the paper [HZ1], because the bounds there relied on an overly optimistic personal communication from T. Spencer. Further refinements of this method are studied in [HZ2, ShZ1, Shi2]. The paper [Shi2] subsumes the main results of the manuscripts [ShZ1, Shi2].

(m) In 2013, Boza [Boza5] using the method of [HWSYZH], which is abbreviated as HW+ in Table I, computed the bounds marked HW+ by starting from better upper bounds for smaller parameters. Most of the currently shown bounds are thus better than those originally listed in [HWSYZH, HZ2]. Five upper bounds not shown in Table I can be obtained similarly but they are larger than 10^5.

2.2. Bounds for $R(k,l)$, higher parameters

k \ l	15	16	17	18	19	20	21	22	23
3	73 WW 87 GoR1	82 Ex21 98 Back1	92 W1+ 109 Back1	99 Ex16 121 Les	106 W1+ 132 Back2	111 Ex16 145 Les	122 W1+ 158 Les	131 W2+ 171 Back2	139 XWCS 185 Back2
4	153 XXR	164 Gerb	200 Lia+	205 2.3.e	213 2.3.g	234 Ex16	242 SLZL	314 LSLW	
5	265 Ex16	289 2.3.h	388 XSR2	396 2.3.g	411 XSR2	424 XSR2	441 2.3.h	485 2.3.h	521 2.3.h
6	401 2.3.h	434 SLLL	548 SLLL	614 SLLL	710 SLLL	878 SLLL		1070 SLLL	
7		609 2.3.h	711 2.3.g	797 2.3.h	908 SLLL		1214 SLLL		
8	861 2.3.h		961 XSR2	1045 2.3.g	1236 2.3.g		1617 2.3.h		

Table IIa. Known bounds for higher two-color Ramsey numbers $R(k,l)$, with references.
Lower and upper bounds are given for $k = 3$, only lower bounds for $k \ge 4$;
Lia+, W1+ and W2+ abbreviate LiaWXS, WWY1 and WSLX2, respectively.

k \ l	24	25	26	27	28	29	30	31
3	143 W1+	154 W2+	159 W1+	167 W1+	173 W2+	184 W2+	190 W2+	199 W2+

k \ l	32	33	34	35	36	37	38	39	40
3	214 W2+	218 Ch+	226 Ch+	231 Ch+	239 Ch+	244 Ch+	256 Ch+		

Table IIb. Known lower bounds for higher Ramsey numbers $R(3,l)$ for $l \geq 24$; W1+, W2+ and Ch+ abbreviate WSLX1, WSLX2 and ChWXSL, respectively.

k	11	12	13	14	15	16	17
lower bound	1597	1639	2557	2989	5485	5605	8917
reference	2.2.c	XSR2	2.2.c	2.2.c	2.2.c	2.2.c	LSL
k	18	19	20	21	22	23	24
lower bound	11005	17885	19069	27077	29941		
reference	LSL	LSL	Lia+	Lia+	Lia+		

Table IIc. Known lower bounds for diagonal Ramsey numbers $R(k,k)$ for $k \geq 11$; Lia+ abreviates LiaWXCS, see also 2.2.c below.

(a) The upper bounds in Tables I and IIa marked [GoR1, Les, Back1] were obtained mainly by deriving lower bounds for several cases of $e(3,k,n)$, which denotes the minimum number of edges in n-vertex triangle-free graphs with independence number less than k. The study of $e(3,k,n)$ was also the main tool for the results obtained in [GrY, GR, RK2, RK3, GoR2].

(b) *Ramsey Calculus* [Back1], is an extensive manuscript by Backelin, which, among other goals, addresses the derivation of $e(3,k,n)$ and the corresponding realisers while avoiding reliance on computer assisted results as far as possible. It achieves the derivation of several lower bounds for $e(3,k+1,n)$ better than those in [GoR1, RK3, RK4] for n close to and above $13k/4$.

(c) The construction by Mathon [Mat] and Shearer [She2] (see also items 2.3.i, 6.2.k and 6.2.l), using the data obtained by Shearer [She4] for primes up to 7000, implies the lower bounds in Table IIc marked 2.2.c. The first two bounds credited in Table IIc to [LSL] also follow similarly from the data in [She4]. The same approach does not improve on the bound $R(12,12) \geq 1639$ [XSR2]. The bounds in [Lia+] were obtained by extending data for Payley graphs beyond [Sha4].

(d) The lower bounds marked [XXR], [XXER], [XSR2], 2.3.e and 2.3.h need not be cyclic. Several of the Cayley colorings from [Ex16] are also non-cyclic. All other lower bounds listed in Table IIab were obtained by construction of cyclic graphs.

(e) The graphs establishing lower bounds marked 2.3.g can be constructed by using appropriately chosen graphs G and H with a common m-vertex induced subgraph, similarly as it was done in several cases in [XXR].

(f) Yu [Yu2] constructed a special class of triangle-free cyclic graphs establishing several lower bounds for $R(3,k)$, for $k \geq 61$. All of these bounds can be improved by the inequalities in 2.3.c and data from Tables I and II.

(g) Unpublished bound $R(4,22) \geq 314$ [LSLW] improves over 282 given in [SL]. [LSLW] includes also $R(4,25) \geq 458$. Not yet published bounds $R(3,23) \geq 139$ [XWCS] and $R(4,17) \geq 200$ [LiaWXS] improve over 137 and 182 obtained in [WSLX2] and [LSS1], respectively.

(h) Two special cases which improve on bounds listed in earlier revisions: $R(9,17) \geq 1411$ is given in [XXR] and $R(10,15) \geq 1265$ can be obtained using 2.3.h.

(i) One can expect that the lower bounds in Table II are weaker than those in Table I, especially smaller ones, in the sense that some of them should not be that hard to improve, in contrast to the bounds in Table I.

2.3. General results on $R(k,l)$

(a) $R(k,l) \leq R(k-1,l)+R(k,l-1)$, with strict inequality when both terms on the right hand side are even [GG]. There are obvious generalizations of this inequality for avoiding graphs other than complete.

(b) $R(k,k) \leq 4R(k,k-2)+2$ [Walk].

(c) Explicit construction for $R(3,3k+1) \geq 4R(3,k+1)-3$, for all $k \geq 2$ [CleDa],
explicit construction for $R(3,4k+1) \geq 6R(3,k+1)-5$, for all $k \geq 1$ [ChCD].

(d) Explicit triangle-free graphs with independence k on $\Omega(k^{3/2})$ vertices [Alon2, CPR]. For other constructive results in relation to $R(3,k)$ see [BBH1, BBH2, Fra1, Fra2, FrLo, GoR1, Gri, KlaM1, Loc, RK2, RK3, RK4, Stat, Yu1]. See also 2.3.(3) and 2.3.(4) below.

(e) The study of bounds for the difference between consecutive Ramsey numbers was initiated in [BEFS], where the bound $R(k,l) \geq R(k,l-1) + 2k - 3$, for $k,l \geq 3$, was established by a construction. In 1980, Erdős and Sós (cf. [Erd2,ChGra2]) asked: If we set $\Delta_{k,l} = R(k,l) - R(k,l-1)$, then is it true that $\Delta_{k,k+1}/k \to \infty$ as $k \to \infty$? Only easy bounds on $\Delta_{k,l}$ are known, in particular $3 \leq \Delta_{3,l} \leq l$ for $k = 3$. For some discussion of the latter see [XSR2, GoR2].

(f) By taking a disjoint union of two critical graphs one can easily see that $R(k,p) \geq s$ and $R(k,q) \geq t$ imply $R(k,p+q-1) \geq s+t-1$. Xu and Xie [XX1] improved this construction to yield better general lower bounds, in particular $R(k,p+q-1) \geq s+t+k-3$.

(g) For $2 \leq p \leq q$ and $3 \leq k$, if (k,p)-graph G and (k,q)-graph H have a common induced subgraph on m vertices without K_{k-1}, then $R(k,p+q-1) > n(G)+n(H)+m$. In particular, this implies the bounds $R(k,p+q-1) \geq R(k,p)+R(k,q)+k-3$ and $R(k,p+q-1) \geq R(k,p)+R(k,q)+p-2$ [XX1, XXR], with further small

improvements in some cases, such as using the term $k-2$ instead of $k-3$ in the previous bound [XSR2].

(h) $R(2k-1,l) \geq 4R(k,l-1)-3$ for $l \geq 5$ and $k \geq 2$, and in particular for $k=3$ we have $R(5,l) \geq 4R(3,l-1)-3$ [XXER].

(i) If the quadratic residues Paley graph Q_p of prime order $p = 4t+1$ contains no K_k, then $R(k,k) \geq p+1$ and $R(k+1,k+1) \geq 2p+3$ [She2, Mat]. Data for larger p was obtained in [LSL]. See also 3.1.c, and items 6.2.k and 6.2.l for similar multicolor results.

(j) Study of Ramsey numbers for large disjoint unions of graphs [Bu1, Bu9], in particular $R(nK_k, nK_l) = n(k+l-1) + R(K_{k-1}, K_{l-1}) - 2$, for n large enough [Bu8].

(k) $R(k,l) \geq L(k,l)+1$, where $L(k,l)$ is the maximal order of any cyclic (k,l)–graph. A compilation of many best cyclic bounds was presented in [HaKr1].

(l) The graphs critical for $R(k,l)$ are $(k-1)$–vertex connected and $(2k-4)$–edge connected, for $k,l \geq 3$ [BePi]. This was improved to vertex connectivity k for $k \geq 5$ and $l \geq 3$ in [XSR2].

(m) All Ramsey-critical (k,l)–graphs are Hamiltonian for $k \geq l-1 \geq 1$ and $k \geq 3$, except when $(k,l) = (3,2)$ [XSR2].

(n) Two-color lower bounds can be obtained by using items 6.2.m, 6.2.n and 6.2.o with $r = 2$. Some generalizations of these were obtained in [ZLLS].

In the last seven items (1)-(7) of this section we only briefly mention some pointers to the literature dealing with asymptotics of Ramsey numbers. This survey was designed mostly for small, finite, and combinatorial results, but still we wish to give the reader some useful and representative references to more traditional papers studying the infinite.

(1) In 1947, Erdős gave a simple probabilistic proof that $R(k,k) \geq ck\,2^{k/2}$ [Erd1]. Spencer [Spe1] improved the constant c to $\sqrt{2}/e$. More probabilistic asymptotic lower bounds for other Ramsey numbers were obtained in [Spe1, Spe2, AlPu].

(2) The limit of $R(k,k)^{1/k}$, if it exists, is between $\sqrt{2}$ and 4 [GRS, GrRö, ChGra2].

(3) In 1995, Kim obtained a breakthrough result by proving that $R(3,k) = \Theta(k^2/\log k)$ [Kim]. The best known lower and upper bounds constants are 1/4 [BohK2] and 1 (implicit in [She1]), respectively. An independent proof of the lower bound constant 1/4 and a conjecture that it is the best possible are presented in [FizGM].

(4) Other asymptotic and general results on triangle-free graphs in relation to $R(3,k)$ can be found in [Boh, AlBK, AjKS, Alon2, CleDa, ChCD, CPR, Gri, FrLo, Loc, She1, She3].

(5) Explicit constructions yielded the lower bounds $R(4,k) \geq \Omega(k^{8/5})$, $R(5,k) \geq \Omega(k^{5/3})$ and $R(6,k) \geq \Omega(k^2)$ [KosPR]. For the same cases of k classical probabilistic arguments give $\Omega((k/\log k)^{5/2})$, $\Omega((k/\log k)^3)$ and $\Omega((k/\log k)^{7/2})$, respectively [Spe2]. These were improved to $\Omega(k^{5/2}/(\log k)^2)$, $\Omega(k^3/(\log k)^{8/3})$ and $\Omega(k^{7/2}/(\log k)^{13/4})$, respectively, in [Boh, BohK1], and in general to $R(s,t) = \Omega(t^{(s+1)/2}/(\log t)^{(s^2-s-4)/(2s-4)})$, for fixed s and large t [BohK1].

(6) Explicit construction of a graph with clique and independence k on $2^{c \log^2 k / \log\log k}$ vertices was presented by Frankl and Wilson [FraWi], and further constructions by Chung [Chu3] and Grolmusz [Grol1, Grol2]. In 2012, the best explicit construction for large k by Barak et al. [BarRSW] improved over [FraWi] by giving such a graph on $2^{2^{(\log\log k)^c}}$ vertices for some $c > 1$, or equivalently, on n vertices, where $\log\log n = (\log\log k)^c$. Explicit constructions such as these are usually weaker than known probabilistic results.

(7) In 2010, Conlon [Con1] obtained the best until now upper bound for the diagonal case:

$$R(k+1, k+1) \leq \binom{2k}{k} k^{-c \log k / \log\log k}$$

Other asymptotic bounds can be found, for example, in [Chu3, McS, Boh, BohK1] (lower bound) and [Tho] (upper bound), and for many other bounds in the general case of $R(k, l)$ consult [Spe2, GRS, GrRö, Chu4, ChGra2, LiRZ1, AlPu, Kriv].

3. Two Colors: $K_n - e$, K_3, $K_{m,n}$

3.1. Dropping one edge from complete graph

This section contains known values and nontrivial bounds for the two color case when the avoided graphs are complete or have the form $K_k - e$, but not both are complete.

G \ H	K_3-e	K_4-e	K_5-e	K_6-e	K_7-e	K_8-e	K_9-e	$K_{10}-e$	$K_{11}-e$
K_3-e	3	5	7	9	11	13	15	17	19
K_3	5	7	11	17	21	25	31	37	42 45
K_4-e	5	10	13	17	28	29 38	34	41	
K_4	7	11	19	30 33	37 52	75	105	139	184
K_5-e	7	13	22	31 39	40 66				
K_5	9	16	30 34	43 67	112	183	277	409	581
K_6-e	9	17	31 39	45 70	59 135				
K_6	11	21	37 53	110	205	373	621	1007	1544
K_7-e	11	28	40 66	59 135	251				
K_7	13	28 30	51 83	193	392	753	1336	2303	3751
K_8	15	42	123	300	657	1349	2558	4722	8200

Table IIIa. Two types of Ramsey numbers $R(G,H)$,
includes all known nontrivial values.

(a) The exact values in Table IIIa involving $K_3 - e$ are obvious, since one can easily see that $R(K_3 - e, K_k) = R(K_3 - e, K_{k+1} - e) = 2k - 1$ for all $k \geq 2$.

(b) More bounds (beyond those shown in Tables IIIa/b) can be easily obtained using Table I, an obvious generalization of the inequality $R(k,l) \leq R(k-1,l) + R(k,l-1)$, and by monotonicity of Ramsey numbers, in this case $R(K_{k-1}, G) \leq R(K_k - e, G) \leq R(K_k, G)$.

(c) If the quadratic residues Paley graph Q_p of prime order $p = 4t + 1$ contains no $K_k - e$, then $R(K_{k+1} - e, K_{k+1} - e) \geq 2p + 1$. In particular, $R(K_{14} - e, K_{14} - e) \geq 2987$ [LiShen]. This was generalized to $K_k - F$ for some small graphs F instead of an edge e $(= K_2)$ [WaLi]. See also item 2.3.i.

G \ H	K_4-e	K_5-e	K_6-e	K_7-e	K_8-e	K_9-e	$K_{10}-e$	$K_{11}-e$
K_3	CH2	Clan	FRS1	GH	Ra1	Ra1	MPR GoR2	WWY2 GoR2
K_4-e	CH1	FRS2	McR	McR	Ea1 HZ2	Ex14	Ex14	
K_4	CH2	EHM1	Boza6 Boza5	Ex14 HZ2	BZ2	BZ2	BZ2	Ea1
K_5-e	FRS2	CE+	Ex14 Ea1	Ex14 HZ2				
K_5	BoH	Ex6 Ex8	Ea1 HZ2	HZ2	BZ2	BZ2	BZ2	BZ2
K_6-e	McR	Ex14 Ea1	Ex14 HZ2	Ex14 HZ2				
K_6	McN/ ShWR	Ex14 BZ1	BZ2	ShZ2	BZ2	BZ2	BZ2	BZ2
K_7-e	McR	Ex14 HZ2	Ex14 HZ2	ShZ1				
K_7	Ea1 BoPo	Ex14 Ea1	Ea1	BZ2	BZ2	BZ2	BZ2	BZ2
K_8	BZ1	BZ1	BZ2	BZ2	BZ2	BZ2	BZ2	BZ2

References for Table IIIa;
CE+ abbreviates CEHMS, for some details on BZ1 and BZ2 see item 3.1.d below.

k	11	12	13	14	15	16
lower bound	42 WWY2	47 Ea1	55 GoR2	59 Ea1	69 WWY2	73 Ea1
upper bound	45 GoR2	53 GoR2	62 GoR2	71 GoR2	80 GoR2	91 GoR2

Table IIIb. Lower and upper bounds for $R(K_3, K_k - e)$ for $11 \le k \le 16$;
lower bounds for $k = 12, 14, 16$ are the same as for $R(K_3, K_{k-1})$.

(d) This item follows personal communication from Boza [Boza5]. The upper bounds marked [BZ1] were obtained until 2012, while ones marked [BZ2] are from 2013. They are implied by [Boza6], the previous work [Boza1, Boza3, BoPo], the method of [HZ2], and the bounds given in [GoR2]. The enumeration of all $(K_6, K_4 - e)$-graphs [ShWR] is used in [BoPo].

(e) All $(K_3, K_k - e)$-graphs were enumerated for $k \le 6$ [Ra1] and $k = 7$ [Fid2, GoR2]. Full sets of $(K_l, K_k - e)$-graphs were posted for the parameters $(K_3, K_k - e)$ for $k \le 7$, $(K_4, K_k - e)$ for $k \le 5$, and $(K_5, K_k - e)$ for $k \le 4$ at [Fid2], and other full and restricted families at [BrCGM, Fuj1].

(f) The number of $(K_3, K_l - e)$-critical graphs for $l = 4, 5$ and 8 is 4, 2 and 9, respectively [MPR]. There are 7 critical graphs for $R(K_3, K_9 - e)$, and at least 40 such graphs for $R(K_3, K_{10} - e)$ [GoR2].

(g) The critical graphs are unique for: $R(K_3, K_l - e)$ for $l = 3$ [Tr], 6 and 7 [Ra1], $R(K_4 - e, K_4 - e)$ [FRS2], $R(K_5 - e, K_5 - e)$ [Ra3] and $R(K_4 - e, K_7 - e)$ [McR].

(h) All of the critical graphs for the cases $R(K_4 - e, K_4)$ [EHM1], $R(K_4 - e, K_5)$ and $R(K_5 - e, K_4)$ [DzFi1] are known, and there are 5, 13 and 6 of them, respectively. The unpublished value of $R(K_4 - e, K_6)$ [McN] was confirmed in [ShWR], where in addition all 24976 critical graphs were found.

(i) It is known that $R(K_4, K_{12} - e) \geq 128$ [Shao] by using one color of the (4,4,4;127)-coloring defined in [HiIr].

(j) $R(K_k - e, K_k - e) \leq 4R(K_{k-2}, K_k - e) - 2$ [LiShen].
For a similar inequality for complete graphs see 2.3.b.

(k) Study of the cases $R(K_m, K_n - K_{1,s})$ and $R(K_m - e, K_n - K_{1,s})$, with several exact values for special parameters [ChaMR].

(l) The upper bounds from [ShZ1, ShZ2] are subsumed by a later article [Shi2].

(m) The upper bounds in [HZ2] were obtained by a reasoning generalizing the bounds for classical numbers in [HZ1]. Several other results from section 2.3 apply, though checking in which situation they do may require looking inside the proofs whether they still hold for $K_n - e$.

3.2. Triangle versus other graphs

(a) $R(3, k) = \Theta(k^2/\log k)$ [Kim].
For more comments on asymptotics see section 2.3 and the item 3.2.o/p below.

(b) Explicit construction for $R(3, 3k+1) \geq 4R(3, k+1) - 3$, for all $k \geq 2$ [CleDa],
explicit construction for $R(3, 4k+1) \geq 6R(3, k+1) - 5$, for all $k \geq 1$ [ChCD].

(c) Explicit triangle-free graphs with independence k on $\Omega(k^{3/2})$ vertices [Alon2, CPR].

(d) $R(K_3, K_7 - 2P_2) = R(K_3, K_7 - 3P_2) = 18$ [SchSch2].

(e) $R(K_3, K_3 + \overline{K}_m) = R(K_3, K_3 + C_m) = 2m + 5$ for $m \geq 212$ [Zhou1].

(f) $R(K_3, K_2 + T_n) = 2n + 3$ for n-vertex trees T_n, for $n \geq 4$ [SonGQ].

(g) $R(K_3, G) = 2n(G) - 1$ for any connected G on at least 4 vertices and with at most $(17n(G) + 1)/15$ edges, in particular for $G = P_i$ and $G = C_i$, for all $i \geq 4$ [BEFRS1].

(h) $R(K_3, Q_n) = 2^{n+1} - 1$ for large n [FizGMSS], where Q_n is the n-dimensional hypercube. For the general case of $R(K_m, Q_n)$ see item 5.15.n.

(i) Relations between $R(3, k)$ and graphs with large $\chi(G)$ [Für],
further detailed study of the relation between $R(3, k)$ and the chromatic gap [GySeT].

(j) $R(K_3, G) \le 2e(G)+1$ for any graph G without isolated vertices [Sid3, GoK].

(k) $R(K_3, G) \le n(G)+e(G)$ for all G, a conjecture [Sid2].

(l) $R(K_3, G)$ for all connected G up to 9 vertices [BBH1, BBH2].

(m) $R(K_3, G)$ for all graphs G on 10 vertices [BrGS], except 10 cases (three of which, including $G = K_{10} - e$, were solved [GoR2]). See also section 8.1.

(n) Formulas for $R(nK_3, mG)$ for all G of order 4 without isolates [Zeng].

(o) For every positive constant c, Δ, and n large enough, there exists graph G with $\Delta(G) \le \Delta$ for which $R(K_3, G) > cn$ [Bra3].

(p) $R(K_3, K_{k,k}) = \Theta(k^2/\log k)$ [LinLi2].

(q) For $R(K_3, K_n)$ see section 2, and for $R(K_3, K_n - e)$ see section 3.1.

(r) Since $B_1 = F_1 = C_3 = W_3 = K_3$, other sections apply. See also [Boh, AjKS, BBH1, BBH2, FrLo, Fra1, Fra2, Für, Gri, GySeT, Loc, KlaM1, LiZa1, RK2, RK3, RK4, She1, She3, Spe2, Stat, Yu1].

3.3. Complete bipartite graphs

Note: This subsection gathers information on Ramsey numbers where specific bipartite graphs are avoided in edge colorings of K_n (as everywhere in this survey), in contrast to the often studied bipartite Ramsey numbers, which are not covered in this survey, where the edges of complete bipartite graphs $K_{n,m}$ are colored.

3.3.1. Numbers

The following Tables IVa and IVb gather information mostly from the surveys by Lortz and Mengersen [LoM3, LoM4]. All cases involving $K_{1,2} = P_3$ are solved by a formula for $R(P_3, G)$, which holds for all isolate-free graphs G, derived in [CH2]. All star versus star numbers are given below in the item 3.3.2.a and in section 5.5.

m,n \ p,q	1, 2	1, 3	1, 4	1, 5	1, 6	2, 2	2, 3	2, 4	2, 5	3, 3	3, 4
2, 2	4 CH2	6 CH2	7 Par3	8 Par3	9 FRS4	6 CH1					
2, 3	5 CH2	7 FRS4	9 Stev	10 FRS4	11 FRS4	8 HaMe4	10 Bu4				
2, 4	6 CH2	8 HaMe3	9 Stev	11 HaMe4	13 LoM4	9 HaMe4	12 ExRe	14 EHM2			
2, 5	7 CH2	9 HaMe3	11 Stev	13 Stev	14 LoM4	11 HaMe4	13 LoM3	16 LoM1	18 EHM2		
2, 6	8 CH2	10 HaMe3	11 Stev	14 Stev	15* Shao	12 HaMe4	14 LoM3	17 LoM3	20 LoM1		
3, 3	7 CH2	8 HaMe3	11 LoM4	12 LoM4	13 LoM4	11 Lortz	13 HaMe3	16 LoM4	18 LoM4	18 HaMe3	
3, 4	7 CH2	9 HaMe3	11 LoM4	13 LoM4	14 LoM4	11 Lortz	14 LoM4	17 Sh1+	≤21 LoM4	≤25 LoM2	≤30 LoM2
3, 5	9 CH2	10 HaMe3	13 Sh1+	15 Sh1+		14 HaMe4	17* Shao		≥21 Sh2+	≤28 LoM2	≤33 LoM2

Table IVa. Ramsey numbers $R(K_{m,n}, K_{p,q})$;
unpublished results are marked with a *, and Sh1+, Sh2+ abbreviate ShaXBP, ShaoWX.

n \ m	2	3	4	5	6	7	8	9	10	11
6	12 HaMe4	14 LoM3	17 LoM3	20 LoM1	21 EHM2					
7	14 HaMe4	17 LoM3	19 LoM3	21 LoM3	24 LoM1	26 EMH2				
8	15 HaMe4	18 LoM3	20 LoM3	22*-23 LoM3	24-25 LoM3	28 LoM1	30 EMH2			
9	16 HaMe4	19 LoM3	22 LoM3	25* Shao	27* Shao	29* Shao	32 LoM1	33 EHM2		
10	17 HaMe4	21 LoM3	24 LoM3	27 LoM3	27-29 LoM3	28-31 LoM3	32-33 LoM3	36 LoM1	38 EHM2	
11	18 HaMe4						≤35 LoM3	36-37 LoM3	40 LoM1	42 EHM2

Table IVb. Known Ramsey numbers $R(K_{2,n}, K_{2,m})$ for $6 \le n \le 11$, $2 \le m \le 11$;
unpublished results improving over [LoM3] are marked with a *.

(a) The next few easily computed values of $R(K_{1,n}, K_{2,2})$, extending data in the first row of Table IVa, are 13, 14, 21 and 22 for n equal to 9, 10, 16 and 17, respectively. See function $f(n)$ in 3.3.2.c of the next subsection below.

(b) Formula for $R(K_{1,n}, K_{k_1,k_2,\ \cdots\ ,k_t,m})$ for m large enough, in particular for $t=1, k_1=2$ with $n \le 5, m \ge 3$ and $n=6, m \ge 11$, for example $R(K_{1,5}, K_{2,7}) = 15$ [Stev].

(c) The values and bounds for higher cases of $R(K_{2,2}, K_{2,n})$ are 20, 22, 22, 24, 25, 26, 27/28, 28/29, 30 and 32 for $12 \le n \le 21$, respectively. All of them were given in [HaMe4], except those for $n = 14$, 15 and 18, which were obtained in [Dyb]. More exact values for prime powers $\lceil \sqrt{n} \rceil$ and $\lceil \sqrt{n} \rceil + 1$ can be found in [HaMe4].

(d) The known values of $R(K_{2,2}, K_{3,n})$ are 15, 16, 17, 20 and 22 for $6 \le n \le 10$ [Lortz], and $R(K_{2,2}, K_{3,12}) = 24$ [Shao]. See Tables IVa and IVb for the smaller cases, and [HaMe4] for upper bounds and values for some prime powers $\lceil \sqrt{n} \rceil$.

(e) $R(K_{2,n}, K_{2,n})$ is equal to 46, 50, 54, 57 and 62 for $12 \le n \le 16$, respectively.
The first open diagonal case is $65 \le R(K_{2,17}, K_{2,17}) \le 66$ [EHM2].
The status of all higher cases for $n < 30$ is listed in [LoM1].

(f) $R(K_{1,4}, K_{4,4}) = R(K_{1,5}, K_{4,4}) = 13$ [ShaXPB]
$R(K_{1,4}, K_{1,2,3}) = R(K_{1,4}, K_{2,2,2}) = 11$ [GuSL]
$R(K_{1,7}, K_{2,3}) = 13$ [Par4, Par6]
$R(K_{1,15}, K_{2,2}) = 20$ [La2]
$R(K_{2,2}, K_{4,4}) = 14$ [HaMe4]
$R(K_{2,2}, K_{4,5}) = 15$ [Shao]
$R(K_{2,2}, K_{4,6}) = 16$ [Shao]
$R(K_{2,2}, K_{5,5}) = R(K_{2,3}, K_{3,5}) = 17$ [Shao]

(g) A number of general upper and lower bounds for $R(K_{s,t}, K_{s,t})$, in particular for small fixed s, and for some slightly off-diagonal cases were obtained in [LoM2]. They can be used to derive the upper bounds for the cases listed in (h) and (i) below.

(h) Several lower bounds of the form $R(K_{s,t}, K_{s,t}) \ge m$ from distance colorings, a slightly more general concept than circular graphs, were presented in [HaKr2] for the following triples (s, t, m): (3,6,38), (3,7,42), (3,8,43), (3,9,54), (4,5,42), (4,6,43), (4,7,54), (5,5,54).

(i) $30 \le R(K_{3,5}, K_{3,5}) \le 38$ [HaKr2][LoM2]
$30 \le R(K_{4,4}, K_{4,4}) \le 62$ [HaKr2][LoM2]

3.3.2. General results

(a) $R(K_{1,n}, K_{1,m}) = n + m - \varepsilon$, where $\varepsilon = 1$ if both n and m are even and $\varepsilon = 0$ otherwise [Har1]. It is also a special case of multicolor numbers for stars obtained in [BuRo1].

(b) $R(K_{1,3}, K_{m,n}) = m + n + 2$ for $m, n \ge 1$ [HaMe3].

(c) $R(K_{1,n}, K_{2,2}) = f(n) \le n + \sqrt{n} + 1$, with $f(q^2) = q^2 + q + 1$ and $f(q^2+1) = q^2 + q + 2$ for every q which is a prime power [Par3]. Furthermore, $f(n) \ge n + \sqrt{n} - 6n^{11/40}$ [BEFRS4]. For more bounds on $f(n)$ see [Par5, Chen, ChenJ, MoCa, WuSZR].

(d) $R(K_{1,n+1}, K_{2,2}) \le R(K_{1,n}, K_{2,2}) + 2$ [Chen].

(e) $R(K_{2,\lambda+1}, K_{1,v-k+1})$ is either $v+1$ or $v+2$ if there exists a (v, k, λ)-difference set. This and other related results are presented in [Par4, Par5]. See also [GoCM, GuLi].

(f) Formulas and bounds on $R(K_{2,2}, K_{2,n})$, and bounds on $R(K_{2,2}, K_{m,n})$. In particular, we have $R(K_{2,2}, K_{2,k}) = n + k\sqrt{n} + c$, for $k = 2, 3, 4$, some prime powers $\lceil \sqrt{n} \rceil$ and $\lceil \sqrt{n} \rceil + 1$, and some $-1 \le c \le 3$ [HaMe4]. An improvement of the latter for some special cases of n was obtained in [Dyb].

(g) $R(K_{2,n}, K_{2,n}) \le 4n - 2$ for all $n \ge 2$, and the equality holds if and only if there exists a strongly regular $(4n-3, 2n-2, n-2, n-1)$-graph [EHM2].

(h) Conjecture that $4n - 3 \le R(K_{2,n}, K_{2,n}) \le 4n - 2$ for all $n \ge 2$. Many special cases are solved and several others are discussed in [LoM1].

(i) $R(K_{2,n-1}, K_{2,n}) \le 4n - 4$ for all $n \ge 3$, with the equality if there exists a symmetric Hadamard matrix of order $4n - 4$. There are only 4 cases in which the equality is still open for $3 \le n \le 58$, namely 30, 40, 44 and 48 [LoM1].

(j) $R(K_{2,n-s}, K_{2,n}) \le 4n - 2s - 3$ for $s \ge 2$ and $n \ge s + 2$, with the equality in many cases involving Hadamard matrices or strongly regular graphs. Asymptotics of $R(K_{2,n}, K_{2,m})$ for $m \gg n$ [LoM3].

(k) Some algebraic lower and upper bounds on $R(K_{s,n}, K_{t,m})$ for various combinations of n, m and $1 \le t, s \le 3$ [BaiLi, BaLX]. A general lower bound $R(K_{m,n}) \ge 2^m (n - n^{0.525})$ for large n [Dong].

(l) Upper bounds for $R(K_{2,2}, K_{m,n})$ for $m, n \ge 2$, with several cases identified for which the equality holds. Special focus on the cases for $m = 2$ [HaMe4].

(m) Bounds for the numbers of the form $R(K_{k,n}, K_{k,m})$, specially for fixed k and close to the diagonal cases. Asymptotics of $R(K_{3,n}, K_{3,m})$ for $m \gg n$ [LoM2].

(n) $R(nK_{1,3}, mK_{1,3}) = 4n + m - 1$ for $n \ge m \ge 1$, $n \ge 2$ [BES].

(o) Asymptotics for $K_{2,m}$ versus K_n [CaLRZ]. Upper bound asymptotics for $K_{k,m}$ versus K_n [LiZa1] and for some bipartite graphs K_n [JiSa].

(p) Special two-color cases apply in the study of asymptotics for multicolor Ramsey numbers for complete bipartite graphs [ChGra1].

4. Two Colors: Numbers Involving Cycles

4.1. Cycles, cycles versus paths and stars

Note: The paper *Ramsey Numbers Involving Cycles* [Ra4] is based on the revision #12 of this survey. It collects and comments on the results involving cycles versus any graphs, in two or more colors. It contains some more details than this survey, but only until 2009.

Cycles

(a) $R(C_3, C_3) = 6$ [GG, Bush],
$R(C_4, C_4) = 6$ [CH1].

(b) $R(C_3, C_n) = 2n-1$ for $n \geq 4$, $R(C_4, C_n) = n+1$ for $n \geq 6$,
$R(C_5, C_n) = 2n-1$ for $n \geq 5$, and $R(C_6, C_6) = 8$ [ChaS].

(c) Result obtained independently in [Ros1] and [FS1], a new simpler proof in [KáRos]:

$$R(C_m, C_n) = \begin{cases} 2n-1 & \text{for } 3 \leq m \leq n,\ m \text{ odd}, (m,n) \neq (3,3), \\ n-1+m/2 & \text{for } 4 \leq m \leq n,\ m \text{ and } n \text{ even}, (m,n) \neq (4,4), \\ \max\{n-1+m/2, 2m-1\} & \text{for } 4 \leq m < n,\ m \text{ even and } n \text{ odd}. \end{cases}$$

(d) Characterization of all graphs critical for $R(C_4, C_n)$ [WuSR].

(e) $R(mC_3, nC_3) = 3n + 2m$ for $n \geq m \geq 1$, $n \geq 2$ [BES].

(f) $R(mC_4, nC_4) = 2n + 4m - 1$ for $m \geq n \geq 1$, $(n,m) \neq (1,1)$ [LiWa1].

(g) Formulas for $R(mC_4, nC_5)$ [LiWa2].

(h) Formulas and bounds for $R(nC_m, nC_m)$ [Den2, Biel1].

(i) Study of $R(S_1, S_2)$, where S_1 and S_2 are sets of cycles [Hans].

(j) Unions of cycles, formulas and bounds for various cases including diagonal, different lengths, different multiplicities [MiSa, Den2], powers of cycles [AllBS], disjoint cycles versus K_n [Fuj2], and their relation to 2-local Ramsey numbers [Biel1].

Cycles versus paths

Result obtained by Faudree, Lawrence, Parsons and Schelp in 1974 [FLPS]:

$$R(C_m, P_n) = \begin{cases} 2n-1 & \text{for } 3 \leq m \leq n,\ m \text{ odd}, \\ n-1+m/2 & \text{for } 4 \leq m \leq n,\ m \text{ even}, \\ \max\{m-1+\lfloor n/2 \rfloor, 2n-1\} & \text{for } 2 \leq n \leq m,\ m \text{ odd}, \\ m-1+\lfloor n/2 \rfloor & \text{for } 2 \leq n \leq m,\ m \text{ even}. \end{cases}$$

For all n and m it holds that $R(P_m, P_n) \leq R(C_m, P_n) \leq R(C_m, C_n)$. Each of the two inequalities can become an equality, and, as derived in [FLPS], all four possible combinations of $<$ and $=$ hold for an infinite number of pairs (m,n). For example, if both m and n are even, and at least one of them is greater than 4, then $R(P_m, P_n) = R(C_m, P_n) = R(C_m, C_n)$.

For related generalizations see [BEFRS2].

Cycles versus stars

Only partial results for C_m versus stars are known. Lawrence [La1] settled the cases for odd m and for long cycles (see also [Clark, Par6]). The case for short even cycles is open, and it is related in particular to bipartite graphs. Partial results for $C_4 = K_{2,2}$ are pointed to in subsections 3.3.1 and 3.3.2. The most known exact result in [La1] is:

$$R(C_m, K_{1,n}) = \begin{cases} 2n+1 & \text{for odd } m \le 2n+1, \\ m & \text{for } m \ge 2n. \end{cases}$$

4.2. Cycles versus complete graphs

Since 1976, it was conjectured that $R(C_n, K_m) = (n-1)(m-1)+1$ for all $n \ge m \ge 3$, except $n = m = 3$ [FS4, EFRS2]. Various parts of this conjecture were proved as follows: for $n \ge m^2 - 2$ [BoEr], for $n > 3 = m$ [ChaS], for $n \ge 4 = m$ [YHZ1], for $n \ge 5 = m$ [BolJY+], for $n \ge 6 = m$ [Schi1], for $n \ge m \ge 7$ with $n \ge m(m-2)$ [Schi1], for $n \ge 7 = m$ [ChenCZ1], and for $n \ge 4m+2$, $m \ge 3$ [Nik]. Open conjectured cases are marked in Table V by "conj."

	C_3	C_4	C_5	C_6	C_7	C_8	C_9	...	C_n for $n \ge m$
K_3	6 GG-Bush	7 ChaS	9 ...	11	13	15	17	... ...	$2n-1$ ChaS
K_4	9 GG	10 CH2	13 He4/JR4	16 JR2	19 YHZ1	22 ...	25	... ...	$3n-2$ YHZ1
K_5	14 GG	14 Clan	17 He2/JR4	21 JR2	25 YHZ2	29 BolJY+	33 ...	... ...	$4n-3$ BolJY+
K_6	18 Kéry	18 Ex2-RoJa1	21 JR5	26 Schi1	31 ...	36	41	... ...	$5n-4$ Schi1
K_7	23 Ka2-GrY	22 RT-JR1	25 Schi2	31 CheCZN	37 CheCZN	43 JaBa/Ch+	49 Ch+	... ...	$6n-5$ Ch+
K_8	28 GR-MZ	26 RT	29-33 JaAl2	36 ChenCX	43 ChenCZ1	50 JaAl1/ZZ3	57 BatJA	... ...	$7n-6$ conj.
K_9	36 Ka2-GR	30 RT-LivLR					65 conj.	... ...	$8n-7$ conj.
K_{10}	40-42 Ex5-GoR1	36 LivLR						...	$9n-8$ conj.
K_{11}	47-50 Ex20-GoR1	39-44 LivLR						...	$10n-9$ conj.

Table V. Known Ramsey numbers $R(C_n, K_m)$;
Ch+ abbreviates ChenCZ1, for comments on joint credits see 4.2.b.

(a) The first column in Table V gives data from the first row in Table I.

(b) Joint credit [He2/JR4] in Table V refers to two cases in which Hendry [He2] announced the values without presenting the proofs, which later were given in [JR4]. The special cases of $R(C_6, K_5) = 21$ [JR2] and $R(C_7, K_5) = 25$ were solved independently in [YHZ2] and [BolJY+]. The double pointer [JaBa/ChenCZ1] refers to two independent papers, similarly as [JaAl1/ZZ3], except that in the latter case [ZZ3] refers to an unpublished manuscript. For joint credits marked in Table V with "-", the first reference is for the lower bound and the second for the upper bound.

(c) Erdős et al. [EFRS2] asked what is the minimum value of $R(C_n, K_m)$ for fixed m, and they suggested that it might be possible that $R(C_n, K_m)$ first decreases monotonically, then attains a unique minimum, then increases monotonically with n.

(d) There exist constants $c_1, c_2 > 0$ such that $c_1(m^{3/2}/\log m) \le R(C_4, K_m) \le c_2(m/\log m)^2$. The lower bound, recently obtained by Bohman and Keevash ([BohK1], see also 4.2.h below) improved over an almost 40 years old bound $c(m/\log m)^{3/2}$ by Spencer [Spe2], using the probabilistic method. The upper bound was reported in a paper by Caro, Li, Rousseau and Zhang [CaLRZ], who in turn give the credit to an unpublished work by Szemerédi from 1980.

(e) Erdős, in 1981, in the Ramsey problems section of the paper [Erd3] formulated a challenge by asking for a proof of $R(C_4, K_m) < m^{2-\varepsilon}$, for some $\varepsilon > 0$. To date, no such proof is known.

(f) Let $C_{\le m}$ be the set of cycles of length at most m, and let the *girth* $g(G)$ be the length of the shortest cycle in graph G. Probabilistic lower bound asymptotics for $R(C_{\le m}, K_k)$ [Spe2] currently is the same as for $R(C_m, K_k)$, for fixed m. However, there are clear differences already for girth 4 and 5 and small k: Backelin [Back1, Back2] found that $R(C_{\le 4}, K_k)$ = 6, 8, 11, 15, 18 for k = 3, 4, 5, 6, 7, and that $R(C_{\le 5}, K_k)$ = 5, 8, 10, 13, 15, also for k = 3, 4, 5, 6, 7, respectively.

(g) Erdős et al. [EFRS2] proved various facts about $R(C_{\le m}, K_k)$, and in particular that it is equal to $2n-1$ for $m \ge 2n-1$, and to $2n$ for $n < m < 2n-1$. The upper asymptotics for $R(C_{\le m}, K_k)$ is implied in the study of independence number in graphs with odd girth m [Den1].

(h) The best known lower bound asymptotics $R(C_n, K_m) = \Omega(m^{(n-1)/(n-2)}/\log m)$, for fixed n and large m, was obtained by Bohman and Keevash [BohK1]. Note that for $n = 4$ it gives the lower bound in 4.2.d above. See also [Spe2, FS4, AlRö] for previous results.

(i) Upper bound asymptotics [BoEr, FS4, EFRS2, CaLRZ, Sud1, LiZa2, AlRö, DoLL2].

4.3. Cycles versus wheels

Note: In this survey the wheel graph $W_n = K_1 + C_{n-1}$ has n vertices,
while some authors use the definition $W_n = K_1 + C_n$ with $n+1$ vertices.
For the cases involving $W_3 = C_3$ versus C_m see sections 3.2 and 4.2.

	C_3	C_4	C_5	C_6	C_7	C_8	C_m	for
W_4	9 GG	10 CH2	13 He4	16 JR2	19 YHZ1	22 ...	$3m-2$ ...	$m \geq 4$ YHZ1
W_5	11 Clan	9 Clan	9 He2	11 JR2	13 SuBB2	15 ...	$2m-1$ ...	$m \geq 5$ SuBB2
W_6	11 BE3	10 JR3	13 ChvS	16 SuBB2	19 ...	22 ...	$3m-2$ ...	$m \geq 4$ SuBB2
W_7	13 BE3	9 Tse1	13 LuLL	11 LuLL	13 LuLL		$2m-1$ ...	$m \geq 10$ Ch1
W_8	15 BE3	11 Tse1	15 LuLL	16 LuLL	19 Ch2	22 ...	$3m-2$ ...	$m \geq 6$ Ch2
W_9	17 BE3	12 Tse1	17 LuLL	13 LuLL	17 LuLL		$2m-1$	$m \geq 13$ Ch1
W_{10}	19 BE3	13 Tse1					$3m-2$	$m \geq 9$ Ch2
W_n for	... $2n-1$ $n \geq 6$ BE3		$2n-1$ $n \geq 19$ Zhou2		$2n-1$ $n \geq 29$ Zhou2		large wheels	cycles

Table VI. Ramsey numbers $R(W_n, C_m)$ for $n \leq 10$, $m \leq 8$;
Ch1 abbreviates ChenCMN, Ch2 abbreviates ChenCNZ.

(a) $R(C_3, W_n) = 2n-1$ for $n \geq 6$ [BE3]. All critical graphs have been enumerated. The critical graphs are unique for $n = 3, 5$, and for no other n [RaJi].

(b) $R(C_4, W_n) = 14, 16, 17$ for $n = 11, 12, 13$, respectively [Tse1], $R(C_4, W_n) = 18, 19, 20, 21$ for $n = 14, 15, 16, 17$, respectively [DyDz2], and several higher values and bounds, including 9 cases of n between 18 and 44 [WuSR, WuSZR].

(c) $R(C_4, W_n) \leq n + \lceil (n-1)/3 \rceil$ for $n \geq 7$ [SuBUB], which was improved to $R(C_4, W_n) \leq n + \sqrt{n-2} + 1$ for $n \geq 11$ [DyDz2].

(d) $R(C_4, W_{q^2+1}) = q^2 + q + 1$ for prime power $q \geq 4$ [DyDz2], exact values of $R(C_4, W_{q^2+2})$ and $R(C_4, W_{q^2-i})$ for special q and small i [WuSZR].

(e) $R(W_n, C_m) = 2n-1$ for odd m with $n \geq 5m-6$ [Zhou2].

(f) $R(W_n, C_m) = 3m-2$ for even $n \geq 4$ with $m \geq n-1$, $m \neq 3$, was conjectured by Surahmat et al. [SuBT1, SuBT2, Sur]. Parts of this conjecture were proved in [SuBT1, ZhaCC1, Shi5], and the proof was completed in [ChenCNZ].

(g) Conjecture that $R(W_n, C_m) = 2m-1$ for odd $n \geq 3$ and all $m \geq 5$ with $m > n$ [Sur]. It was proved for $2m \geq 5n-7$ [SuBT1], and further for $2m \geq 3n-1$ [ChenCMN]. See also [Shi5].

(h) Observe apparently four distinct situations with respect to parity of m and n.

(i) Cycles are Ramsey unsaturated for some wheels [AliSur], see also comments on [BaLS] in subsection 5.16.

(j) Study of cycles versus generalized wheels $W_{k,n}$ [Sur, SuBTB, Shi5].

4.4. Cycles versus books

	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_m	for
B_2	7 RS1	7 F*al*6	9 C*al*	11 F*al*8	13 ...	15	17	19	21	$2m-1$ ...	$m \geq 4$ F*al*8
B_3	9 RS1	9 F*al*6	10 F*al*8	11 JR2	13 Shi5	15 F*al*8	17 ...	19	21	$2m-1$ ...	$m \geq 6$ F*al*8
B_4	11 RS1	11 F*al*6	11 F*al*8	12 S*al*1	13 S*al*1	15 Shi5	17 Shi5	19 F*al*8	21 ...	$2m-1$ ...	$m \geq 7$ F*al*8
B_5	13 RS1	12 F*al*6	13 F*al*8	14 S*al*1	15 S*al*1	15 S*al*2	17 S*al*2	19 Shi5	21 Shi5	$2m-1$ ...	$m \geq 8$ F*al*8
B_6	15 RS1	13 F*al*6	15 F*al*8	16 S*al*2	17 S*al*2	18 S*al*2	18 S*al*2		21 Shi5	$2m-1$ ...	$m \geq 11$ Shi5
B_7	17 RS1	16 F*al*6	17 F*al*8	16 S*al*2	19 S*al*2	20 S*al*2	21 S*al*2			$2m-1$	$m \geq 13$ Shi5
B_8	19 RS1	17 Tse1	19 F*al*8	17 S*al*2	19 S*al*2	22 S*al*2	≥ 23 S*al*2			$2m-1$	$m \geq 14$ Shi5
B_9	21 RS1	18 Tse1	21 F*al*8	18 S*al*2			≥ 25 S*al*2	≥ 26 S*al*2		$2m-1$	$m \geq 16$ Shi5
B_{10}	23 RS1	19 Tse1	23 F*al*8	19 S*al*2				≥ 28 S*al*2		$2m-1$	$m \geq 17$ Shi5
B_{11}	25 RS1	20 Tse1	25 F*al*8							$2m-1$	$m \geq 19$ Shi5
B_n for	... $2n+3$ $n \geq 2$ RS1	 $\approx n$ some (c)	... $2n+3$ $n \geq 4$ F*al*8		$2n+3$ $n \geq 15$ F*al*8		$2n+3$ $n \geq 23$ F*al*8		$2n+3$ $n \geq 31$ F*al*8	 large books	cycles

Table VII. Ramsey numbers $R(B_n, C_m)$ for $n, m \leq 11$;
et al. abbreviations: F*al*/FRS, C*al*/CRSPS, S*al*1/ShaXBP, S*al*2/ShaXB.

(a) For the cases of $B_1 = K_3$ versus C_m see section 4.2.
The exact values for the cases (3,7), (4,8), (4,9), (5,10), (5,11) were obtained independently in [S*al*1, S*al*2]/[ShaXBP, ShaXB] using computer algorithms.

(b) $R(C_4, B_{12}) = 21$ [Tse1], $R(C_4, B_{13}) = 22$, $R(C_4, B_{14}) = 24$ [Tse2].
$R(C_4, B_8) = 17$ [Tse2] (it was reported incorrectly in [FRS7] to be 16).

(c) $q^2 + q + 2 \le R(C_4, B_{q^2 - q + 1}) \le q^2 + q + 4$ for prime power q [FRS7]. B_n is a subgraph of B_{n+1}, hence likely $R(C_4, B_n) = n + O(\sqrt{n})$ (compare to $R(C_4, K_{2,n})$ in section 3.3).

(d) $R(B_n, C_m) = 2n + 3$ for odd $m \ge 5$ with $n \ge 4m - 13$ [FRS9].

(e) $R(B_n, C_m) = 2m - 1$ for $n \ge 1$, $m \ge 2n + 2$ [FRS9]. The range of m was extended to $m \ge 2n - 1 \ge 7$ in [ShaXB], and to $m > (6n + 7)/4$ in [Shi5].

(f) $R(B_n, C_n) \ge 3n - 2$ and $R(B_{n-1}, C_n) \ge 3n - 4$ for $n \ge 3$ [ShaXB].

(g) More theorems on $R(B_n, C_m)$ in [FRS7, FRS9, NiRo4, Zhou1].

(h) Cycles versus some generalized books [Shi5].

4.5. Cycles versus other graphs

(a) C_4 versus stars [Par3, Par4, Par5, BEFRS4, Chen, ChenJ, GoMC, MoCa, WuSZR]. For several exact results see $K_{2,2}$ in Tables IVa and IVb, and for general results see items 3.3.1.a, 3.3.2.c and 3.3.2.d.

(b) C_4 versus unions of stars [HaABS, Has]

(c) C_4 versus trees [EFRS4, Bu7, BEFRS4, Chen]

(d) C_4 versus all graphs on six vertices [JR3]

(e) C_4 versus various types of complete bipartite graphs, see section 3.3

(f) $R(C_4, G) \le 2q + 1$ for any isolate-free graph G with q edges [RoJa2]

(g) $R(C_4, G) \le p + q - 1$ for any connected graph G on p vertices and q edges [RoJa2]

(h) $R(C_5, K_6 - e) = 17$ [JR4]

(i) $R(C_5, K_4 - e) = 9$ [CRSPS]

(j) C_5 versus all graphs on six vertices [JR4]

(k) $R(C_6, K_5 - e) = 17$ [JR2]

(l) C_6 versus all graphs on five vertices [JR2]

(m) $R(C_{2m+1}, G) = 2n - 1$ for sufficiently large sparse graphs G on n vertices, in particular $R(C_{2m+1}, T_n) = 2n - 1$ for all $n > 1512m + 756$, for n-vertex trees T_n [BEFRS2].

(n) $R(C_n, G) \le 2q + \lfloor n/2 \rfloor - 1$, for $3 \le n \le 5$, for any isolate-free graph G with $q > 3$ edges. It is conjectured that it also holds for other n [RoJa2].

(o) Cycles versus trees [BEFRS2, FSS1]

(p) Monotone paths and cycles [Lef]

(q) Cycles versus $K_{n,m}$ and multipartite complete graphs [BoEr]

(r) Cycles versus generalized books and wheels [Shi5, Sur, SuBTB], and versus other special graphs of the form $K_n + G$ with small $n \le 3$ and sparse G [Shi5].

5. General Graph Numbers in Two Colors

This section includes data with respect to general graph results. We tried to include all nontrivial values and identities regarding exact results, or references to them, but only those out of general bounds and other results which, in our opinion, may have a direct connection to the evaluation of specific numbers. If some small value cannot be found below, it may be covered by the cumulative data gathered in section 8, or be a special case of a general result listed in this section. Note that $P_2 = K_2$, $B_1 = F_1 = C_3 = W_3 = K_3$, $B_2 = K_4 - e$, $P_3 = K_3 - e$, $W_4 = K_4$ and $C_4 = K_{2,2}$ imply other identities not mentioned explicitly.

5.1. Paths

$R(P_m, P_n) = n + \lfloor m/2 \rfloor - 1$ for all $n \geq m \geq 2$ [GeGy]

Stripes mP_2 [CocL1, CocL2, Lor]

Disjoint unions of paths (also called linear forests) [BuRo2, FS2]

5.2. Wheels

Note: In this survey the wheel graph $W_n = K_1 + C_{n-1}$ has n vertices, while some authors use the definition $W_n = K_1 + C_n$ with $n+1$ vertices.

m \ n	3	4	5	6	7
3	6	9 GG	11 Clan	11 BE3	13 BE3
4		18 GG	17 He3	19 FM	
5			15 He2	17 FM	
6				17 FM	

Table VIII. Ramsey numbers $R(W_m, W_n)$ for $m \leq n \leq 7$.

(a) $R(W_3, W_n) = 2n-1$ for all $n \geq 6$ [BE3],
All critical colorings for $R(W_3, W_n)$ for all $n \geq 3$ [RaJi].

(b) The value $R(W_5, W_5) = 15$ was given in the Hendry's table [He2] without a proof. Later the proof was published in [HaMe2].

(c) All critical colorings (2, 1 and 2) for $R(W_n, W_6)$, for n = 4, 5, 6 [FM].

(d) $R(W_6, W_6)$ = 17, $R(4,4)$ = 18 and $\chi(W_6) = 4$ give a counterexample $G = W_6$ to the Erdős conjecture (Erd2, see also [GRS]) that $R(G, G) \geq R(K_{\chi(G)}, K_{\chi(G)})$.

5.3. Books

m \ n	1	2	3	4	5	6	7
1	6	7 CH2	9 Clan	11 RS1	13 RS1	15 RS1	17 RS1
2		10 CH1	11 Clan	13 Rou	16 RS1	17 Rou	18 BlLR
3			14 RS1	15 Sh1+	17 RS1		
4				18 RS1	≤ 20 RS1	22 RS1	
5					21 RS1		
6						26 RS1	

Table IX. Ramsey numbers $R(B_m, B_n)$ for $m, n \leq 7$;
Sh1+ abbreviates ShaXBP.

(a) $254 \leq R(B_{37}, B_{88}) \leq 255$ [Par6].

(b) Unpublished result $R(B_2, B_6) = 17$ [Rou] was confirmed in [BlLR].

(c) There are 4 Ramsey-critical graphs for $R(B_2, B_3)$, a unique graph for $R(B_3, B_4)$ [ShaXBP], 3 for $R(B_2, B_6)$ and 65 for $R(B_2, B_7)$ [BlLR].

(d) $R(B_1, B_n) = 2n + 3$ for all $n > 1$ [RS1].

(e) $R(B_n, B_m) = 2n + 3$ for all $n \geq cm$ for some $c < 10^6$ [NiRo2, NiRo3].

(f) $R(B_n, B_n) = (4 + o(1))n$ [RS1, NiRS].

(g) In general, $R(B_n, B_n) = 4n + 2$ for $4n + 1$ a prime power. Several other specific values (like $R(B_{62}, B_{65}) = 256$) and general equalities and bounds for $R(B_n, B_m)$ can be found in [RS1, FRS8, Par6, NiRS, LiRZ2].

5.4. Trees and forests

In this subsection T_n and F_n denote n-vertex tree and forest, respectively.

(a) $R(T_n, T_n) \leq 4n + 1$ [EG].

(b) $R(T_n, T_n) \geq \lfloor (4n - 1)/3 \rfloor$ [BE2], see also section 5.15.

(c) Conjecture that $R(T_n, T_n)$ is at most $2n - 2$ for even n and $2n - 3$ for odd n [BE2]. Note that this is the same as asking if $R(T_n, T_n) \leq R(K_{1,n-1}, K_{1,n-1})$. Zhao proved that $R(T_n, T_n) \leq 2n - 2$ and thus confirmed the conjecture for even n. Independently, Ajtai

et al. [AjKSS] announced a full proof for large n. This recent progress subsumes some of the results pointed to in items (d)-(l) below.

(d) For general discussion of related problems see [Bu7, FSS1, ChGra2], in particular of the conjecture that $R(T_m, T_n) \le n+m-2$ holds for all trees [FSS1].

(e) If $\Delta(T_m) = m-2$ and $\Delta(T_n) = n-2$ then the exact values of $R(T_m, T_n)$ are known, and they are between $n+m-5$ and $n+m-3$ depending on n and m. In particular, for $n = 2k+1$ we have $R(T_{2k+1}, T_{2k+1}) = 2n-5$ [GuoV].

(f) Examples of families T_m and T_n (including P_n) for which $R(T_m, T_n) = n+m-c$, $c = 3, 4, 5$ [SunZ], extending the results in [GuoV].

(g) View tree T as a bipartite graph with parts t_1 and t_2, $t_2 \ge t_1$. Define $b(T) = \max\{2t_1+t_2-1, 2t_2-1\}$. Then the bound $R(T,T) \ge b(T)$ holds always, $R(T,T) = b(T)$ holds for many classes of trees [EFRS3, GeGy], and asymptotically [HaŁT], but cases for nonequality have been found [GHK].

(h) Comments in [BaLS] about some conjectures on Ramsey saturation of non-star trees, which would imply that $R(T_n, T_n) \le 2n-2$ holds for sufficiently large n.

(i) Formulas for $R(T_m, T_n)$ for some subcases of when T_m and T_n satisfy $\Delta(T_m) = m-3$ and $\Delta(T_n) \le n-3$ [SunWW].

(j) $R(T_m, K_{1,n}) \le m+n-1$, with equality for $(m-1) \mid (n-1)$ [Bu1].

(k) $R(T_m, K_{1,n}) = m+n-1$ for sufficiently large n for almost all trees T_m [Bu1]. Many cases were identified for which $R(T_m, K_{1,n}) = m+n-2$ [Coc, ZZ1], see also [Bu1].

(l) $R(T_m, K_{1,n}) \le m+n$ if T_n is not a star and $(m-1) \nmid (n-1)$,
some classes of trees and stars for which the equality holds [GuoV].

(m) $R(F_n, F_n) > n + \log_2 n - O(\log\log n)$ [BE2], forests are tight for this bound [CsKo].

(n) Forests, linear forests (unions of paths) [BuRo2, FS3, CsKo].

(o) Paths versus trees [FSS1], see also other parts of this survey involving special graphs, in particular sections 5.5, 5.6, 5.10, 5.12 and 5.15.

5.5. Stars, stars versus other graphs

$R(K_{1,n}, K_{1,m}) = n+m-\varepsilon$, where $\varepsilon = 1$ for even n and m, and $\varepsilon = 0$ otherwise [Har1]. This is also a special case of multicolor numbers for stars 6.6.e obtained in [BuRo1].

$R(K_{1,n}, K_m) = n(m-1)+1$ by Chvátal's theorem [Chv].

Stars versus C_4 [Par3, Par4, Par5, BEFRS4, Chen, ChenJ, GoMC, MoCa, WuSZR]
Stars versus $K_{2,n}$ [Par4, GoMC]
Stars versus $K_{n,m}$ [Stev, Par3, Par4]
See also section 3.3

$R(K_{1,4}, B_4) = 11$ [RS2]
$R(K_{1,4}, K_{1,2,3}) = R(K_{1,4}, K_{2,2,2}) = 11$ [GuSL]

Stars versus paths [Par2, BEFRS2]
Stars versus cycles [La1, Clark], see also [Par6] and section 4.1
Stars versus $2K_2$ [MeO]
Stars versus stripes mP_2 [CocL1, CocL2, Lor]

Stars versus W_5 and W_6 [SuBa1]
$nK_{1,m}$ versus W_5 [BaHA]
Stars versus W_9 [Zhang2, ZhaCZ1]
Stars versus wheels [HaBA1, ChenZZ2, Kor]
Stars versus books [CRSPS, RS2]
Stars versus trees [Bu1, Cheng, Coc, GuoV, SunZ, ZZ1]
Stars versus $K_n - tK_2$ [Hua1, Hua2]
Union of two stars [Gros2]
Unions of stars versus C_4 and W_5 [HaABS, Has]
Unions of stars versus wheels [BaHA, HaBA2, SuBAU1]

5.6. Paths versus other graphs

Note: for cycles versus P_n see section 4.1.

P_3 versus all isolate-free graphs [CH2]
Paths versus stars [Par2, BEFRS2]
Paths versus trees [FS4, FSS1, SunZ]
Paths versus books [RS2]
Paths versus K_n [Par1]
Paths versus $2K_n$ [SuAM, SuAAM]
Paths versus $K_{n,m}$ [Häg]
Paths versus some balanced complete multipartite graphs [Pokr]
Paths versus W_5 and W_6 [SuBa1]
Paths versus W_7 and W_8 [Bas]
Paths versus wheels [BaSu, ChenZZ1, SaBr3, Zhang1]
$R(P_n, mW_4) = 2n + m - 2$ [Sudar]
Paths versus beaded wheels [AliBT2]
Paths versus powers of paths [Pokr, AllBS]
Paths versus fans [SaBr2]
Paths versus $K_1 + P_m$ [SaBr1, SaBr4]
Paths and cycles versus trees [FSS1]

Powers of paths [AllBS]
Unions of paths [BuRo2]
Paths and unions of paths versus Jahangir graphs [AliBas, AliBT1, AliSur]
Paths and unions of paths versus $K_{2m} - mK_2$ [AliBB]

Sparse graphs versus paths and cycles [BEFRS2]
Graphs with long tails [Bu2, BG]
Monotone paths and cycles [Lef]

5.7. Fans, fans versus other graphs

$R(F_1, F_n) = R(K_3, F_n) = 4n+1$ for $n \geq 2$, and bounds for $R(F_m, F_n)$ [LiR2, GGS]

$R(F_2, F_n) = 4n+1$ for $n \geq 2$ and $R(F_m, F_n) \leq 4n+2m$ for $n \geq m \geq 2$ [LinLi1]

$R(K_4, F_n) = 6n+1$ for $n \geq 3$ [SuBB3]

Fans versus paths, formulas for a number of cases including $R(P_6, F_n)$ [SaBr2].
Missing case $R(P_6, F_4) = 12$ solved in [Shao].

Fans versus cycles [Shi5]
Fans versus K_n [LiR2]
Lower bounds on $R(F_2, K_n)$ from cyclic graphs for $n \leq 9$ [Shao]

5.8. Wheels versus other graphs

Notes: In this survey the wheel graph $W_n = K_1 + C_{n-1}$ has n vertices,
while some authors use the definition $W_n = K_1 + C_n$ with $n+1$ vertices.
For cycles versus W_n see section 4.3.

$R(W_5, K_5 - e) = 17$ [He2][YH]
$R(W_5, K_5) = 27$ [He2][RST]
$R(W_5, K_6) \geq 33$, $R(W_5, K_7) \geq 43$ [Shao, ShaoWX]
W_5 and W_6 versus stars and paths [SuBa1]
W_5 versus $nK_{1,m}$ [BaHA]
W_5 versus unions of stars [Has]
W_5 and W_6 versus trees [BaSNM]
W_7 and W_8 versus paths [Bas]

W_7 versus trees T_n with $\Delta(T_n) \geq n-3$, other special trees T,
and for $n \leq 8$ [ChenZZ3, ChenZZ5, ChenZZ6]

W_7 and W_8 versus trees [ChenZZ4, ChenZZ5]
W_9 versus stars [Zhang2, ZhaCZ1, ZhaCC2]
W_9 versus trees of high degree [ZhaCZ2]

Wheels versus stars [HaBA1, ChenZZ2, Kor]
Wheels W_n, for even n, versus star-like trees [SuBB1]
Wheels versus paths [BaSu, ChenZZ1, SaBr3, Zhang1]
Wheels versus books [Zhou3]
Wheels versus unions of stars [BaHA, HaBA2, SuBAU1]
Wheels versus linear forests (disjoint unions of paths) [SuBa2]
Generalized wheels versus cycles [Shi5]
Upper asymptotics for $R(W_n, K_m)$ [Song5, SonBL]
Upper asymptotics for generalized wheels versus K_n [Song9]

5.9. Books versus other graphs

Note: for cycles versus B_n see section 4.4.

$R(B_3, K_4) = 14$ [He3]
$R(B_3, K_5) = 20$ [He2][BaRT]
$R(B_4, K_{1,4}) = 11$ [RS2]

Cyclic lower bounds for $R(B_m, K_n)$ for $m \le 7$, $n \le 9$
and for $R(B_3, K_n - e)$ for $n \le 7$ [Shao, ShaoWX]

Books versus paths [RS2]
Books versus stars [CRSPS, RS2]
Books versus trees [EFRS7]
Books versus K_n [LiR1, Sud2]
Books versus wheels [Zhou3]
Books versus $K_2 + C_n$ [Zhou3]
Books and $(K_1 + tree)$ versus K_n [LiR1]
Generalized books $K_3 + qK_1$ versus cycles [Shi5]
Generalized books $K_r + qK_1$ versus K_n [NiRo1, NiRo4]

5.10. Trees and forests versus other graphs

In this subsection T_n and F_n denote n-vertex tree and forest, respectively.

$R(T_n, K_m) = (n-1)(m-1)+1$ [Chv]

$R(T_n, C_{2m+1}) = 2n-1$ for all $n > 1512m + 756$ [BEFRS2]

$R(T_n, B_m) = 2n-1$ for all $n \ge 3m - 3$ [EFRS7]

$R(F_{nk}, K_m) = (n-1)(m-2)+nk$ for all forests F_{nk} consisting of k trees with n vertices each, also exact formula for all other cases of forests versus K_m [Stahl]

Exact results for almost all small $(n(G) \le 5)$ connected graphs G versus all trees [FRS4]

Trees versus stars [Bu1, Cheng, Coc, GuoV, ZZ1]
Trees versus paths [FS4, FSS1]
Trees versus C_4 [EFRS4, Bu7, BEFRSS5, Chen]
Trees versus cycles [FSS1, EFRS6]
Trees versus books [EFRS7]
Trees versus W_5 and W_6 [BaSNM]
Trees versus W_7 and W_8 [ChenZZ4, ChenZZ5]

Trees T_n with $\Delta(T_n) \ge n-3$, other special trees T,
and for $n \le 8$ versus W_7 [ChenZZ3, ChenZZ5, ChenZZ6]
Trees T_n with $\Delta(T_n) \ge n-4$ versus W_9 [ZhaCZ2]

Star-like trees versus odd wheels [SuBB1, ChenZZ3]
Trees versus $K_n + \overline{K}_m$ [RS2, FSR]
Trees versus bipartite graphs [BEFRS4, EFRS6]

Trees versus almost complete graphs [GoJa2]
Trees versus multipartite complete graphs [EFRS8, BEFRSGJ]

Linear forests versus $3K_3$ and $2K_4$ [SuBAU2]
Linear forests versus $2K_m$ [SuAAM]
Linear forests versus wheels [SuBa2]
Forests versus almost complete graphs [ChGP]
Forests versus complete graphs [BE1, Stahl, BaHA]

Study of graphs G for which all or almost all trees are G-good [BF, BEFRSGJ], see also section 5.15 and 5.16, item [Bu2], for the definition and more pointers.

See also various parts of this survey for special trees, and section 5.4.

5.11. Cases for $n(G)$, $n(H) \le 5$

Clancy [Clan], in 1977, presented a table of $R(G,H)$ for all isolate-free graphs G with $n(G) = 5$ and H with $n(H) \le 4$, except 5 entries. All five of the open entries have been solved as follows:

$R(B_3, K_4) = 14$	[He3]
$R(K_5, K_4 - e) = 16$	[BoH]
$R(W_5, K_4) = 17$	[He2]
$R(K_5 - e, K_4) = 19$	[EHM1]
$R(K_5, K_4) = R(4,5) = 25$	[MR4]

An interesting case in [Clan] is

$$R(K_4, K_5 - P_3) = R(K_4, K_4 + e) = R(4,4) = 18.$$

Hendry [He2], in 1989, presented a table of $R(G,H)$ for all graphs G and H on 5 vertices without isolates, except 7 entries. Five of the open entries have been solved:

$R(K_5, K_4 + e) = R(4,5) = 25$	[Ka1][MR4]
$R(K_5, K_5 - P_3) = 25$	[Ka1][Boza2, CalSR]
$R(K_5, B_3) = 20$	[He2][BaRT]
$R(K_5, W_5) = 27$	[He2][RST]
$R(W_5, K_5 - e) = 17$	[He2][YH]

The still open cases for K_5 versus $K_5 - e$ and K_5 are:

$30 \le R(K_5, K_5 - e) \le 34$	[Ex6][Ex8]
$43 \le R(K_5, K_5) \le 49$	[Ex4][MR5]

All critical colorings for the case $R(C_5 + e, K_5) = 17$ were found by Hendry [He5].

5.12. Mixed cases

$12 \le R(Q_3, Q_3)$, where Q_3 is the 8-vertex 3-dimensional cube graph,
$19 \le R(P, P)$, where P is the 10-vertex Petersen graph,
$30 \le R(K_{2,2,2}, K_{2,2,2})$, where $K_{2,2,2}$ is the octahedron [HaKr2].

Unicyclic graphs [Gros1, Köh, KrRod]
$K_{2,m}$ and C_{2m} versus K_n [CaLRZ]
$K_{2,n}$ versus any graph [RoJa2]
Union of two stars [Gros2]
Double stars* [GHK, BahS]
Brooms+ [EFRS3]
Graphs with bridge versus K_n [Li1]
Multipartite complete graphs [BFRS, FRS3, Stev]
Multipartite complete graphs versus trees [EFRS8, BEFRSGJ]
Multipartite complete graphs versus sparse graphs [EFRS4]
Graphs with long tails [Bu2, BG]

5.13. Multiple copies of graphs, disconnected graphs

(a) $2K_2$ versus all isolate-free graphs [CH2]

(b) nK_2 versus mK_2, in particular $R(nK_2, nK_2) = 3n - 1$ for $n \ge 1$ [CocL1, CocL2, Lor]

(c) nK_3 versus mK_3, in particular $R(nK_3, nK_3) = 5n$ for $n \ge 2$ [BES], see also section 4.1

(d) nK_3 versus mK_4 [LorMu]

(e) $nK_{1,m}$ versus W_5 [BaHA]

(f) $R(nK_4, nK_4) = 7n + 4$ for large n [Bu8]

(g) Stripes mP_2 [CocL1, CocL2, Lor]

(h) $R(G, H)$ for all disconnected isolate-free graphs H on at most 6 vertices versus all G on at most 5 vertices, except 3 cases [LoM5]. Missing cases were completed in [KroMe].

(i) $R(F, G \cup H) \le \max\{ R(F, G) + n(H), R(F, G) \}$ [Par6]

(j) $R(mG, nH) \le (m-1)n(G) + (n-1)n(H) + R(G, H)$ [BES],
Formulas for $R(nK_3, mG)$ for all isolate-free graphs G on 4 vertices [Zeng],
Variety of results for numbers $R(nG, mH)$ [Bu1, BES, HaBA2, SuBAU1].

(k) Disjoint unions of paths (linear forests) [BuRo2, FS2]
Linear forests versus $3K_3 \cup 2K_4$ [SuBAU2]

(l) Forests versus K_n [Stahl, BaHA] and W_n [BaHA]. Generalizations to forests versus other graphs G in terms of $\chi(G)$ and the chromatic surplus of G [Biel4], and for linear forests versus $2K_n$ [SuAM].

---

* double star is a union of two stars with their centers joined by an edge

+ broom is a star with a path attached to its center

(m) Disconnected graphs versus other graphs [BE1, GoJa1]

(n) See section 4.1 for cases involving unions of cycles

(o) See also [Bu9, BE1, LorMu, MiSa, Den2, Biel1, Biel2]

5.14. General results for special graphs

(a) $R(K_m^p, K_n^q) = R(K_m, K_n)$ for $m, n \geq 3$, $m + n \geq 8$, $p \leq m/(n-1)$ and $q \leq n/(m-1)$, where K_s^t is a K_s with additional vertex connected to it by t edges [BEFS]. Some applications can be found in [BlLR].

(b) $R(K_{2,k}, G) \leq kq + 1$, for $k \geq 2$, for isolate-free graphs G with $q \geq 2$ edges [RoJa2].

(c) $R(W_6, W_6) = 17$ and $\chi(W_6) = 4$ [FM]. This gives a counterexample $G = W_6$ to the Erdős conjecture (see [GRS]) $R(G,G) \geq R(K_{\chi(G)}, K_{\chi(G)})$, since $R(4,4) = 18$.

(d) $R(G + K_1, H) \leq R(K_{1,R(G,H)}, H)$ [BE1].

(e) $R(\overline{K}_2 + G, \overline{K}_2 + G) \leq 4R(G, \overline{K}_2 + G) - 2$ [LiShen].

(f) Study of $R(G + K_1, nH + K_1)$ [LinLD].

(g) $R(K_{p+1}, B_q^r) = p(q + r - 1) + 1$ for generalized books $B_q^r = K_r + qK_1$, for sufficiently large q [NiRo1].

(h) Study of the cases $R(K_m, K_n - K_{1,s})$ and $R(K_m - e, K_n - K_{1,s})$, with several exact values for special parameters [ChaMR].

(i) Study of $R(T + K_1, K_n)$ for trees T [LiR1]. Asymptotic upper bounds for $R(T + K_2, K_n)$ [Song7], see also [SonGQ].

(j) Bounds on $R(H + \overline{K}_n, K_n)$ for general H [LiR3]. Also, for fixed k and m, as $n \to \infty$, $R(K_k + \overline{K}_m, K_n) \leq (m + o(1))\, n^k/(\log n)^{k-1}$ [LiRZ1].

(k) Asymptotics of $R(H + \overline{K}_n, K_n)$. In particular, the order of magnitude of $R(K_{m,n}, K_n)$ is $n^{m+1}/(\log n)^m$ [LiTZ]. Upper asymptotics for $R(K_s + K_{m,n}, K_k)$ [Song9].

(l) Study of the largest k such that if the star $K_{1,k}$ is removed from K_r, $r = R(G,H)$, any edge 2-coloring of the remaining part still contains monochromatic G or H, as for K_r, for various special G and H [HoIs].

(m) Let G'' be a graph obtained from G by deleting two vertices with adjacent edges. Then $R(G,H) \leq A + B + 2 + 2\sqrt{(A^2 + AB + B^2)/3}$, where $A = R(G'', H)$ and $B = R(G, H'')$ [LiRZ2].

5.15. General results for sparse graphs

(a) $R(K_n, T_m) = (n-1)(m-1) + 1$ for any tree T_m on m vertices [Chv].

(b) Graphs yielding $R(K_n, G) = (n-1)(n(G)-1) + 1$, called Ramsey n-good [BE3], and related results [EFRS5]. An extensive survey and further study of n-goodness appeared in [NiRo4].

(c) $R(C_{2m+1}, G) = 2n - 1$ for sufficiently large sparse graphs G on n vertices, little more complicated formulas for P_{2m+1} instead of C_{2m+1} [BEFRS2].

(d) $R(G,G) \le c_d n(G)$ for all G, where constant c_d depends only on the maximum degree d in G [CRST]. The constant was improved in [GRR1, FoxSu1]. Tight lower and upper bounds for bipartite G [GRR2, Con2]. Further improvements of the constant c_d in general were obtained in [ConFS4], and for graphs with bounded bandwidth in [AllBS].

(e) Study of L-sets, which are sets of pairs of graphs whose Ramsey numbers are linear in the number of vertices. Conjecture that Ramsey numbers grow linearly for d-degenerate graphs (graph is d-degenerate if all its subgraphs have minimum degree at most d) [BE1]. Progress towards this conjecture was obtained by several authors, including [KoRö1, KoRö2, KoSu, FoxSu1, FoxSu2].

(f) $R(G,G) \le c_d n$ for all d-arrangeable graphs G on n vertices, in particular with the same constant for all planar graphs [ChenS]. The constant c_d was improved in [Eaton]. An extension to graphs not containing a subdivision of K_d [RöTh].

(g) Conjecture that $R(G,G) \le 12n(G)$ for all planar G, for sufficiently large n [AllBS].

(h) Ramsey numbers grow linearly for degenerate graphs versus some sparser graphs, arrangeable graphs, crowns, graphs with bounded maximum degree, planar graphs, and graphs without any topological minor of a fixed clique [Shi3].

(i) Discussion of various old and new classes of Ramsey linear graphs [NeOs].

(j) Study of graphs G, called *Ramsey size linear*, for which there exists a constant c_G such that for all H with no isolates $R(G,H) \le c_G e(H)$ [EFRS9]. An overview and further results were given in [BaSS].

(k) $R(G,G) < 6n$ for all n-vertex graphs G, in which no two vertices of degree at least 3 are adjacent [LiRS]. This improves the result $R(G,G) \le 12n$ in [Alon1]. In an early paper by Burr and Erdős [BE1] it was proved that if any two points of degree at least 3 are at distance at least 3 then $R(G,G) \le 18n$.

(l) $R(G_{a,b}, G_{a,b}) = (3/2 + o(1))ab$, where $G_{a,b}$ is the rectangular $a \times b$ grid graph. Other similar results follow for bipartite planar graphs with bounded degree and grids of higher dimension [MoSST].

(m) $R(Q_n, Q_n) \le 2^{(3+\sqrt{5})n/2 + o(n)}$, for the n-dimensional hypercube Q_n with 2^n vertices [Shi1]. This bound can also be derived from a theorem in [KoRö1]. An improvement was obtained in [Shi4], and a further one to $R(Q_n, Q_n) \le 2^{2n+5}n$ in [FoxSu1]. A lower bound construction for $12 \le R(Q_3, Q_3)$ was presented in [HaKr2].

(n) $R(K_m, Q_n) = (m-1)(2^n - 1) + 1$ for every fixed m and sufficiently large n [FizGMSS].

(o) Conjecture that $R(G,G) = 2n(G) - 1$ if G is unicyclic of odd girth [Gros1]. Further support for the conjecture was given in [Köh, KrRod].

(p) See also earlier subsections 5.* for various specific sparse graphs.

5.16. General results

[CH2] $R(G,H) \geq (\chi(G)-1)(c(H)-1)+1$, where $\chi(G)$ is the chromatic number of G, and $c(H)$ is the size of the largest connected component of H.

[CH3] $R(G,G) > (s\,2^{e(G)-1})^{1/n(G)}$, where s is the number of automorphisms of G. Hence $R(K_{n,n},K_{n,n}) > 2^n$, see also item 6.7.i.

[BE2] $R(G,G) \geq \lfloor (4n(G)-1)/3 \rfloor$ for any connected G, and $R(G,G) \geq 2n-1$ for any connected nonbipartite G. These bounds can be achieved for all $n \geq 4$.

[Bu2] Graphs H yielding $R(G,H) = (\chi(G)-1)(n(H)-1)+s(G)$, where $s(G)$ is a chromatic surplus of G, defined as the minimum number of vertices in some color class under all vertex colorings in $\chi(G)$ colors (such H's are called G-good). This idea, initiated in [Bu2], is a basis of a number of exact results for $R(G,H)$ for large and sparse graphs H [BG, BEFRS2, BEFRS3, Bu5, FS, EFRS4, FRS3, BEFSRGJ, BF, LiR4, Biel2, SuBAU3, Song6, AllBS]. Surveys of this area appeared in [FRS5, NiRo4].

[BaLS] Graph G is Ramsey saturated if $R(G+e,G+e) > R(G,G)$ for every edge e in $\overline{G}$. This paper contains several theorems involving cycles, cycles with chords and trees on Ramsey saturated and unsaturated graphs, and also seven conjectures including one stating that almost all graphs are Ramsey unsaturated. Some classes of graphs were proved to be Ramsey unsaturated [Ho]. Special cases involving cycles and Jahangir graphs were studied in [AliSur].

[Für] Relations between $R(3,k)$ and graphs with large $\chi(G)$. Further detailed study of the relation between $R(3,k)$ and the chromatic gap [GySeT].

[Bra3] $R(G,H) > h(G,d)\,n(H)$ for all nonbipartite G and almost every d-regular H, for some h unbounded in d.

[DoLL1] Lower asymptotics of $R(G,H)$ depending on the average degree of G and the size of H. This continues the study initiated in [EFRS5], later much enhanced for both lower and upper bounds in [Sud3].

[LiZa1] Lower bound asymptotics of $R(G,H)$ for large dense H.

[Erd4] A conjecture posed by Erdős in 1983 that there exists a constant c such that $R(G,G) \leq 2^{c\sqrt{e(G)}}$ for all isolate-free graphs G. Discussion of this conjecture and partial results, proof for bipartite graphs and progress in other cases are included in [AlKS]. In 2011, Sudakov [Sud4] completed the proof of this conjecture. An extension of the latter to some off-diagonal cases is presented in [MaOm], and an improvement of the constant for bipartite graphs is given in [JoPe]. For the multicolor case see item 6.7.i.

[Kriv] Lower bound on $R(G,K_n)$ depending on the density of subgraphs of G. This construction for $G = K_m$ produces a bound similar to the best known probabilistic lower bound by Spencer [Spe2]. Further lower and upper bounds on $R(G,K_n)$ in terms of n and $e(G)$ can be found in [Sud3].

[Con3] Upper bounds on $R(G, K_n)$ for dense graphs G.

[BE1] Relations between the cases of G or $G + K_1$ versus H or $H + K_1$.

[HaKr1] Study of cyclic graphs yielding lower bounds for Ramsey numbers. Exact formulas for paths and cycles, and values for small complete graphs and for graphs with up to five vertices.

[Par3] Relations between some Ramsey graphs and block designs. See also [Par4].

[Li2] Relations between the Shannon capacity of noisy communication channels and graph Ramsey numbers. See also section 6 in [Ros2], and [XuR3].

[Bu6] Given integer m and graphs G and H, determining whether $R(G, H) \leq m$ holds is NP – hard. Further complexity results related to Ramsey theory were presented in [Bu10].

[Scha] Ramsey arrowing is Π_2^p – complete, a rare natural example of a problem higher than NP in the polynomial hierarchy of computational complexity theory.

[-] Special cases of multicolor results listed in section 6.

[-] See also surveys listed in section 8.

6. Multicolor Ramsey Numbers

The only known value of a multicolor classical Ramsey number:

$R_3(3) = R(3,3,3) = R(3,3,3;2) = 17$ [GG]

2 critical colorings (on 16 vertices) [KaSt, LayMa]
2 colorings on 15 vertices [Hein]
115 colorings on 14 vertices [PR1]

6.1. Bounds for classical numbers

General upper bound, implicit in [GG]:

$$R(k_1,\dots,k_r) \le 2 - r + \sum_{i=1}^{r} R(k_1,\dots,k_{i-1},k_i-1,k_{i+1},\dots,k_r) \qquad \text{(a)}$$

Inequality in (a) is strict if the right hand side is even and at least one of the terms in the summation is even. It is suspected that this upper bound is never tight for $r \ge 3$ and $k_i \ge 3$, except for $r = k_1 = k_2 = k_3 = 3$. However, only two cases are known to improve over (a), namely $R_4(3) \le 62$ [FKR] and $R(3,3,4) \le 31$ [PR1, PR2], for which (a) produces the bounds of 66 and 34, respectively.

Diagonal Cases

r \ m	3	4	5	6	7	8	9
3	17 GG	128 HiIr	417 Ex16	1070 Mat	3214 XuR1	6079 XSR2	13761 XXER
4	51 Chu1	634 XXER	3049 Xu	15202 XXER	62017 XXER		
5	162 Ex10	3416 XXER	26912 Xu				
6	538 FreSw						
7	1682 FreSw						

Table X. Known nontrivial lower bounds for diagonal multicolor Ramsey numbers $R_r(m)$, with references.

The best published bounds corresponding to the entries in Table X marked as personal communications [Ex16] and [Xu] are $415 \le R_3(5)$, $2721 \le R_4(5)$ and $26082 \le R_5(5)$ [XXER].

The most studied and intriguing open case is

$$[\text{Chu1}] \quad 51 \le R_4(3) = R(3,3,3,3) \le 62 \quad [\text{FKR}]$$

The construction for $51 \le R_4(3)$ as described in [Chu1] is correct, but be warned of a typo found by Christopher Frederick in 2003 (there is a triangle (31,7,28) in color 1 in the displayed matrix). The inequality 6.1.a implies $R_4(3) \le 66$, Folkman [Fol] in 1974 improved this bound to 65, and Sánchez-Flores [San] in 1995 proved $R_4(3) \le 64$.

The upper bounds in $162 \le R_5(3) \le 307$, $538 \le R_6(3) \le 1838$, $1682 \le R_7(3) \le 12861$, $128 \le R_3(4) \le 236$ and $634 \le R_4(4) \le 6474$ are implied by 6.1.a (we repeat lower bounds from Table X just to see easily the ranges). All the latter and other upper bounds obtainable from known smaller bounds and 6.1.a can be computed with the help of a LISP program written by Kerber and Rowat [KerRo].

Off-Diagonal Cases

Three colors:

k \ m	4	5	6	7	8	9	10	11	12	13	14
3	30 Ka2	45 Ex2	60 Rob3	85 Ex18	103 Ex18	129 Ex18	147 Ex18	162 Ex18	185 6.2.f	212 LSS2	233 6.2.f
4	55 KLR	89 Ex17	117 Ex17	145 Ex17	193 6.2.f	229 6.2.f					
5	89 Ex17	139 Ex17	181 Ex17	237 6.2.f							

Table XI. Known nontrivial lower bounds for 3-color Ramsey numbers of the form $R(3,k,m)$, with references.

In addition, the bounds $303 \le R(3,6,6)$, $609 \le R(3,7,7)$ and $1689 \le R(3,9,9)$ were derived in [XXER] (used there for building other lower bounds for some diagonal cases).

The other most studied, and perhaps the only open case of a classical multicolor Ramsey number, for which we can anticipate exact evaluation in the not-too-distance future is

$$[\text{Ka2}] \quad 30 \le R(3,3,4) \le 31 \quad [\text{PR1, PR2}]$$

In [PR1] it is conjectured that $R(3,3,4) = 30$, and the results in [PR2] eliminate some cases which could give $R(3,3,4) = 31$. The upper bounds in $45 \le R(3,3,5) \le 57$, $55 \le R(3,4,4) \le 79$, and $89 \le R(3,4,5) \le 160$ are implied by 6.1.a. We repeat lower bounds from Table XI to show explicitly the current ranges.

Four colors:

$97 \le R(3,3,3,4) \le 153$	[Ex17], 6.1.a
$171 \le R(3,3,4,4) \le 462$	[Ex15, XXER], 6.1.a
$381 \le R(3,4,4,4) \le 1619$	6.2.j, 6.1.a
$162 \le R(3,3,3,5)$	[XXER]
$565 \le R(3,3,3,11)$	6.2.f
$681 \le R(3,4,5,5)$	[XXER]

Lower bounds for higher numbers can be obtained by using general constructive results from section 6.2 below. For example, the bounds $261 \le R(3,3,15)$ and $247 \le R(3,3,3,7)$ were not published explicitly but are implied by 6.2.f and 6.2.g, respectively.

6.2. General results for complete graphs

(a) $R(k_1,\dots,k_r) \le 2 - r + \sum_{i=1}^{r} R(k_1,\dots,k_{i-1},k_i-1,k_{i+1},\dots,k_r)$ [GG].

(b) $R_r(3) \ge 3R_{r-1}(3) + R_{r-3}(3) - 3$ [Chu1].

(c) $R_r(m) \ge c_m(2m-3)^r$, and some slight improvements of this bound for small values of m were described in [AbbH, Gi1, Gi2, Song2]. For $m = 3$, the best known lower bound is $R_r(3) \ge (3.199...)^r$ [XXER].

(d) $R_r(3) \le r!(e - e^{-1} + 3)/2 \approx 2.67r!$ [Wan], which improves the classical $3r!$ [GRS].

(e) The limit $L = \lim_{r\to\infty} R_r(3)^{1/r}$ exists, though it can be infinite [ChGri].
It is known that $3.199 < L$, as implied by (c) above. For more related results, mostly on the asymptotics of $R_r(3)$, see [AbbH, Fre, Chu2, GRS, GrRö].

(f) $R(3,k,l) \ge 4R(k,l-1) - 3$ for $k \ge 3$, $l \ge 5$, and in general for $r \ge 2$ and $k_i \ge 2$ it holds
$R(3,k_1,\dots,k_r) \ge 4R(k_1-1,k_2,\dots,k_r) - 3$ for $k_1 \ge 5$, and
$R(k_1,2k_2-1,k_3,\dots,k_r) \ge 4R(k_1-1,k_2,\dots,k_r) - 3$ for $k_1 \ge 5$ [XX2, XXER].

(g) $R(3,3,3,k_1,\dots,k_r) \ge 3R(3,3,k_1,\dots,k_r) + R(k_1,\dots,k_r) - 3$ [Rob2].

(h) For $r+1$ colors, avoiding K_3 in the first r colors and avoiding K_m in the last color, $R(3,\dots,3,m) \le r!\,m^{r+1}$ [Sár].

(i) $R(k_1,\dots,k_r) \ge S(k_1,\dots,k_r) + 2$, where $S(k_1,\dots,k_r)$ is the generalized Schur number [AbbH, Gi1, Gi2]. In particular, the special case $k_1 = \dots = k_r = 3$ has been widely studied [Fre, FreSw, Ex10, Rob3].

(j) $R(k_1,\dots,k_r) \ge L(k_1,\dots,k_r) + 1$, where $L(k_1,\dots,k_r)$ is the maximal order of any cyclic $(k_1,\dots,k_r)$–coloring, which can be considered a special case of Schur partitions defining (symmetric) Schur numbers. Many lower bounds for Ramsey numbers were established

by cyclic colorings. The following recurrence can be used to derive lower bounds for higher parameters. For $k_i \geq 3$ [Gi2],

$$L(k_1, \ldots, k_r, k_{r+1}) \geq (2k_{r+1} - 3)L(k_1, \ldots, k_r) - k_{r+1} + 2.$$

(k) $R_r(m) \geq p+1$ and $R_r(m+1) \geq r(p+1)+1$ if there exists a K_m–free cyclotomic $r-$class association scheme of order p [Mat].

(l) If the quadratic residues Paley graph Q_p of prime order $p = 4t+1$ contains no K_k, then $R(s, k+1, k+1) \geq 4ps - 6p + 3$ [XXER].

(m) $R_r(pq+1) > (R_r(p+1)-1)(R_r(q+1)-1)$ [Abb1]

(n) $R_r(pq+1) > R_r(p+1)(R_r(q+1)-1)$ for $p \geq q$ [XXER]

(o) $R(p_1q_1+1, \ldots, p_rq_r+1) > (R(p_1+1, \ldots, p_r+1)-1)(R(q_1+1, \ldots, q_r+1)-1)$ [Song3]

(p) $R_{r+s}(m) > (R_r(m)-1)(R_s(m)-1)$ [Song2]

(q) $R(k_1, k_2, \ldots, k_r) > (R(k_1, \ldots, k_i) - 1)(R(k_{i+1}, \ldots, k_r) - 1)$ in [Song1], see [XXER].

(r) $R(k_1, k_2, \ldots, k_r) > (k_1 + 1)(R(k_2 - k_1 + 1, k_3, \ldots, k_r) - 1)$ [Rob4]

(s) Further lower bound constructions, though with more complicated assumptions, were presented in [XX2, XXER].

(t) Grolmusz [Grol1] generalized the classical constructive lower bound by Frankl and Wilson [FraWi] (item 2.3.6) to more colors and to hypergraphs [Grol3] (item 7.4.k).

(u) Exact asymptotics of a very special but important case is known, namely $R(3, 3, n) = \Theta(n^3 \text{ poly–log } n)$ [AlRö]. For general upper bounds and more asymptotics see in particular [Chu4, ChGra2, ChGri, GRS, GrRö].

All lower bounds in (b) through (t) above are constructive. Item (g) generalizes (b), (o) generalizes both (m) and (q), and (q) generalizes (p). (n) is stronger than (m). Finally, we note that the construction in (o) with $q_1 = \ldots = q_i = 1 = p_{i+1} = \ldots = p_r$ is the same as (q).

6.3. Cycles

Note: The paper *Ramsey Numbers Involving Cycles* [Ra4] is based on the revision #12 of this survey. It collects and comments on the results involving cycles versus any graphs, in two or more colors. It contains some more details than this survey, but only until 2009.

6.3.1. Three colors

(a) One long cycle.

The first larger paper in this area by Erdős, Faudree, Rousseau and Schelp [EFRS1] appeared in 1976. It gives several formulas and bounds for $R(C_m, C_n, C_k)$ and $R(C_m, C_n, C_k, C_l)$ for large m. For three colors [EFRS1] includes:

$R(C_m, C_{2p+1}, C_{2q+1}) = 4m - 3$ for $p \geq 2$, $q \geq 1$,
$R(C_m, C_{2p}, C_{2q+1}) = 2(m+p) - 3$ and
$R(C_m, C_{2p}, C_{2q}) = m + p + q - 2$ for $p, q \geq 1$ and large m.

$m\ n\ k$	$R(C_m, C_n, C_k)$	references	general results
3 3 3	**17**	GG	page 36
3 3 4	17	ExRe	
3 3 5	21	Sun1+/Tse3	$5k-4$ for $k \geq 5$, $m = n = 3$ [Sun1+]
3 3 6	26	Sun1+	
3 3 7	31	Sun1+	
3 4 4	12	Schu	
3 4 5	13	Sun1+/Rao/Tse3	
3 4 6	13	Sun1+/Tse3	
3 4 7	15	Sun1+/Tse3	
3 5 5	≥ 17	Tse3	
3 5 6	21	Sun1+	
3 5 7	25	Sun1+	
3 6 6			
3 6 7	21	Sun1+	
3 7 7			
4 4 4	**11**	BiaS	
4 4 5	12	Sun2+/Tse3	
4 4 6	12	Sun2+/Tse3	$k+2$ for $k \geq 11$, $m = n = 4$ [Sun2+]
4 4 7	12	Sun2+/Tse3	values for $k = 8, 9, 10$ are 12, 13, 13 [Sun2+]
4 5 5	13	Tse3	
4 5 6	13	Sun1+	
4 5 7	15	Sun1+	
4 6 6	11	Tse3	
4 6 7	13	Sun1+/Tse3	
4 7 7			
5 5 5	**17**	YR1	
5 5 6	21	Sun1+	
5 5 7	25	Sun1+	
5 6 6			
5 6 7	21	Sun1+	
5 7 7			
6 6 6	**12**	YR2	$R_3(C_{2q}) \geq 4q$ for $q \geq 2$ [DzNS]
6 6 7	15	Sun1+	see 6.3.1.a for larger parameters
6 7 7			see 6.3.1.a for larger parameters
7 7 7	**25**	FSS2	$R_3(C_{2q+1}) = 8q+1$ for large q [KoSS1, KoSS2]
8 8 8	**16**	Sun/SunY	$R_3(C_{2q}) = 4q$ for large q [BenSk]

Table XII. Ramsey numbers $R(C_m, C_n, C_k)$ for $m, n, k \leq 7$ and $m = n = k = 8$;
Sun1+ abbreviates SunYWLX, Sun2+ abbreviates SunYLZ2,
the work in [SunYWLX] and [SunYLZ2] is independent from [Tse3].

(b) Triple even cycles.

$R_3(C_{2m}) \geq 4m$ for all $m \geq 2$ [DzNS], see also 6.3.2.d/e/f. It was proven that $R(C_n, C_n, C_n) = (2+o(1))n$ for even n [FiŁu1, GyRSS], which was improved to exactly $2n$, for large n, by Benevides and Skokan [BenSk]. In 2005, Dzido [Dzi1] conjectured that $R_3(C_{2m}) = 4m$ for all $m \geq 3$. The first open case is for $R_3(C_{10})$, known to be at least 20. A more general result holds for slightly off-diagonal cases [FiŁu1]:

$$R(C_{2\lfloor \alpha_1 n \rfloor}, C_{2\lfloor \alpha_2 n \rfloor}, C_{2\lfloor \alpha_3 n \rfloor}) = (\alpha_1 + \alpha_2 + \alpha_3 + \max\{\alpha_1, \alpha_2, \alpha_3\} + o(1))n, \quad \textit{for all} \quad \alpha_1, \alpha_2, \alpha_3 > 0.$$

The conjectured equality $R_3(C_{2m}) = 4m$, whenever true, implies $R_3(P_{2m+1}) = 4m+1$ [DyDR] (see also section 6.4).

(c) Triple odd cycles.

Bondy and Erdős conjectured that $R(C_n, C_n, C_n) \le 4n-3$ for all $n \ge 4$ (see for example [Erd2]). If true, then for all odd $n \ge 5$ we have $R(C_n, C_n, C_n) = 4n-3$. The first open case is for $R_3(C_9)$, known to be at least 33. Erdős [Erd3] and other authors credit this conjecture to Bondy and Erdős, often pointing to a 1973 paper [BoEr]. Interestingly, however, the conjecture is not mentioned in this paper.

Łuczak proved that $R(C_n, C_n, C_n) \le (4+o(1))n$, with equality for odd n [Łuc]. The result $R_3(C_{2m+1}) = 8m+1$ for all sufficiently large m, or equivalently $R(C_n, C_n, C_n) = 4n-3$ for large odd n, was announced with an outline of the proof by Kohayakawa, Simonovits and Skokan [KoSS1], followed by the full proof in [KoSS2].

(d) $R(C_3, C_3, C_k) = 5k-4$ for $k \ge 5$ [SunYWLX], and $R(C_4, C_4, C_k) = k+2$ for $k \ge 11$ [SunYLZ2]. All exceptions to these formulas for small k are listed in Table XII.

(e) Asymptotics for triples of cycles of mixed parity similar in form to (b) [FiŁu2].

(f) Almost all of the off-diagonal cases in Table XII required the use of computers.

6.3.2. More colors

k \ m	3	4	5	6	7	8
3	17	11	17	12	25	16
4	51 62	18	33 158	18 20	49	20
5	162 307	27 29	65	26	97	28
6	538 1838	32 43	129		193	

Table XIII. Known values and bounds for $R_k(C_m)$ for small k, m;

(a) For the entries in the row $k = 3$ and in the column $m = 3$ in Table XIII, more details and all corresponding references are in sections 6.3.1 and 6.1, respectively. The lower bounds for $m = 5, 7$ are implied by 6.3.2.k, $R_k(C_m) \le 158$ follows from 6.3.2.j, and references to other cases with $k, m \ge 4$ can be found below in this section.

$R_4(C_4) = 18$	[Ex2] [SunYLZ1]
$18 \le R_4(C_6) \le 20$	[SunYJLS][ZhaSW]
$27 \le R_5(C_4) \le 29$	[LaWo1]
$R_5(C_6) = 26$	[SunYJLS] [SunYW]
$24 \le R(C_3, C_4, C_4, C_4) \le 27$	[DyDz1] [XuR2]
$30 \le R(C_3, C_3, C_4, C_4) \le 36$	[DyDz1] [XuR2]
$49 \le R(C_3, C_3, C_3, C_4)$	6.7.e
$18 \le R(C_4, C_6, C_6, C_6) \le 20$	[ZhaSW]
$18 \le R(C_4, C_4, C_6, C_6) \le 20$	[ZhaSW]
$R(C_4, C_4, C_4, C_6) = 19$	[ZhaSW]

(b) $R_k(C_4) \le k^2 + k + 1$ for all $k \ge 1$, $R_k(C_4) \ge k^2 - k + 2$ for all $k - 1$ which is a prime power [Ir, Chu2, ChGra1], and $R_k(C_4) \ge k^2 + 2$ for odd prime power k [LaWo1]. The latter was extended to any prime power k in [Ling, LaMu].

(c) Formulas for $R(C_m, C_n, C_k, C_l)$ for large m [EFRS1].

Bounds in (d) through (i) below cover different situations and each is best in some respect.

(d) $R_k(C_{2m}) \ge (k+1)m$ for odd k and $m \ge 2$, and
$R_k(C_{2m}) \ge (k+1)m - 1$ for even k and $m \ge 2$ [DzNS].

(e) $R_k(C_{2m}) \ge 2(k-1)(m-1) + 2$ [SunYXL].

(f) $R_k(C_{2m}) \ge k^2 + 2m - k$ for $2m \ge k+1$ and prime power k [SunYJLS].

(g) $R_k(C_{2m}) = \Theta(k^{m/(m-1)})$ for fixed $m = 2, 3$ and 5 [LiLih].

(h) $R_k(C_{2m}) \le 201km$ for $k \le 10^m/201m$ [EG].

(i) $R_k(C_{2m}) \le 2km + o(m)$ for all fixed $k \ge 2$ [ŁucSS].

(j) $R_k(C_5) < \sqrt{18^k k!}/10$ [Li4].

(k) $2^k m < R_k(C_{2m+1}) \le (k+2)!(2m+1)$ [BoEr].
Better upper bound $R_k(C_{2m+1}) < 2(k+2)!m$ was obtained in [EG].
Much better upper bound $R_k(C_{2m+1}) \le (c^k k!)^{1/m}$, for some positive constant c,
if all Ramsey-critical colorings for C_{2m+1} are not far from regular, was obtained in [Li4].

(l) Conjecture that $R_k(C_{2m+1}) = 2^k m + 1$ for all $m \ge 2$ was credited by several authors to Bondy and Erdős [BoEr], though only lower bound, not the conjecture, is in this paper.

(m) $R(C_n, C_{l_1}, \ldots, C_{l_k}) = 2^k(n-1) + 1$ for all l_i's odd with $l_i > 2^i$, and sufficiently large n, and support for the conjecture that $R_k(C_n) = 2^{k-1}(n-1) + 1$ for large odd n [AllBS].

(n) $R_k(C_{2m+1}) \le k2^k(2m+1) + o(m)$ for all fixed $k \ge 4$ [ŁucSS].

(o) Asymptotic bounds for $R_k(C_n)$ [Bu1, GRS, ChGra2, Li4, LiLih, ŁucSS].

(p) Survey of multicolor cycle cases [Li3].

6.3.3. Cycles versus other graphs

(a) Some cases involving C_4:

$20 \le R(C_4, C_4, K_4) \le 22$	[DyDz1] [XSR1]
$27 \le R(C_3, C_4, K_4) \le 32$	[DyDz1] [XSR1]
$52 \le R(C_4, K_4, K_4) \le 72$	[XSR1]
$34 \le R(C_4, C_4, C_4, K_4) \le 50$	[DyDz1] [XSR1]
$43 \le R(C_3, C_4, C_4, K_4) \le 76$	[DyDz1] [XSR1]
$87 \le R(C_4, C_4, K_4, K_4) \le 179$	[XSR1]
$R(K_{1,3}, C_4, K_4) = 16$	[KlaM2]
$R(C_4, C_4, K_4 - e) = 16$	[DyDz1]
$R(C_4, C_4, C_4, T) = 16$ for $T = P_4$ and $T = K_{1,3}$	[ExRe]

(b) Study of $R(C_n, K_{t_1}, \dots, K_{t_k})$ and $R(C_n, K_{t_1, s_1}, \dots, K_{t_k, s_k})$ for large n [EFRS1].

(c) $R(C_n, K_{t_1}, \dots, K_{t_k}) = (n-1)(r-1)$ for $n \ge 4r+2$, where $r = R(K_{t_1}, \dots, K_{t_k})$ [OmRa2].

(d) Study of asymptotics for $R(C_m, \dots, C_m, K_n)$, in particular for any fixed number of colors $k \ge 4$ we have $R(C_4, C_4, \dots, C_4, K_n) = \Theta(n^2/\log^2 n)$ [AlRö].

(e) Study of asymptotics for $R(C_{2m}, C_{2m}, K_n)$ for fixed m [AlRö, ShiuLL], in particular $R(C_4, C_4, K_n) = \Theta(n^2 \text{ poly–log } n)$ [AlRö].

(f) Monotone paths and cycles [Lef].

(g) For combinations of C_3 and K_n see sections 2.2, 3.2, 4.2, 6.1 and 6.2.

6.4. Paths, paths versus other graphs

In 2007, Gyárfás, Ruszinkó, Sárközy and Szemerédi [GyRSS] established that for all n large enough we have

$$R(P_n, P_n, P_n) = 2n - 2 + (n \bmod 2).$$

Faudree and Schelp [FS2] conjectured that the latter holds for all $n \ge 1$. It is true for $n \le 9$ (see (c) below), and the first open case is that for P_{10}. The conjectured equality $R(C_{2m}, C_{2m}, C_{2m}) = 4m$ (see 6.3.1.a), whenever true, implies the above for three paths P_{2m+1} [DyDR].

6.4.1. Three color path and path-cycle cases

(a) $R(P_m, P_n, P_k) = m + \lfloor n/2 \rfloor + \lfloor k/2 \rfloor - 2$ for $m \ge 6(n+k)^2$ [FS2],
the equality holds asymptotically for $m \ge n \ge k$ with an extra term $o(m)$ [FiŁu1], extensions of the range of m, n, k for which (a) holds were obtained in [Biel3].

(b) $R(P_3, P_m, P_n) = m + \lfloor n/2 \rfloor - 1$ for $m \ge n$ and $(m, n) \ne (3,3), (4,3)$ [MaORS2].

(c) $R_3(P_3) = 5$ [Ea1], $R_3(P_4) = 6$ [Ir],
$R(P_m, P_n, P_k) = 5$ for other m–n–k combinations with $3 \le m, n, k \le 4$ [ArKM],
$R_3(P_5) = 9$ [YR1], $R_3(P_6) = 10$ [YR1], and $R_3(P_7) = 13$ [YY],
$R_3(P_8) = 14$, $R_3(P_9) = 17$ [DyDR].

(d) $R(P_4, P_4, P_{2n}) = 2n + 2$ for $n \ge 2$,
$R(P_5, P_5, P_5) = R(P_5, P_5, P_6) = 9$,
$R(P_5, P_5, P_n) = n + 2$ for $n \ge 7$,
$R(P_5, P_6, P_n) = R(P_4, P_6, P_n) = n + 3$ for $n \ge 6$,
$R(P_6, P_6, P_{2n}) = R(P_4, P_8, P_{2n}) = 2n + 4$ for $n \ge 14$ [OmRa1].

(e) $R(P_m, P_n, C_k) = 2n + 2\lfloor m/2 \rfloor - 3$ for large n and odd $m \ge 3$ [DzFi2],
improvements on the range of m, n, k [Biel3, Fid1].

(f) $R(P_3, P_3, C_m) = 5, 6, 6$, for $m = 3, 4$ [ArKM], 5,
$R(P_3, P_3, C_m) = m$ for $m \ge 6$ [Dzi2].
$R(P_3, P_4, C_m) = 7$ for $m = 3, 4$ [ArKM] and 5,
$R(P_3, P_4, C_m) = m + 1$ for $m \ge 6$ [Dzi2].
$R(P_4, P_4, C_m) = 9, 7, 9$ for $m = 3, 4$ [ArKM] and 5 [Dzi2],
$R(P_4, P_4, C_m) = m + 2$ for $m \ge 6$ [DzKP].

(g) $R(P_3, P_5, C_m) = 9, 7, 9, 7, 9$ for $m = 3, 4, 5, 6, 7$ [Dzi2, DzFi2],
$R(P_3, P_5, C_m) = m + 1$ for $m \ge 8$ [DzKP].
A table of $R(P_3, P_k, C_m)$ for all $3 \le k \le 8$ and $3 \le m \le 9$ [DzFi2].

(h) $R(P_4, P_5, C_m) = 11, 7, 11, 11$, and $m + 2$ for $m = 3, 4, 5, 7$ and $m \ge 23$,
$R(P_4, P_6, C_m) = 13, 8, 13, 13$, and $m + 3$ for $m = 3, 4, 5, 7$ and $m \ge 18$ [ShaXSP].

(i) $R(P_3, P_n, C_4) = n + 1$ for $n \ge 6$ [DzFi2],
$R(P_3, P_n, C_6) = n + 2$ for $n \ge 6$,
$R(P_3, P_n, C_8) = n + 3$ for $n \ge 7$ [Fid1],
$R(P_3, P_n, C_k) = 2n - 1$, and
$R(P_4, P_n, C_k) = 2n + 1$ for odd $k \ge 3$ and $n \ge k$ [DzFi2].

(j) $R(P_3, P_6, C_m) = m + 2$ for $m \ge 23$,
$R(P_6, P_6, C_m) = R(P_4, P_8, C_m) = m + 4$ for $m \ge 27$,
$R(P_6, P_7, C_m) = m + 4$ for $m \ge 57$,
$R(P_4, P_n, C_4) = R(P_5, P_n, C_4) = n + 2$ for $n \ge 5$ [OmRa1].

(k) $R(P_3, C_3, C_3) = 11$ [BE3], $R(P_3, C_4, C_4) = 8$ [ArKM], $R(P_3, C_6, C_6) = 9$ [Dzi2],
$R(P_3, C_m, C_m) = R(C_m, C_m) = 2m - 1$ for odd $m \ge 5$ [DzKP] (for $m = 5, 7$ [Dzi2]),

(l) $R(P_3, C_n, C_m) = R(C_n, C_m)$ for $n \ge 7$ and odd m, $5 \le m \le n$, and
some values and bounds on $R(P_3, C_n, C_m)$ in other cases [Fid1].

(m) $R(P_3, C_3, C_4) = 8$ [ArKM], $R(P_3, C_3, C_5) = 9$, $R(P_3, C_3, C_6) = 11$,
$R(P_3, C_3, C_7) = 13$, $R(P_3, C_4, C_5) = 8$, $R(P_3, C_4, C_6) = 8$,

$R(P_3, C_4, C_7) = 8$, $R(P_3, C_5, C_6) = 11$, $R(P_3, C_5, C_7) = 13$ and $R(P_3, C_6, C_7) = 11$ [Dzi2].

(n) Formulas for $R(pP_3, qP_3, rP_3)$ and $R(pP_4, qP_4, rP_4)$ [Scob].

(o) $R(P_3, K_4 - e, K_4 - e) = 11$ [Ex7]. All colorings which can form any color neighborhood for the open case $R_3(K_4 - e)$ (see section 6.5) were found in [Piw2].

6.4.2. More colors

(a) $R_k(P_3) = k + 1 + (k \bmod 2)$, $R_k(2P_2) = k + 3$ for all $k \geq 1$ [Ir].

(b) $R_k(P_4) = 2k + c_k$ for all k and some $0 \leq c_k \leq 2$. If k is not divisible by 3 then $c_k = 3 - k \bmod 3$ [Ir]. Wallis [Wall] showed $R_6(P_4) = 13$, which already implied $R_{3t}(P_4) = 6t + 1$, for all $t \geq 2$. Independently, the case $R_k(P_4)$ for $k \neq 3^m$ was completed by Lindström in [Lind], and later Bierbrauer proved $R_{3^m}(P_4) = 2(3^m) + 1$ for all $m > 1$. $R_3(P_4) = 6$ [Ir].

(c) Formula for $R(P_{n_1}, \ldots, P_{n_k})$ for large n_1 [FS2], and some extensions [Biel3].
Conjectures about $R(P_{n_1}, \ldots, P_{n_k})$ when all or all but one of n_i's are even [OmRa1].

(d) Formulas for $R(P_{n_1}, \ldots, P_{n_k}, C_m)$ for some cases, for large m [OmRa1].

(e) Formula for $R(n_1P_2, \ldots, n_kP_2)$, in particular $R(nP_2, nP_2, nP_2) = 4n - 2$ [CocL1].
Note how close the latter is to $R(C_{2n}, C_{2n}, C_{2n}) = 4n$, see an earlier item 6.3.1b.

(f) Cockayne and Lorimer [CocL1] found the exact formula for $R(n_1P_2, \ldots, n_kP_2)$, and later Lorimer [Lor] extended it to a more general case of $R(K_m, n_1P_2, \ldots, n_kP_2)$. More general cases of the latter, with multiple copies of the complete graph, paths, stars and forests, were studied in [Stahl, LorSe, LorSo, GyRSS]. A special 3-color case $R(P_3, mP_2, nP_2) = 2m + n - 1$ for $m \geq n \geq 3$ is given in [MaORS2].

(g) Multicolor cases for one large path or cycle involving small paths, cycles, complete and complete bipartite graphs [EFRS1].

(h) See sections 6.5 and 8.2, especially [ArKM, BoDD], for a number of cases for triples of small graphs.

6.5. Special cases

$R_3(K_3 + e) = R_3(K_3)$ [=17]	[YR3, ArKM], where $K_3 + e = K_4 - P_3$
$R(K_3 + e, K_3 + e, K_4 - e) = 17$	[ShWR]
If $R_4(K_3) = 51$ then $R_4(K_3 + e) = 52$, and	
if $R_4(K_3) > 51$ then $R_4(K_3 + e) = R_4(K_3)$	[ShWR]

$28 \le R_3(K_4-e) \le 30$	[Ex7] [Piw2]
$R(P_3, K_4-e, K_4-e) = 11$	[Ex7], all colorings [Piw2]
$472 \le R_3(K_6-e)$	[HeDL]
$1102 \le R_3(K_7-e)$	[HeDL]
$21 \le R(K_3, K_4-e, K_4-e) \le 27$	[ShWR]
$33 \le R(K_4, K_4-e, K_4-e) \le 60$	[ShWR]
$55 \le R(K_4, K_4, K_4-e) \le 113$	[Ea1][BoDD]
$R(C_4, P_4, K_4-e) = 11$	[ArKM]
$R(C_4, P_4, K_4) = 14$	[BoDD]
$R(C_4, C_4, K_4-e) = 16$	[DyDz1]
$R(C_4, K_3, K_4-e) = 17$	[BoDD]
$R(C_4, K_4-e, K_4-e) = 19$	[BoDD]
$28 \le R(C_4, K_4, K_4-e) \le 36$	[BoDD]
$30 \le R(K_3, K_4, K_4-e) \le 41$	[Ea1][BoDD]
$33 \le R(K_4, K_4-e, K_4-e) \le 59$	[Ea1][BoDD]

See also section 8.2 for pointers to cumulative data for three colors.

6.6. General results for special graphs

(a) Formulas for $R_k(G)$, where G is one of the graphs P_3, $2K_2$ and $K_{1,3}$, for all k, and for P_4 if k is not divisible by 3 [Ir]. For some details see section 6.4.2b.

(b) $tk^2+1 \le R_k(K_{2,t+1}) \le tk^2+k+2$, where the upper bound is general, and the lower bound holds when both t and k are prime powers [ChGra1, LaMu].

(c) $(m-1)\lfloor (k+1)/2 \rfloor < R_k(T_m) \le 2km+1$ for any tree T_m with m edges [EG], see also [GRS]. The lower bound can be improved for special large k [EG, GRS]. The upper bound was improved to $R_k(T_m) < (m-1)(k+\sqrt{k(k-1)})+2$ in [GyTu].

(d) $k(\sqrt{m}-1)/2 < R_k(F_m) < 4km$ for any forest F_m with m edges [EG], see [GRS]. See also pointers in items (p) and (r) below.

(e) $R(S_1, \dots, S_k) = n+\varepsilon$, where S_i's are arbitrary stars, $n = n(S_1)+\dots+n(S_k)-2k$, and we set $\varepsilon = 1$ if n is even and some $n(S_i)$ is odd, and $\varepsilon = 2$ otherwise [BuRo1]. See also [GauST, Par6].

(f) Formula for $R(S_1, \dots, S_k, K_n)$, where S_i's are arbitrary stars [Jac]. It was generalized to a formula for $R(S_1, \dots, S_k, K_{k_1}, \dots, K_{k_r})$ expressed in terms of $R(k_1, \dots, k_r)$ and star orders [BoCGR]. A much shorter proof of the latter was presented in [OmRa2].

(g) Formula for $R(S_1, \dots, S_k, nK_2)$, where S_i's are arbitrary stars [CocL2], and a formula for $R(n_1K_2, \dots, n_kK_2)$ [CocL1]. See also cases involving P_2 in section 6.4.2.

(h) Formula for $R(S_1,\dots,S_k,T)$, where S_i's are stars and T is a tree [ZZ1].

(i) Formulas for $R(S_1,\dots,S_k)$, where each S_i's is a star or m_iK_2 [ZZ2, EG], formula for the case $R(S,mK_2,nK_2)$ [GySá2].

(j) Bounds on $R_k(G)$ for unicyclic graphs G of odd girth.
Some exact values for special graphs G, for $k=3$ and $k=4$ [KrRod].

(k) For prime $p=3q+1$, if the cubic residues Paley graph Q_p contains no K_k-e, then $R_3(K_{k+1}-e)>3p$ [HeDL]. The cases $k=5,6$ give two bounds listed in section 6.5.

(l) $R_k(K_{3,3})=(1+o(1))k^3$ [AlRóS].

(m) Bounds on $R_k(K_{s,t})$, in particular for $K_{2,2}=C_4$ and $K_{2,t}$ [ChGra1, AxFM]. Asymptotics of $R_k(K_{s,t})$ for fixed k and s [DoLi, LiTZ]. Upper bounds on $R_k(K_{s,t})$ [SunLi].

(n) Exact asymptotics $R(K_{t,s},K_{t,s},K_{t,s},K_m)=\Theta(m^t/\log^t m)$, for any fixed $t>1$ and large $s\ge(t-1)!+1$ [AlRö].

(o) Bounds on $R_k(G)$ for trees, forests, stars and cycles [Bu1].

(p) Bounds for trees $R_k(T)$ and forests $R_k(F)$ [EG, GRS, BierB, GyTu, Bra1, Bra2, SwPr].

(q) $R_3(G_{a,b})=(2+o(1))ab$, where $G_{a,b}$ is the rectangular $a\times b$ grid graph. Lower and upper bounds on $R_3(G)$ for graphs G with small bandwidth and bounded $\Delta(G)$ [MoSST].

(r) Study of the case $R(K_m,n_1P_2,\dots,n_kP_2)$ [Lor]. Other similar results include $R(P_3,mK_2,nK_2)=2m+n-1$ for $m\ge n\ge 3$ [MaORS2] and $R(S_n,nK_2,nK_2)=3n-1$ [GySá2]. More general cases, with multiple copies of the complete graph, stars and forests, were investigated in [Stahl, LorSe, LorSo, GyRSS]. See also section 6.4.2.

(s) See section 8.2, especially [ArKM, BoDD], for a number of cases for other small graphs, similar to those listed in sections 6.3 and 6.4.

6.7. General results

(a) Szemerédi's Regularity Lemma [Szem] states that the vertices of every large graph can be partitioned into similar size parts so that the edges between these parts behave almost randomly. This lemma has been used extensively in various forms to prove the upper bounds, including those studied in [BenSk, GyRSS, GySS1, HaŁP1+, HaŁP2+, KoSS1, KoSS2].

(b) $R(m_1G_1,\dots,m_kG_k)\le R(G_1,\dots,G_k)+\sum_{i=1}^{k} n(G_i)(m_i-1)$, exercise 8.3.28 in [West].

(c) If G is connected and $R(K_k,G)=(k-1)(n(G)-1)+1$, in particular if G is any n-vertex tree, then $R(K_{k_1},\dots,K_{k_r},G)=(R(k_1,\dots,k_r)-1)(n-1)+1$ [BE3]. A generalization for connected $G_1,\dots,G_n$ in place of G appeared in [Jac].

(d) If F,G,H are connected graphs then $R(F,G,H)\ge(R(F,G)-1)(\chi(H)-1)+\min\{R(F,G),s(H)\}$, where $s(G)$ is the chromatic surplus of G (see item [Bu2] in section 5.16). This leads to several formulas and bounds for F and G being stars and/or

trees when $H = K_n$ [ShiuLL].

(e) $R(K_{k_1},\dots,K_{k_r},G_1,\dots,G_s) \geq (R(k_1,\dots,k_r)-1)(R(G_1,\dots,G_s)-1)+1$ for arbitrary graphs $G_1,\dots,G_s$ [Bev]. This generalizes 6.2.q.

(f) Constructive bound $R(G_1,\dots,G_{t^{n-1}}) \geq t^n+1$ for decompositions of K_{t^n} [LaWo1, LaWo2].

(g) $R(G_1,\dots,G_k) \leq 32\Delta k^{\Delta} n$, where $n \geq n(G_i)$ and $\Delta \geq \Delta(G_i)$ for all $1 \leq i \leq k$,
$R(G_1,\dots,G_k) \leq k^{2k\Delta q} n$, where $q \geq \chi(G_i)$ for all $1 \leq i \leq k$ [FoxSu1].

(h) $R_k(G) \leq k^{6e(G)^{2/3}k}$ for all isolate-free graphs G and $k \geq 3$ [JoPe].
For the original two-color conjecture, now a theorem, see item [Erd4] in section 5.16.

(i) $R_k(G) > (sk^{e(G)-1})^{1/n(G)}$, where s is the number of automorphisms of G [CH3]. Other general bounds for $R_k(G)$ [CH3, Par6].

(j) Study of $R(G_1,\dots,G_k,G)$ for large sparse G [EFRS1, Bu3].

(k) Study of asymptotics for $R(C_n,\dots,C_n,K_m)$ [AlRö]. See also sections 6.3.3.d/e.

(l) Relations between the Shannon capacity of noisy communication channels and graph Ramsey numbers. A lower bound construction for $R_k(m)$ implying that supremum of the Shannon capacity over all graphs with bounded independence cannot be achieved by any finite graph power [XuR3]. For some other links between Shannon capacity and Ramsey numbers see section 6 in [Ros2], and [Li2].

(m) See surveys listed in section 8.

7. Hypergraph Numbers

7.1. Values and bounds for numbers

The only known value of a classical Ramsey number for hypergraphs:

$R(4,4;3) = 13$,
more than 200000 critical colorings [MR1]

The computer evaluation of $R(4,4;3)$ in 1991 consisted of an improvement of the upper bound from 15 to 13. This result followed an extensive theoretical study of this number by several authors [Gi4, Isb1, Sid1].

(a) $33 \le R(4,5;3)$ [Ex13]
$58 \le R(4,6;3)$ [Ex18]
$82 \le R(5,5;3)$ [Ex18]
$56 \le R(4,4,4;3)$ [Ex8]
$34 \le R(5,5;4)$ [Ex11]

(b) $R(K_4-t, K_4-t;3) = 7$ [Ea2]
$R(K_4-t, K_4;3) = 8$ [Sob, Ex1, MR1]
$14 \le R(K_4-t, K_5;3)$ [Ex1]
$13 \le R(K_4-t, K_4-t, K_4-t;3) \le 16$ [Ex1] [Ea3]

(c) The first bound on $R(4,5;3) \ge 24$ was obtained by Isbell [Isb2]. Shastri [Shas] gave a weak bound $R(5,5;4) \ge 19$ (now 34 in [Ex11]), nevertheless his lemmas, the stepping-up lemmas by Erdős and Hajnal (see [GRS, GrRö], also 7.4.a below), and others in [Ka3, Abb2, GRS, GrRö, HuSo, SonYL] can be used to derive better lower bounds for higher numbers.

(d) Several lower bound constructions for 3-uniform hypergraphs were presented in [HuSo]. Study of lower bounds on $R(p,q;4)$ can be found in [Song3] and [SonYL, Song4] (the latter two papers are almost the same in contents). Most of the concrete lower bounds in these papers can be easily improved by using the same techniques, but starting with better constructions for small parameters as listed above.

(e) $R(p,q;4) \ge 2R(p-1,q;4) - 1$ for $p, q > 4$, and
$R(p,q;4) \ge (p-1)R(p-1,q;4) - p + 2$ for $p \ge 5$, $q \ge 7$ [SonYL].
Lower bound asymptotics for $R(p,q;4)$ [SonLi].

(f) $R(K_{1,1,c}, K_{1,1,c};3) = c + 2$ for $2 \le c \le 4$, and
a conjecture that this equality also holds for all $c \ge 5$ [MiPal].

7.2. Cycles and paths

Definitions. $P_n^{r,s}$ is called an *s-path* in an r-uniform hypergraph H, if it consists of n hyperedges $\{e_1, ..., e_n\}$ in $E(H)$, such that $|e_i \cap e_{i+1}| = s$ for all $1 \le i < n$, and all other vertices in e_j's are distinct [Peng]. An *s-cycle* $C_n^{r,s}$ is defined analogously. Several authors use the terms of *loose* paths and *loose* cycles, which are 1-path and 1-cycles, and *tight* paths and

tight cycles, the latter most often for 3-uniform hypergraphs when they are 2-paths and 2-cycles, respectively. A 3-uniform *Berge* cycle is formed by n distinct vertices, such that all consecutive pairs of vertices are in an edge of the cycle, and all of the cycle edges are distinct. Berge cycles are not determined uniquely.

In the following items (b) to (i), when $r = 3$ or r is implied by the context, we write C_n and P_n for the r-uniform loose cycles and paths, $C_n^{r,1}$ and $P_n^{r,1}$, respectively. In other cases special comments are added.

(a) Tetrahedron is formed by four triples on the set of four points. The Ramsey number of tetrahedron is $R(4,4;3) = 13$ [MR1].

(b) For loose cycles and paths, $R(C_3, C_3; 3) = 7$, $R(C_4, C_4; 3) = 9$, and for the r-uniform case we have in general $R(P_3, P_3; r) = R(P_3, C_3; r) = R(C_3, C_3; r) + 1 = 3r - 1$ and $R(P_4, P_4; r) = R(P_4, C_4; r) = R(C_4, C_4; r) + 1 = 4r - 2$, for $r \geq 3$. These results and discussion of several related cases were presented in [GyRa].

(c) $R(P_m, P_n; 3) = R(C_m, C_n; 3) + 1 = R(P_m, C_n; 3) = 2m + \lfloor (n+1)/2 \rfloor$, for all $m \geq n$, and $R(C_m, P_n; 3) = 2m + \lfloor (n-1)/2 \rfloor$, for $m > n$ [MaORS1, OmSh].

(d) For loose cycles, $R(C_{2n}, C_{2n}; 3) > 5n - 2$ and $R(C_{2n+1}, C_{2n+1}; 3) > 5n + 1$, and asymptotically these lower bounds are tight [HaŁP1+]. Generalizations to r-uniform hypergraphs and graphs other than cycles appeared in [GySS1].

(e) For tight cycles, $R(C_{3n}, C_{3n}; 3) \approx 4n$ and $R(C_{3n+i}, C_{3n+i}; 3) \approx 6n$ for $i = 1$ or 2, and for tight paths $R(P_n, P_n; 3) \approx 4n/3$ [HaŁP2+]. Some related results are discussed in [PoRRS].

(f) Exact values for Ramsey numbers involving s-paths for even r and $s = r/2$, in particular for $P_n^{r,s}$ versus $P_3^{r,s}$ and $P_4^{r,s}$, when this value is $(n+1)s + 1$ [Peng].

(g) For 3-uniform Berge cycles and two colors, $R(C_n, C_n; 3) = n$ for $n \geq 5$ [GyLSS].

(h) For loose cycles, $R(C_3, C_3, C_3; 3) = 8$, and in general for $k \geq 4$ colors Gyárfás and Raeisi established the bounds $k + 5 \leq R_k(C_3; 3) \leq 3k$ [GyRa].

(i) For 3-uniform Berge cycles, $R_3(C_n; 3) = (1 + o(1))5n/4$ [GySá1].

(j) Lower and upper asymptotic bounds for $R(C_3^{3,1}, K_m; 3)$ and $R(C_3^{r,1}, K_m; r)$ [KosMV2].

(k) Gyárfás, Sárközy and Szemerédi proved that, for sufficiently large n, every 2-coloring of the edges of the complete 4-uniform hypergraph K_n contains a monochromatic 3-tight Berge cycle C_n [GySS2]. Special multicolor cases for r-uniform hypergraphs were studied in [GyLSS].

(l) Study of $R(G, nH; r)$ and $R(mG, nH; r)$ for loose/tight path, cycles and stars, including several exact results for large m or n [OmRa3].

7.3. General results for 3-uniform hypergraphs

(a) $2^{cn^2} < R(n,n;3) < 2^{2^n}$ is credited to Erdős, Hajnal and Rado (see [ChGra2] p. 30).

(b) For some a, b the numbers $R(m,a,b;3)$ are at least exponential in m [AbbS].

(c) Improved lower and upper asymptotics for $R(s,n;3)$ for fixed s and large n, proof of related Erdős and Hajnal conjecture on the growth of $R(4,n;3)$, and the lower bound $2^{n^{c\ln n}} < R(n,n,n;3)$ [ConFS2].

(d) $R(G,G;3) \le cn(H)$ for some constant c depending only on the maximum degree of a 3-uniform hypergraph H [CooFKO1, NaORS]. Similar results were proved for r-uniform hypergraphs in [KüCFO, Ishi, CooFKO2, ConFS1], see also item 7.4.g.

(e) Asymptotic lower bounds for $R(K_{a,b,c}, K_{a,b,c};3)$, where $K_{a,b,c}$ is formed by all abc triples on sets of orders a, b, c [MiPal].

(f) If G is a 3-uniform H-free hypergraph, then G contains a complete or empty tripartite subgraph with parts of order $(\log n(H))^{c+1/2}$, where $c > 0$ depends only on H. Furthermore, for $k \ge 4$ no analogue of it can hold for k-uniform hypergraphs [ConFS5].

(g) Asymptotic or exact values of $R_k(H;3)$ when H is a bow $\{abc, ade\}$, kite $\{abc, abd\}$, tight path $P_3^{3,2} = \{abc, bcd, cde\}$, or windmill $\{abc, bde, cef, bce\}$, and, among others, a special case $R_6(kite;3) = 8$ [AxGLM].

(h) $R_k(K_3) \le R_{4k}(K_4 - t;3) \le R_{4k}(K_3) + 1$ [AxGLM].

(i) Upper bounds on $R_k(H;3)$ for complete multipartite 3-uniform hypergraphs H, a 4-color case, and some other general and special cases [ConFS1, ConFS2, ConFS3]. $R_k(H;3)$ ranges from $\sqrt{6k}\,(1+o(1))$ to double exponential in k [AxGLM].

7.4. General results

(a) If $R(n,n;r) > m$ then $R(2n+r-4, 2n+r-4; r+1) > 2^m$, for $n > r \ge 3$ (see [GRS] p. 106). This is the so-called stepping-up lemma, usually credited to Erdős and Hajnal. An improvement of the stepping-up lemma implying better lower bounds for a few types of hypergraph Ramsey numbers were obtained by Conlon, Fox and Sudakov [ConFS6].

(b) Lower bounds on $R_k(n;r)$ are discussed in [AbbW, DLR].

(c) General lower bounds for large number of colors were given in an early paper by Hirschfeld [Hir], and some of them were later improved in [AbbL].

(d) Lower and upper asymptotics of $R(s,n;k)$ for fixed s [ConFS2].

(e) Exact and asymptotic results generalizing 7.2.d to r-uniform case for cycles, and 2- and 3-color cases for all r-uniform diamond matchings [GySS1].

(f) Study of $R(G,nH;r)$ and $R(mG,nH;r)$ for loose/tight path and cycles (possibly with some additions), stars, r-partite hypergraphs, including several exact results for large m or n [OmRa3].

(g) $R(H,H;r) \le cn(H)^{1+\varepsilon}$, for some constant $c = c(\Delta, r, \varepsilon)$ depending only on the maximum degree of H, r and $\varepsilon > 0$ [KoRö3]. The proofs of the linear bound $cn(H)$ were

obtained independently in [KüCFO] and [Ishi], the latter including the multicolor case, and then without regularity lemma in [ConFS1]. More discussion of lower and upper bounds for various cases can be found in [ConFS1, ConFS2, ConFS3, CooFKO2].

(h) Let T_r be an r-uniform hypergraph with r edges containing a fixed $(r-1)$-vertex set S and the $(r+1)$-st edge intersecting all former edges in one vertex outside S. Then $R(T_r, K_t; r) = O(t^r/\log t)$ [KosMV1].

(i) Let $H^r(s,t)$ be the complete r-partite r-uniform hypergraph with $r-2$ parts of size 1, one part of size s, and one part of size t (for example, for $r=2$ it is the same as $K_{s,t}$). For the multicolor numbers, Lazebnik and Mubayi [LaMu] proved that

$$tk^2 - k + 1 \le R_k(H^r(2, t+1); r) \le tk^2 + k + r,$$

where the lower bound holds when both t and k are prime powers. For the general case of $H^r(s,t)$, more bounds are presented in [LaMu].

(j) $R_k(H; r)$ is polynomial in k when a fixed r-uniform H is r-partite, otherwise it is at least exponential in k [AxGLM].

(k) Grolmusz [Grol1] generalized the classical constructive lower bound by Frankl and Wilson [FraWi] (item 2.3.6) to more colors and to hypergraphs [Grol3].

(l) Lower and upper asymptotics, and other theoretical results on hypergraph numbers, are gathered in [GrRö, GRS, ConFS1, ConFS2, ConFS3, Song8].

8. Cumulative Data and Surveys

8.1. Cumulative data for two colors

[CH1] $R(G,G)$ for all graphs G without isolates on at most 4 vertices.

[CH2] $R(G,H)$ for all graphs G and H without isolates on at most 4 vertices.

[Clan] $R(G,H)$ for all graphs G on at most 4 vertices and H on 5 vertices, except five entries (now all solved, see section 5.11). All critical colorings for the isolate-free graphs G and H studied in [Clan] were found in [He4].

[Bu4] $R(G,G)$ for all graphs G without isolates and with at most 6 edges.

[He1] $R(G,G)$ for all graphs G without isolates and with at most 7 edges.

[HaMe2] $R(G,G)$ for all graphs G on 5 vertices and with 7 or 8 edges.

[He2] $R(G,H)$ for all graphs G and H on 5 vertices without isolates, except 7 entries (2 still open, see 5.11 and the paragraph at the end of this section).

[LoM5] $R(G,H)$ for all disconnected isolate-free graphs H on at most 6 vertices versus all G on at most 5 vertices, except 3 cases. Missing cases were completed in [KroMe].

[HoMe] $R(G,H)$ for $G = K_{1,3}+e$ and $G = K_4 - e$ versus all connected graphs H on 6 vertices, except $R(K_4-e, K_6)$. The result $R(K_4-e, K_6)=21$ was claimed by McNamara [McN, unpublished], now confirmed in [ShWR].

[Boza4] $R(G,H)$ for some graphs G with 4 vertices versus all graphs H with 7 vertices.

[FRS4] $R(G,T)$ for all connected graphs G with $n(G)\le 5$, and almost all trees T.

[FRS1] $R(K_3,G)$ for all connected graphs G on 6 vertices.

[Jin] $R(K_3,G)$ for all connected graphs G on 7 vertices.
Some errors in [Jin] were found [SchSch1].

[Zeng] Formulas for $R(nK_3,mG)$ for all G of order 4 without isolates.

[Brin] $R(K_3,G)$ for all connected graphs G on at most 8 vertices. The numbers for K_3 versus sets of graphs with fixed number of edges, on at most 8 vertices, were presented in [KlaM1].

[BBH1] $R(K_3,G)$ for all connected graphs G on 9 vertices. See also [BBH2].

[BrGS] $R(K_3,G)$ for all graphs G on 10 vertices, except 10 cases (three of which, including $G=K_{10}-e$, were solved [GoR2]).

[JR3] $R(C_4,G)$ for all graphs G on at most 6 vertices.

[JR4] $R(C_5,G)$ for all graphs G on at most 6 vertices.

[JR2] $R(C_6,G)$ for all graphs G on at most 5 vertices.

[LoM3] $R(K_{2,n},K_{2,m})$ for all $2\le n, m\le 10$ except 8 cases, for which lower and upper bounds are given. Further data for other complete bipartite graphs are gathered in section 3.3 and [LoMe4].

[HaKr1] All best lower bounds up to 102 from cyclic graphs. Formulas for best cyclic lower bounds for paths and cycles, and values for small complete graphs and for graphs with up to five vertices.

Chvátal and Harary [CH1, CH2] formulated several simple but very useful observations on how to discover values of some numbers. All five missing entries in the tables of Clancy [Clan] have been solved (section 5.11). Out of 7 open cases in [He2] 5 have been solved, including $R(4,5)=R(G_{19},G_{23})=25$ and other cases listed in section 5.11. The 2 cases still open are for K_5 versus K_5 (section 2.1) and K_5 versus K_5-e (section 3.1). Many extremal and other Ramsey graphs for various parameters are available at [BrCGM, McK, Ex18, Fid2, Fuj1], see section 8.3 below.

8.2. Cumulative data for three colors

[YR3] $R_3(G)$ for all graphs G with at most 4 edges and no isolates.

[YR1] $R_3(G)$ for all graphs G with 5 edges and no isolates, except K_4-e.
The case of $R_3(K_4-e)$ remains open (see section 6.5).

[YY] $R_3(G)$ for all graphs G with 6 edges and no isolates, except 10 cases.

[ArKM] $R(F,G,H)$ for many triples of isolate-free graphs with at most 4 vertices.
Some of the missing cases completed in [KlaM2].

[BoDD] Extension of [ArKM] to most triples of graphs with at most 4 vertices.

[DzFi2] $R(P_3, P_k, C_m)$ for all $3 \le k \le 8$ and $3 \le m \le 9$.

8.3. Electronic Resources

(a) W. Gasarch [Gas] maintains a website gathering over 60 pointers to literature on applications of Ramsey theory in computer science, http://www.cs.umd.edu/~gasarch/ramsey/ramsey.html.

(b) Many of the Ramsey graph constructions found by G. Exoo [Ex1-Ex20] are posted at http://ginger.indstate.edu/ge/RAMSEY.

(c) G. Brinkmann, K. Coolsaet, J. Goedgebeur and H. Mélot, *House of Graphs: A database of interesting graphs* [BrCGM], http://hog.grinvin.org.

(d) B.D. McKay, presents some graphs related to classical Ramsey numbers [McK], http://cs.anu.edu.au/people/bdm/data/ramsey.html.

(e) R. Fidytek, presents some Ramsey graphs of type $(K_n, K_m - e)$ [Fid2], see also 3.1.f, http://fidytek.inf.ug.edu.pl/ramsey.

(f) H. Fujita, some Ramsey graphs [Fuj1], http://opal.inf.kyushu-u.ac.jp/~fujita/ramsey.html.

(g) Ramsey@Home [RaHo] is a distributed computing project at the University of Wisconsin-Oshkosh designed to find new lower bounds for various Ramsey numbers. Join and help! http://www.ramseyathome.com/ramsey.

8.4. Surveys

(1974) A general survey of results in Ramsey graph theory by S.A. Burr [Bu1]

(1978) A general survey of results in Ramsey graph theory by T.D. Parsons [Par6]

(1980) Survey of results and new problems on multiplicities and Ramsey multiplicities by S.A. Burr and V. Rosta [BuRo3]

(1981) Summary of progress by Frank Harary [Har2]

(1983) A survey of bounds and values by F.R.K. Chung and C.M. Grinstead [ChGri]

(1983) Special volume of the *Journal of Graph Theory* [JGT]

(1984) A review of Ramsey graph theory for newcomers by F.S. Roberts [Rob1]

(1987) What can we hope to accomplish in generalized Ramsey Theory? [Bu7]

(1987) Survey of asymptotic problems by R.L. Graham and V. Rödl [GrRö]

(1990) *Ramsey Theory* by R.L. Graham, B.L. Rothschild and J.H. Spencer [GRS]

(1991) Survey by R.J. Faudree, C.C. Rousseau and R.H. Schelp of graph goodness results, i.e. conditions for the formula $R(G, H) = (\chi(G) - 1)(n(H) - 1) + s(G)$ [FRS5]

(1996) A chapter in *Handbook of Combinatorics* by J. Nešetřil [Neš]

(1996) Survey of zero-sum Ramsey theory by Y. Caro [Caro]

(1997) Among 114 open problems and conjectures of Paul Erdős, presented and commented by F.R.K. Chung, 31 are concerned directly with Ramsey numbers [Chu4]. 216 references are given. An extended version of this work was prepared jointly with R.L. Graham [ChGra2] in 1998.

(2001) An extensive chapter on Ramsey theory in a widely used student textbook and researcher's guide of graph theory by D. West [West]

(2002) *Ramsey Theory and Paul Erdős* by R.L. Graham and J. Nešetřil [GrNe]

(2003) Special issue of *Combinatorics, Probability and Computing* [CoPC]

(2004) Dynamic survey of Ramsey theory applications by V. Rosta [Ros2]. A website maintained by W. Gasarch [Gas] gathers over 60 pointers to literature on applications of Ramsey theory in computer science.

(2009) History, results and people of Ramsey theory. The mathematical coloring book, mathematics of coloring and the colorful life of its creators by A. Soifer [Soi1].

(2011) *Ramsey Theory. Yesterday, Today and Tomorrow*, a special volume in the series *Progress in Mathematics* [Soi2]. A survey of Ramsey numbers involving cycles by the author is included in this volume [Ra4].

(2013) *Problems in Graph Theory from Memphis,* "a summary of problems and results coming out of the 20 year collaboration between Paul Erdős and the authors", by R.J. Faudree, C.C. Rousseau and R.H. Schelp [FRS6].

The surveys by S.A. Burr [Bu1] and T.D. Parsons [Par6] contain extensive chapters on general exact results in graph Ramsey theory. F. Harary presented the state of the theory in 1981 in [Har2], where he also gathered many references including seven to other early surveys of this area. More than two decades ago, Chung and Grinstead in their survey paper [ChGri] gave less data than in this work, but included a broad discussion of different methods used in Ramsey computations in the classical case. S. A. Burr, one of the most experienced researchers in Ramsey graph theory, formulated in [Bu7] seven conjectures on Ramsey numbers for sufficiently large and sparse graphs, and reviewed the evidence for them found in the literature. Three of them have been refuted in [Bra3].

For newer extensive presentations see [GRS, GrRö, FRS5, Neš, Chu4, ChGra2], though these focus on asymptotic theory not on the numbers themselves. A very welcome addition is the 2004 compilation of applications of Ramsey theory by V. Rosta [Ros2]. This survey could not be complete without recommending special volumes of the *Journal of Graph Theory* [JGT, 1983] and *Combinatorics, Probability and Computing* [CoPC, 2003], which, besides a number of research papers, include historical notes and present to us Frank P. Ramsey (1903-1930) as a person. Finally, read a colorful book by A. Soifer [Soi1, 2009] on history and results in Ramsey theory, followed by a collection of essays and technical papers based on presentations from the 2009 Ramsey theory workshop at DIMACS [Soi2, 2011].

The historical perspective and, in particular, the timeline of progress on prior best bounds, can be obtained by checking all the previous versions of this survey since 1994 at http://www.cs.rit.edu/~spr/ElJC/eline.html.

9. Concluding Remarks

This compilation does not include information on numerous variations of Ramsey numbers, nor related topics, like size Ramsey numbers, zero-sum Ramsey numbers, irredundant Ramsey numbers, induced Ramsey numbers, planar Ramsey numbers, bipartite Ramsey numbers, on-line Ramsey numbers, mixed Ramsey numbers, local Ramsey numbers, rainbow Ramsey numbers, connected Ramsey numbers, chromatic Ramsey numbers, avoiding sets of graphs in some colors, coloring graphs other than complete, or the so called Ramsey multiplicities. Interested readers can find such information in some of the surveys listed in section 8 here.

Readers may be interested in knowing that the US patent 6965854 B2 issued on November 15, 2005 claims a method of using Ramsey numbers in "Methods, Systems and Computer Program Products for Screening Simulated Traffic for Randomness." Check the original document at http://www.uspto.gov/patft if you wish to find out whether your usage of Ramsey numbers is covered by this patent.

Acknowledgements

In addition to the many individuals who helped to improve consecutive versions of this survey, the author would like to specially thank Brendan McKay, Geoffrey Exoo and Heiko Harborth for their help in gathering data for the first versions. Thanks to many other individuals who over the years have helped me in the development and improvement of new revisions.

The author apologizes for any omissions or other errors in reporting results belonging to the scope of this work. Suggestions for any kind of corrections or additions will be greatly appreciated and considered for inclusion in the next revision of this survey.

References

Out of 661 references gathered below, most appeared in about 100 different periodicals, among which most articles were published in: *Discrete Mathematics* 68, *Journal of Combinatorial Theory* (old, Series A and B) 55, *Journal of Graph Theory* 50, *Electronic Journal of Combinatorics* 32, *Ars Combinatoria* 28, *Journal of Combinatorial Mathematics and Combinatorial Computing* 27, *European Journal of Combinatorics* 20, *Utilitas Mathematica* 18, *Combinatorica* 15, *Graphs and Combinatorics* 14, *Australasian Journal of Combinatorics* 14, *Discrete Applied Mathematics* 12, *Congressus Numerantium* 12, and *Combinatorics, Probability and Computing* 11. The results of 143 references depend on computer algorithms.

The references are ordered alphabetically by the last name of the first author, and where multiple papers have the same first author they are ordered by the last name of the second author, etc. We preferred that all work by the same author be in consecutive positions. Unfortunately, this causes that some of the abbreviations are not in alphabetical order. For example, [BaRT] is earlier on the list than [BaLS]. We also wish to explain a possible confusion with respect to the order of parts and spelling of Chinese names. We put them without any abbreviations, often with the last name written first as is customary in original. Sometimes this is different from the citations in other sources. One can obtain all variations of writing any specific name by consulting the authors database of *Mathematical Reviews* at http://www.ams.org/mathscinet/search, or *zbMATH* (formerly *Zentralblatt für Mathematik*) at http://www.zbmath.org/authors.

Papers containing results obtained with the help of computer algorithms have been marked with stars. We identify two such categories of papers: those marked with * involving some use of computers where the results are easily verifiable with some computations, and those marked with ** where cpu intensive algorithms have to be implemented to replicate or verify the results. The first category contains mostly constructions done by algorithms, while the second mostly nonexistence results or claims of complete enumerations of special classes of graphs.

A, Ba, Br	page 57
Ca, Cl, D, E	page 62
F, Ga, Gu, H	page 68
I, J, K, La, Lo	page 74
M, N, O, P, Q, R	page 79
Sa, Si, Su	page 84
T, U, V, W, X, Y, Z	page 90 - page 94

A

[Abb1] H.L. Abbott, Some Problems in Combinatorial Analysis, *Ph.D. thesis,* Department of Mathematical and Statistical Sciences, University of Alberta, Edmonton, 1965.

[Abb2] H.L. Abbott, A Theorem Concerning Higher Ramsey Numbers, in *Infinite and Finite Sets,* (A. Hajnal, R. Rado and V.T. Sós eds.) Vol. 1, 25-28, Colloq. Math. Soc. Janos Bolyai, Vol. 10, North-Holland, Amsterdam, 1975.

[AbbH] H.L. Abbott and D. Hanson, A Problem of Schur and Its Generalizations, *Acta Arithmetica,* **20** (1972) 175-187.

[AbbL] H.L. Abbott and Andy Liu, Remarks on a Paper of Hirschfeld Concerning Ramsey Numbers, *Discrete Mathematics,* **39** (1982) 327-328.

[AbbS] H.L. Abbott and M.J. Smuga-Otto, Lower Bounds for Hypergraph Ramsey Numbers, *Discrete Applied Mathematics,* **61** (1995) 177-180.

[AbbW] H.L. Abbott and E.R. Williams, Lower Bounds for Some Ramsey Numbers, *Journal of Combinatorial Theory,* Series A, **16** (1974) 12-17.

[-] Adiwijaya, see [SuAM, SuAAM].

[AjKS] M. Ajtai, J. Komlós and E. Szemerédi, A Note on Ramsey Numbers, *Journal of Combinatorial Theory,* Series A, **29** (1980) 354-360.

[AjKSS] M. Ajtai, J. Komlós, M. Simonovits and E. Szemerédi, Erdős-Sós Conjecture, *in preparation* (2013).

[AliBB] K. Ali, A.Q. Baig and E.T. Baskoro, On the Ramsey Number for a Linear Forest versus a Coctail Party Graph, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **71** (2009) 173-177.

[AliBas] K. Ali and E.T. Baskoro, On the Ramsey Numbers for a Combination of Paths and Jahangirs, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **65** (2008) 113-119.

[AliBT1] K. Ali, E.T. Baskoro and I. Tomescu, On the Ramsey Numbers for Paths and Generalized Jahangir Graphs $J_{s,m}$, *Bull. Math. Soc. Sci. Math. R. S. Roumanie (N.S.),* **51(99)** (2008) 177-182.

[AliBT2] K. Ali, E.T. Baskoro and I. Tomescu, On the Ramsey Number for Paths and Beaded Wheels, *Journal of Prime Research in Mathematics,* **5** (2009) 133-138.

[AliSur] K. Ali and Surahmat, A Cycle or Jahangir Ramsey Unsaturated Graphs, *Journal of Prime Research in Mathematics,* **2** (2006) 187-193.

[AllBS] P. Allen, G. Brightwell and J. Skokan, Ramsey-Goodness - and Otherwise, *Combinatorica,* **33** (2013) 125-160.

[Alon1] N. Alon, Subdivided Graphs Have Linear Ramsey Numbers, *Journal of Graph Theory,* **18** (1994) 343-347.

[Alon2] N. Alon, Explicit Ramsey Graphs and Orthonormal Labelings, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #R12, **1** (1994), 8 pages.

[AlBK] N. Alon, S. Ben-Shimon and M. Krivelevich, A Note on Regular Ramsey Graphs, *Journal of Graph Theory,* **64** (2010) 244-249.

[AlKS] N. Alon, M. Krivelevich and B. Sudakov, Turán Numbers of Bipartite Graphs and Related Ramsey-Type Questions, *Combinatorics, Probability and Computing,* **12** (2003) 477-494.

[AlPu] N. Alon and P. Pudlák, Constructive Lower Bounds for off-diagonal Ramsey Numbers, *Israel Journal of Mathematics,* **122** (2001) 243-251.

[AlRö] N. Alon and V. Rödl, Sharp Bounds for Some Multicolor Ramsey Numbers, *Combinatorica,* **25** (2005) 125-141.

[AlRóS] N. Alon, L. Rónyai and T. Szabó, Norm-Graphs: Variations and Applications, *Journal of Combinatorial Theory,* Series B, **76** (1999) 280-290.

[-] B.M.N. Alzaleq, see [BatJA, JaAl1, JaAl2].

[ArKM] J. Arste, K. Klamroth and I. Mengersen, Three Color Ramsey Numbers for Small Graphs, *Utilitas Mathematica,* **49** (1996) 85-96.

[-] H. Assiyatun, see [HaABS, HaBA1, HaBA2, BaHA, SuAAM, SuBAU1, SuBAU2, SuBAU3].

[AxFM] M. Axenovich, Z. Füredi and D. Mubayi, On Generalized Ramsey Theory: the Bipartite Case, *Journal of Combinatorial Theory,* Series B, **79** (2000) 66-86.

[AxGLM] M. Axenovich, A. Gyárfás, Homg Liu and D. Mubayi, Multicolor Ramsey Numbers for Triple Systems, *preprint,* arXiv, http://arxiv.org/abs/1302.5304 (2013).

Ba - Bo

[BaRT]* A. Babak, S.P. Radziszowski and Kung-Kuen Tse, Computation of the Ramsey Number $R(B_3, K_5)$, *Bulletin of the Institute of Combinatorics and its Applications*, **41** (2004) 71-76.

[Back1] J. Backelin, Contributions to a Ramsey Calculus, *manuscript* 2000-2012.

[Back2] J. Backelin, *personal communication* (2013).

[BahS] P. Bahls and T.S. Spencer, On the Ramsey Numbers of Trees with Small Diameter, *Graphs and Combinatorics*, **29** (2013) 39-44.

[BaiLi] Bai Lufeng and Li Yusheng, Algebraic Constructions and Applications in Ramsey Theory, *Advances in Mathematics*, **35** (2006) 167-170.

[BaLX] Bai Lufeng, Li Yusheng and Xu Zhiqiang, Algebraic Constructions and Applications in Ramsey Theory, *Journal of Mathematical Study (China)*, **37** (2004) 245-249.

[-] Bai Lufeng, see also [SonBL].

[-] A.Q. Baig, see [AliBB].

[BaLS] P.N. Balister, J. Lehel and R.H. Schelp, Ramsey Unsaturated and Saturated Graphs, *Journal of Graph Theory*, **51** (2006) 22-32.

[BaSS] P.N. Balister, R.H. Schelp and M. Simonovits, A Note on Ramsey Size-Linear Graphs, *Journal of Graph Theory*, **39** (2002) 1-5.

[-] A.M.M. Baniabedalruhman, see [JaBa].

[-] Qiquan Bao, see [ShaXB, ShaXBP].

[BarRSW] B. Barak, A. Rao, R. Shaltiel and A. Widgerson, 2-Source Dispersers for $n^{o(1)}$ Entropy, and Ramsey Graphs Beating the Frankl-Wilson Construction, *Annals of Mathematics* (2), **176** (2012) 1483-1544.

[Bas] E.T. Baskoro, The Ramsey Number of Paths and Small Wheels, *Majalah Ilmiah Himpunan Matematika Indonesia*, MIHMI, **8** (2002) 13-16.

[BaHA] E.T. Baskoro, Hasmawati and H. Assiyatun, The Ramsey Numbers for Disjoint Unions of Trees, *Discrete Mathematics*, **306** (2006) 3297-3301.

[BaSu] E.T. Baskoro and Surahmat, The Ramsey Number of Paths with respect to Wheels, *Discrete Mathematics*, **294** (2005) 275-277.

[BaSNM] E.T. Baskoro, Surahmat, S.M. Nababan and M. Miller, On Ramsey Graph Numbers for Trees versus Wheels of Five or Six Vertices, *Graphs and Combinatorics*, **18** (2002) 717-721.

[-] E.T. Baskoro, see also [AliBB, AliBas, AliBT1, AliBT2, HaABS, HaBA1, HaBA2, SuBa1, SuBa2, SuBAU1, SuBAU2, SuBAU3, SuBB1, SuBB2, SuBB3, SuBB4, SuBT1, SuBT2, SuBTB, SuBUB].

[BatJA] M.S.A. Bataineh, M.M.M. Jaradat and L.M.N. Al-Zaleq, The Cycle-Complete Graph Ramsey Number $r(C_9, K_8)$, *International Scholarly Research Network - Algebra*, Article ID 926191, (2011), 10 pages.

[BenSk] F.S. Benevides and J. Skokan, The 3-Colored Ramsey Number of Even Cycles, *Journal of Combinatorial Theory*, Series B, **99** (2009) 690-708.

[-] S. Ben-Shimon, see [AlBK].

[Bev] D. Bevan, *personal communication* (2002).

[BePi] A. Beveridge and O. Pikhurko, On the Connectivity of Extremal Ramsey Graphs, *Australasian Journal of Combinatorics*, **41** (2008) 57-61.

[BiaS] A. Bialostocki and J. Schönheim, On Some Turán and Ramsey Numbers for C_4, in *Graph Theory and Combinatorics* (ed. B. Bollobás), Academic Press, London, (1984) 29-33.

[Biel1] H. Bielak, Ramsey and 2-local Ramsey Numbers for Disjoint Unions of Cycles, *Discrete Mathematics*, **307** (2007) 319-330.

[Biel2] H. Bielak, Ramsey Numbers for a Disjoint Union of Some Graphs, *Applied Mathematics Letters*, **22** (2009) 475-477.

[Biel3] H. Bielak, Multicolor Ramsey Numbers for Some Paths and Cycles, *Discussiones Mathematicae Graph Theory*, **29** (2009) 209-218.

[Biel4] H. Bielak, Ramsey Numbers for a Disjoint Union of Good Graphs, *Discrete Mathematics*, **310** (2010) 1501-1505.

[Bier] J. Bierbrauer, Ramsey Numbers for the Path with Three Edges, *European Journal of Combinatorics*, **7** (1986) 205-206.

[BierB] J. Bierbrauer and A. Brandis, On Generalized Ramsey Numbers for Trees, *Combinatorica,* **5** (1985) 95-107.

[BlLR]* K. Black, D. Leven and S.P. Radziszowski, New Bounds on Some Ramsey Numbers, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **78** (2011) 213-222.

[Boh] T. Bohman, The Triangle-Free Process, *Advances in Mathematics,* **221** (2009) 1653-1677.

[BohK1] T. Bohman and P. Keevash, The Early Evolution of the H-Free Process, *Inventiones Mathematicae,* **181** (2010) 291-336.

[BohK2] T. Bohman and P. Keevash, Dynamic Concentration of the Triangle-Free Process, *preprint,* arXiv, http://arxiv.org/abs/1302.5963 (2013).

[BolJY+] B. Bollobás, C.J. Jayawardene, Yang Jian Sheng, Huang Yi Ru, C.C. Rousseau, and Zhang Ke Min, On a Conjecture Involving Cycle-Complete Graph Ramsey Numbers, *Australasian Journal of Combinatorics,* **22** (2000) 63-71.

[BoH] R. Bolze and H. Harborth, The Ramsey Number $r(K_4-x, K_5)$, in *The Theory and Applications of Graphs,* (Kalamazoo, MI, 1980), John Wiley & Sons, New York, (1981) 109-116.

[BoEr] J.A. Bondy and P. Erdős, Ramsey Numbers for Cycles in Graphs, *Journal of Combinatorial Theory*, Series B, **14** (1973) 46-54.

[Boza1] L. Boza, Nuevas Cotas Superiores de Algunos Números de Ramsey del Tipo $r(K_m, K_n - e)$, in proceedings of the *VII Jornada de Matemática Discreta y Algoritmica*, JMDA 2010, Castro Urdiales, Spain, July 2010.

[Boza2] L. Boza, The Ramsey Number $r(K_5 - P_3, K_5)$, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #P90, **18**(1) (2011), 10 pages.

[Boza3]* L. Boza, Upper Bounds for Some Ramsey Numbers of $K_n - e$ versus K_m, *manuscript* (2012).

[Boza4]* L. Boza, Números de Ramsey de Algunos Grafos de 4 Vértices y Todods los Grafos de 7 Vértices, in proceedings of the *VIII Jornada de Matemática Discreta y Algoritmica*, JMDA 2012, Almeria, Spain, July 2012.

[Boza5]* L. Boza, *personal communication* (2013).

[Boza6]* L. Boza, Sobre el Número de Ramsey $R(K_4, K_6 - e)$, *VIII Encuentro Andaluz de Matemática Discreta*, Sevilla, Spain, October 2013.

[BoCGR] L. Boza, M. Cera, P. Garcia-Vázquez and M.P. Revuelta, On the Ramsey Numbers for Stars versus Complete Graphs, *European Journal of Combinatorics*, **31** (2010) 1680-1688.

[BoDD]* L. Boza, J. Dybizbański and T. Dzido, Three Color Ramsey Numbers for Graphs With at Most 4 Vertices, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #P47, **19**(4) (2012), 16 pages.

[BoPo]* L. Boza and J.R. Portillo, Sobre el Número de Ramsey $R(K_4 - e, K_7)$, in proceedings of the *VIII Jornada de Matemática Discreta y Algoritmica*, JMDA 2012, Almeria, Spain, July 2012.

Br - Bu

[-] A. Brandis, see [BierB].

[Bra1] S. Brandt, Subtrees and Subforests in Graphs, *Journal of Combinatorial Theory*, Series B, **61** (1994) 63-70.

[Bra2] S. Brandt, Sufficient Conditions for Graphs to Contain All Subgraphs of a Given Type, *Ph.D. thesis*, Freie Universität Berlin, 1994.

[Bra3] S. Brandt, Expanding Graphs and Ramsey Numbers, *preprint No. A 96-24*, ftp://ftp.math.fu-berlin.de/pub/math/publ/pre/1996 (1996).

[BBH1]** S. Brandt, G. Brinkmann and T. Harmuth, All Ramsey Numbers $r(K_3, G)$ for Connected Graphs of Order 9, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #R7, **5** (1998), 20 pages.

[BBH2]** S. Brandt, G. Brinkmann and T. Harmuth, The Generation of Maximal Triangle-Free Graphs, *Graphs and Combinatorics*, **16** (2000) 149-157.

[-] G. Brightwell. see [AllBS].

[Brin]** G. Brinkmann, All Ramsey Numbers $r(K_3, G)$ for Connected Graphs of Order 7 and 8, *Combinatorics, Probability and Computing*, **7** (1998) 129-140.

[BrCGM]* G. Brinkmann, K. Coolsaet, J. Goedgebeur and H. Mélot, House of Graphs: A database of interesting graphs, *Discrete Applied Mathematics*, **161** (2013) 311-314.

[BrGS]** G. Brinkmann, J. Goedgebeur and J.C. Schlage-Puchta, Ramsey Numbers $R(K_3, G)$ for Graphs of Order 10, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #P36, **19**(4) (2012), 23 pages.

[-] G. Brinkmann, see also [BBH1, BBH2].

[-] H.J. Broersma, see [SaBr1, SaBr2, SaBr3, SaBr4, SuBB1, SuBB2, SuBB3, SuBB4, SuBTB, SuBUB].

[BR]* J.P. Burling and S.W. Reyner, Some Lower Bounds of the Ramsey Numbers $n(k, k)$, *Journal of Combinatorial Theory*, Series B, **13** (1972) 168-169.

[Bu1] S.A. Burr, Generalized Ramsey Theory for Graphs - a Survey, in *Graphs and Combinatorics* (R. Bari and F. Harary eds.), Springer LNM **406**, Berlin, (1974) 52-75.

[Bu2] S.A. Burr, Ramsey Numbers Involving Graphs with Long Suspended Paths, *Journal of the London Mathematical Society* (2), **24** (1981) 405-413.

[Bu3] S.A. Burr, Multicolor Ramsey Numbers Involving Graphs with Long Suspended Path, *Discrete Mathematics*, **40** (1982) 11-20.

[Bu4] S.A. Burr, Diagonal Ramsey Numbers for Small Graphs, *Journal of Graph Theory*, **7** (1983) 57-69.

[Bu5] S.A. Burr, Ramsey Numbers Involving Powers of Sparse Graphs, *Ars Combinatoria*, **15** (1983) 163-168.

[Bu6] S.A. Burr, Determining Generalized Ramsey Numbers is NP-Hard, *Ars Combinatoria*, **17** (1984) 21-25.

[Bu7] S.A. Burr, What Can We Hope to Accomplish in Generalized Ramsey Theory?, *Discrete Mathematics*, **67** (1987) 215-225.

[Bu8] S.A. Burr, On the Ramsey Numbers $r(G, nH)$ and $r(nG, nH)$ When n Is Large, *Discrete Mathematics*, **65** (1987) 215-229.

[Bu9] S.A. Burr, On Ramsey Numbers for Large Disjoint Unions of Graphs, *Discrete Mathematics*, **70** (1988) 277-293.

[Bu10] S.A. Burr, On the Computational Complexity of Ramsey-type Problems, Mathematics of Ramsey Theory, *Algorithms and Combinatorics*, **5**, Springer, Berlin, 1990, 46-52.

[BE1] S.A. Burr and P. Erdős, On the Magnitude of Generalized Ramsey Numbers for Graphs, in *Infinite and Finite Sets*, (A. Hajnal, R. Rado and V.T. Sós eds., Keszthely 1973) Vol. 1, 215-240, Colloq. Math. Soc. Janos Bolyai, Vol. 10, North-Holland, Amsterdam, 1975.

[BE2] S.A. Burr and P. Erdős, Extremal Ramsey Theory for Graphs, *Utilitas Mathematica*, **9** (1976) 247-258.

[BE3] S.A. Burr and P. Erdős, Generalizations of a Ramsey-Theoretic Result of Chvátal, *Journal of Graph Theory*, **7** (1983) 39-51.

[BEFRS1] S.A. Burr, P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, An Extremal Problem in Generalized Ramsey Theory, *Ars Combinatoria,* **10** (1980) 193-203.

[BEFRS2] S.A. Burr, P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Ramsey Numbers for the Pair Sparse Graph-Path or Cycle, *Transactions of the American Mathematical Society,* **269** (1982) 501-512.

[BEFRS3] S.A. Burr, P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, The Ramsey Number for the Pair Complete Bipartite Graph-Graph of Limited Degree, in *Graph Theory with Applications to Algorithms and Computer Science,* (Y. Alavi et al. eds.), John Wiley & Sons, New York, (1985) 163-174.

[BEFRS4] S.A. Burr, P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Some Complete Bipartite Graph-Tree Ramsey Numbers, *Annals of Discrete Mathematics,* **41** (1989) 79-89.

[BEFRSGJ] S.A. Burr, P. Erdős, R.J. Faudree, C.C. Rousseau, R.H. Schelp, R.J. Gould and M.S. Jacobson, Goodness of Trees for Generalized Books, *Graphs and Combinatorics,* **3** (1987) 1-6.

[BEFS] S.A. Burr, P. Erdős, R.J. Faudree and R.H. Schelp, On the Difference between Consecutive Ramsey Numbers, *Utilitas Mathematica,* **35** (1989) 115-118.

[BES] S.A. Burr, P. Erdős and J.H. Spencer, Ramsey Theorems for Multiple Copies of Graphs, *Transactions of the American Mathematical Society,* **209** (1975) 87-99.

[BF] S.A. Burr and R.J. Faudree, On Graphs G for Which All Large Trees Are G-good, *Graphs and Combinatorics,* **9** (1993) 305-313.

[BFRS] S.A. Burr, R.J. Faudree, C.C. Rousseau and R.H. Schelp, On Ramsey Numbers Involving Starlike Multipartite Graphs, *Journal of Graph Theory,* **7** (1983) 395-409.

[BG] S.A. Burr and J.W. Grossman, Ramsey Numbers of Graphs with Long Tails, *Discrete Mathematics,* **41** (1982) 223-227.

[BuRo1] S.A. Burr and J.A. Roberts, On Ramsey Numbers for Stars, *Utilitas Mathematica,* **4** (1973) 217-220.

[BuRo2] S.A. Burr and J.A. Roberts, On Ramsey Numbers for Linear Forests, *Discrete Mathematics,* **8** (1974) 245-250.

[BuRo3] S.A. Burr and V. Rosta, On the Ramsey Multiplicities of Graphs - Problems and Recent Results, *Journal of Graph Theory,* **4** (1980) 347-361.

[Bush] L.E. Bush, The William Lowell Putnam Mathematical Competition (question #2 in Part I asks for the proof of $R(3,3) \le 6$), *American Mathematical Monthly,* **60** (1953) 539-542.

Ca - Ch

[CaET]* N.J. Calkin, P. Erdős and C.A. Tovey, New Ramsey Bounds from Cyclic Graphs of Prime Order, *SIAM Journal of Discrete Mathematics,* **10** (1997) 381-387.

[CalSR]* J.A. Calvert and M.J. Schuster and S.P. Radziszowski, Computing the Ramsey Number $R(K_5 - P_3, K_5)$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **82** (2012) 131-140.

[Car] D. Cariolaro, On the Ramsey Number $R(3,6)$, *Australasian J. of Combinatorics,* **37** (2007) 301-304.

[Caro] Y. Caro, Zero-Sum Problems - A Survey, *Discrete Mathematics,* **152** (1996) 93-113.

[CaLRZ] Y. Caro, Li Yusheng, C.C. Rousseau and Zhang Yuming, Asymptotic Bounds for Some Bipartite Graph - Complete Graph Ramsey Numbers, *Discrete Mathematics,* **220** (2000) 51-56.

[-] M. Cera, see [BoCGR].

[ChaMR] J. Chapppelon, L.P. Montejano and J.L. Ramirez Alfonsin, Upper Bounds and Values for $r(K_m, K_n - K_{1,s})$ and $r(K_m - e, K_n - K_{1,s})$, *preprint,* arXiv, http://arxiv.org/abs/1310.3786 (2013).

[ChGP] G. Chartrand, R.J. Gould and A.D. Polimeni, On Ramsey Numbers of Forests versus Nearly Complete Graphs, *Journal of Graph Theory,* **4** (1980) 233-239.

[CRSPS] G. Chartrand, C.C. Rousseau, M.J. Stewart, A.D. Polimeni and J. Sheehan, On Star-Book Ramsey Numbers, in *Proceedings of the Fourth International Conference on the Theory and Applications of Graphs*, (Kalamazoo, MI 1980), John Wiley & Sons, (1981) 203-214.

[ChaS] G. Chartrand and S. Schuster, On the Existence of Specified Cycles in Complementary Graphs, *Bulletin of the American Mathematical Society*, **77** (1971) 995-998.

[Chen] Chen Guantao, A Result on C_4-Star Ramsey Numbers, *Discrete Mathematics*, **163** (1997) 243-246.

[ChenS] Chen Guantao and R.H. Schelp, Graphs with Linearly Bounded Ramsey Numbers, *Journal of Combinatorial Theory*, Series B, **57** (1993) 138-149.

[ChWXSL]* Chen Hong, Wu Kang, Xu Xiaodong, Su Wenlong and Liang Wenzhong, New Lower Bound for Nine Classical Ramsey Numbers $R(3,t)$ (in Chinese), *Journal of Mathematics*, **31** (2011) 582-586.

[-] Chen Hong, see also [LiaWXCS, XWCS].

[ChenJ] Chen Jie, The Lower Bound of Some Ramsey Numbers (in Chinese), *Journal of Liaoning Normal University, Natural Science*, **25** (2002) 244-246.

[ChenCMN] Yaojun Chen, T.C. Edwin Cheng, Zhengke Miao and C.T. Ng, The Ramsey Numbers for Cycles versus Wheels of Odd Order, *Applied Mathematics Letters*, **22** (2009) 1875-1876.

[ChenCNZ] Yaojun Chen, T.C. Edwin Cheng, C.T. Ng and Yunqing Zhang, A Theorem on Cycle-Wheel Ramsey Number, *Discrete Mathematics*, **312** (2012) 1059-1061.

[ChenCX] Yaojun Chen, T.C. Edwin Cheng and Ran Xu, The Ramsey Number for a Cycle of Length Six versus a Clique of Order Eight, *Discrete Applied Mathematics*, **157** (2009) 8-12.

[ChenCZ1] Yaojun Chen, T.C. Edwin Cheng and Yunqing Zhang, The Ramsey Numbers $R(C_m, K_7)$ and $R(C_7, K_8)$, *European Journal of Combinatorics*, **29** (2008) 1337-1352.

[ChenZZ1] Chen Yaojun, Zhang Yunqing and Zhang Ke Min, The Ramsey Numbers of Paths versus Wheels, *Discrete Mathematics*, **290** (2005) 85-87.

[ChenZZ2] Chen Yaojun, Zhang Yunqing and Zhang Ke Min, The Ramsey Numbers of Stars versus Wheels, *European Journal of Combinatorics*, **25** (2004) 1067-1075.

[ChenZZ3] Chen Yaojun, Zhang Yunqing and Zhang Ke Min, The Ramsey Numbers $R(T_n, W_6)$ for $\Delta(T_n) \geq n-3$, *Applied Mathematics Letters*, **17** (2004) 281-285.

[ChenZZ4] Chen Yaojun, Zhang Yunqing and Zhang Ke Min, The Ramsey Numbers of Trees versus W_6 or W_7, *European Journal of Combinatorics*, **27** (2006) 558-564.

[ChenZZ5] Chen Yaojun, Zhang Yunqing and Zhang Ke Min, The Ramsey Numbers $R(T_n, W_6)$ for Small n, *Utilitas Mathematica*, **67** (2005) 269-284.

[ChenZZ6] Chen Yaojun, Zhang Yunqing and Zhang Ke Min, The Ramsey Numbers $R(T_n, W_6)$ for T_n without Certain Deletable Sets, *Journal of Systems Science and Complexity*, **18** (2005) 95-101.

[-] Chen Yaojun, see also [CheCZN, ZhaCC1, ZhaCC2, ZhaCZ1, ZhaCZ2].

[Cheng] Cheng Ying, On Graphs Which Do Not Contain Certain Trees, *Ars Combinatoria*, **19** (1985) 119-151.

[CheCZN] T.C. Edwin Cheng, Yaojun Chen, Yunqing Zhang and C.T. Ng, The Ramsey Numbers for a Cycle of Length Six or Seven versus a Clique of Order Seven, *Discrete Mathematics*, **307** (2007) 1047-1053.

[-] T.C. Edwin Cheng, see also [ChenCMN, ChenCNZ, ChenCX, ChenCZ1, ZhaCC1, ZhaCC2].

[Chu1] F.R.K. Chung, On the Ramsey Numbers $N(3,3,...,3;2)$, *Discrete Mathematics*, **5** (1973) 317-321.

[Chu2] F.R.K. Chung, On Triangular and Cyclic Ramsey Numbers with k Colors, in *Graphs and Combinatorics* (R. Bari and F. Harary eds.), Springer LNM **406**, Berlin, (1974) 236-241.

[Chu3] F.R.K. Chung, A Note on Constructive Methods for Ramsey Numbers, *Journal of Graph Theory*, **5** (1981) 109-113.

[Chu4] F.R.K. Chung, Open problems of Paul Erdős in Graph Theory, *Journal of Graph Theory*, **25** (1997) 3-36.

[ChCD] F.R.K. Chung, R. Cleve and P. Dagum, A Note on Constructive Lower Bounds for the Ramsey Numbers $R(3,t)$, *Journal of Combinatorial Theory*, Series B, **57** (1993) 150-155.

[ChGra1] F.R.K. Chung and R.L. Graham, On Multicolor Ramsey Numbers for Complete Bipartite Graphs, *Journal of Combinatorial Theory*, Series B, **18** (1975) 164-169.

[ChGra2] F.R.K. Chung and R.L. Graham, *Erdős on Graphs, His Legacy of Unsolved Problems*, A K Peters, Wellesley, Massachusetts (1998).

[ChGri] F.R.K. Chung and C.M. Grinstead, A Survey of Bounds for Classical Ramsey Numbers, *Journal of Graph Theory*, **7** (1983) 25-37.

[Chv] V. Chvátal, Tree-Complete Graph Ramsey Numbers, *Journal of Graph Theory*, **1** (1977) 93.

[CH1] V. Chvátal and F. Harary, Generalized Ramsey Theory for Graphs, II. Small Diagonal Numbers, *Proceedings of the American Mathematical Society*, **32** (1972) 389-394.

[CH2] V. Chvátal and F. Harary, Generalized Ramsey Theory for Graphs, III. Small Off-Diagonal Numbers, *Pacific Journal of Mathematics*, **41** (1972) 335-345.

[CH3] V. Chvátal and F. Harary, Generalized Ramsey Theory for Graphs, I. Diagonal Numbers, *Periodica Mathematica Hungarica*, **3** (1973) 115-124.

[CRST] V. Chvátal, V. Rödl, E. Szemerédi and W.T. Trotter Jr., The Ramsey Number of a Graph with Bounded Maximum Degree, *Journal of Combinatorial Theory*, Series B, **34** (1983) 239-243.

[ChvS] V. Chvátal and A. Schwenk, On the Ramsey Number of the Five-Spoked Wheel, in *Graphs and Combinatorics* (R. Bari and F. Harary eds.), Springer LNM **406**, Berlin, (1974) 247-261.

Cl - Cs

[Clan] M. Clancy, Some Small Ramsey Numbers, *Journal of Graph Theory*, **1** (1977) 89-91.

[Clap] C. Clapham, The Ramsey Number $r(C_4, C_4, C_4)$, *Periodica Mathematica Hungarica*, **18** (1987) 317-318.

[CEHMS] C. Clapham, G. Exoo, H. Harborth, I. Mengersen and J. Sheehan, The Ramsey Number of $K_5 - e$, *Journal of Graph Theory*, **13** (1989) 7-15.

[Clark] L. Clark, On Cycle-Star Graph Ramsey Numbers, *Congressus Numerantium*, **50** (1985) 187-192.

[CleDa] R. Cleve and P. Dagum, A Constructive $\Omega(t^{1.26})$ Lower Bound for the Ramsey Number $R(3,t)$, *International Computer Science Institute*, TR-89-009, Berkeley, CA, 1989.

[-] R. Cleve, see also [ChCD].

[Coc] E.J. Cockayne, Some Tree-Star Ramsey Numbers, *Journal of Combinatorial Theory*, Series B, **17** (1974) 183-187.

[CocL1] E.J. Cockayne and P.J. Lorimer, The Ramsey Number for Stripes, *Journal of the Australian Mathematical Society*, Series A, **19** (1975) 252-256.

[CocL2] E.J. Cockayne and P.J. Lorimer, On Ramsey Graph Numbers for Stars and Stripes, *Canadian Mathematical Bulletin*, **18** (1975) 31-34.

[CPR] B. Codenotti, P. Pudlák and G. Resta, Some Structural Properties of Low-Rank Matrices Related to Computational Complexity, *Theoretical Computer Science*, **235** (2000) 89-107.

[CoPC] Special issue on Ramsey theory of *Combinatorics, Probability and Computing*, **12** (2003), Numbers 5 and 6.

[Con1] D. Conlon, A New Upper Bound for Diagonal Ramsey Numbers, *Annals of Mathematics*, **170** (2009) 941-960.

[Con2] D. Conlon, Hypergraph Packing and Sparse Bipartite Ramsey Numbers, *Combinatorics, Probability and Computing*, **18** (2009) 913-923.

[Con3] D. Conlon, The Ramsey Number of Dense Graphs, *Bulletin of the London Mathematical Society* **45** (2013) 483-496.

[ConFS1] D. Conlon, J. Fox and B. Sudakov, Ramsey Numbers of Sparse Hypergraphs, *Random Structures and Algorithms*, **35** (2009) 1-14.

[ConFS2] D. Conlon, J. Fox and B. Sudakov, Hypergraph Ramsey Numbers, *Journal of the American Mathematical Society*, **23** (2010) 247-266.

[ConFS3] D. Conlon, J. Fox and B. Sudakov, Large Almost Monochromatic Subsets in Hypergraphs, *Israel Journal of Mathematics*, **181** (2011) 423-432.

[ConFS4] D. Conlon, J. Fox and B. Sudakov, On Two Problems in Graph Ramsey Theory, *Combinatorica*, **32** (2012) 513-535.

[ConFS5] D. Conlon, J. Fox and B. Sudakov, Erdős-Hajnal-type Theorems in Hypergraphs, *Journal of Combinatorial Theory*, Series B, **102** (2012) 1142-1154.

[ConFS6] D. Conlon, J. Fox and B. Sudakov, An Improved Bound for the Stepping-Up Lemma, *Discrete Applied Mathematics*, **161** (2013) 1191-1196.

[CooFKO1] O. Cooley, N. Fountoulakis, D. Kühn and D. Osthus, 3-Uniform Hypergraphs of Bounded Degree Have Linear Ramsey Numbers, *Journal of Combinatorial Theory*, Series B, **98** (2008) 484-505.

[CooFKO2] O. Cooley, N. Fountoulakis, D. Kühn and D. Osthus, Embeddings and Ramsey Numbers of Sparse k-uniform Hypergraphs, *Combinatorica*, **29** (2009) 263-297.

[-] O. Cooley, see also [KüCFO].

[-] K. Coolsaet, see [BrCGM].

[CsKo] R. Csákány and J. Komlós, The Smallest Ramsey Numbers, *Discrete Mathematics*, **199** (1999) 193-199.

D

[-] P. Dagum, see [ChCD, CleDa].

[Den1] T. Denley, The Independence Number of Graphs with Large Odd Girth, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #R9, **1** (1994), 12 pages.

[Den2] T. Denley, The Ramsey Numbers for Disjoint Unions of Cycles, *Discrete Mathematics*, **149** (1996) 31-44.

[Dong] Dong Lin, A Note on a Lower Bound for $r(K_{m,n})$, *Journal of Tongji University (Natural Science)*, **38** (2010) 776,778.

[DoLi] Lin Dong and Yusheng Li, A Construction for Ramsey Numbers for $K_{m,n}$, *European Journal of Combinatorics*, **31** (2010) 1667-1670.

[DoLL1] Lin Dong, Yusheng Li and Qizhong Lin, Ramsey Numbers Involving Graphs with Large Degrees, *Applied Mathematics Letters*, **22** (2009) 1577-1580.

[DoLL2] Dong Lin, Li Yusheng and Lin Qizhong, Ramsey Numbers of Cycles vs. Large Complete Graph, *Advances in Mathematics (China)*, **39** (2010) 700-702.

[-] Dong Lin, see also [HeLD, LinLD].

[DuHu] Duan Chanlun and Huang Wenke, Lower Bound of Ramsey Number $r(3,10)$ (in Chinese), *Acta Scientiarum Naturalium Universitatis Nei Mongol*, **31** (2000) 468-470.

[DLR] D. Duffus, H. Lefmann and V. Rödl, Shift Graphs and Lower Bounds on Ramsey Numbers $r_k(l;r)$, *Discrete Mathematics*, **137** (1995) 177-187.

[Dyb]* J. Dybizbański, On Some Ramsey Numbers of C_4 versus $K_{2,n}$, *Journal of Combinatorial Mathematics and Combinatorial Computing*. **87** (2013) 137-145.

[DyDz1]* J. Dybizbański and T. Dzido, On Some Ramsey Numbers for Quadrilaterals, *Electronic Journal of Combinatorics*, http:// www.combinatorics.org, #P154, **18**(1) (2011), 12 pages.

[DyDz2] J. Dybizbański and T. Dzido, On Some Ramsey Numbers for Quadrilaterals versus Wheels, to appear in *Graphs and Combinatorics*, published online 28 Feb 2013.

[DyDR] J. Dybizbański, T. Dzido and S.P. Radziszowski, On Some Three-Color Ramsey Numbers for Paths, *manuscript* (2013).

[-] J. Dybizbański, see also [BoDD].

[Dzi1]* T. Dzido, Ramsey Numbers for Various Graph Classes (in Polish), *Ph.D. thesis*, University of Gdańsk, Poland, November 2005.

[Dzi2]* T. Dzido, Multicolor Ramsey Numbers for Paths and Cycles, *Discussiones Mathematicae Graph Theory*, **25** (2005) 57-65.

[DzFi1]* T. Dzido and R. Fidytek, The Number of Critical Colorings for Some Ramsey Numbers, *International Journal of Pure and Applied Mathematics*, ISSN 1311-8080, **38** (2007) 433-444.

[DzFi2]* T. Dzido and R. Fidytek, On Some Three Color Ramsey Numbers for Paths and Cycles, *Discrete Mathematics*, **309** (2009) 4955-4958.

[DzKP] T. Dzido, M. Kubale and K. Piwakowski, On Some Ramsey and Turán-type Numbers for Paths and Cycles, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #R55, **13** (2006), 9 pages.

[DzNS] T. Dzido, A. Nowik and P. Szuca, New Lower Bound for Multicolor Ramsey Numbers for Even Cycles, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #N13, **12** (2005), 5 pages.

[-] T. Dzido, see also [BoDD, DyDz1, DyDz2, DyDR].

E

[Ea1] Easy to obtain by simple combinatorics from other results, in particular by using graphs establishing lower bounds with smaller parameters.

[Ea2] Unique 2-(6,3,2) design gives lower bound 7, upper bound is easy.

[Ea3] Every edge (3,3,3;2)-coloring of K_{15} has 35 edges in each color [Hein], and since the number of triangles in K_{16} is not divisible by 3, hence no required triangle-coloring of K_{16} exists.

[Eaton] N. Eaton, Ramsey Numbers for Sparse Graphs, *Discrete Mathematics*, **185** (1998) 63-75.

[Erd1] P. Erdős, Some Remarks on the Theory of Graphs, *Bulletin of the American Mathematical Society*, **53** (1947) 292-294.

[Erd2] P. Erdős, Some New Problems and Results in Graph Theory and Other Branches of Combinatorial Mathematics, *Combinatorics and Graph Theory* (Calcutta 1980), Berlin-NY Springer, LNM **885** (1981) 9-17.

[Erd3] P. Erdős, On the Combinatorial Problems Which I Would Most Like to See Solved, *Combinatorica*, **1** (1981) 25-42.

[Erd4] P. Erdős, On Some Problems in Graph Theory, Combinatorial Analysis and Combinatorial Number Theory, *Graph Theory and Combinatorics*, (Cambridge 1983), 1-17, Academic Press, London-New York, 1984.

[EFRS1] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Generalized Ramsey Theory for Multiple Colors, *Journal of Combinatorial Theory*, Series B, **20** (1976) 250-264.

[EFRS2] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, On Cycle-Complete Graph Ramsey Numbers, *Journal of Graph Theory*, **2** (1978) 53-64.

[EFRS3] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Ramsey Numbers for Brooms, *Congressus Numerantium*, **35** (1982) 283-293.

[EFRS4] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Multipartite Graph-Sparse Graph Ramsey Numbers, *Combinatorica*, **5** (1985) 311-318.

[EFRS5] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, A Ramsey Problem of Harary on Graphs with Prescribed Size, *Discrete Mathematics*, **67** (1987) 227-233.

[EFRS6] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Extremal Theory and Bipartite Graph-Tree Ramsey Numbers, *Discrete Mathematics*, **72** (1988) 103-112.

[EFRS7] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, The Book-Tree Ramsey Numbers, *Scientia*, Series A: Mathematical Sciences, Valparaíso, Chile, **1** (1988) 111-117.

[EFRS8] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Multipartite Graph-Tree Graph Ramsey Numbers, in *Graph Theory and Its Applications: East and West, Proceedings of the First China-USA International Graph Theory Conference,* Annals of the New York Academy of Sciences, **576** (1989) 146-154.

[EFRS9] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Ramsey Size Linear Graphs, *Combinatorics, Probability and Computing,* **2** (1993) 389-399.

[EG] P. Erdős and R.L. Graham, On Partition Theorems for Finite Sets, in *Infinite and Finite Sets,* (A. Hajnal, R. Rado and V.T. Sós eds.) Vol. 1, 515--527, Colloq. Math. Soc. Janos Bolyai, Vol. 10, North Holland, 1975.

[-] P. Erdős, see also [BoEr, BE1, BE2, BE3, BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRSGJ, BEFS, BES, CaET].

[Ex1]* G. Exoo, Ramsey Numbers of Hypergraphs, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **2** (1987) 5-11.

[Ex2]* G. Exoo, Constructing Ramsey Graphs with a Computer, *Congressus Numerantium,* **59** (1987) 31-36.

[Ex3]* G. Exoo, Applying Optimization Algorithm to Ramsey Problems, in *Graph Theory, Combinatorics, Algorithms, and Applications* (Y. Alavi ed.), SIAM Philadelphia, (1989) 175-179.

[Ex4]* G. Exoo, A Lower Bound for $R(5,5)$, *Journal of Graph Theory,* **13** (1989) 97-98.

[Ex5]* G. Exoo, On Two Classical Ramsey Numbers of the Form $R(3,n)$, *SIAM Journal of Discrete Mathematics,* **2** (1989) 488-490.

[Ex6]* G. Exoo, A Lower Bound for $r(K_5-e,K_5)$, *Utilitas Mathematica,* **38** (1990) 187-188.

[Ex7]* G. Exoo, Three Color Ramsey Number of K_4-e, *Discrete Mathematics,* **89** (1991) 301-305.

[Ex8]* G. Exoo, Indiana State University, *personal communication* (1992).

[Ex9]* G. Exoo, Announcement: On the Ramsey Numbers $R(4,6)$, $R(5,6)$ and $R(3,12)$, *Ars Combinatoria,* **35** (1993) 85. The construction of a graph proving $R(4,6)\geq 35$ is presented in detail at http://ginger.indstate.edu/ge/RAMSEY (2001).

[Ex10]* G. Exoo, A Lower Bound for Schur Numbers and Multicolor Ramsey Numbers of K_3, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #R8, **1** (1994), 3 pages.

[Ex11]* G. Exoo, Indiana State University, *personal communication* (1997).

[Ex12]* G. Exoo, Some New Ramsey Colorings, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #R29, **5** (1998), 5 pages. The constructions are available electronically from http://ginger.indstate.edu/ge/RAMSEY.

[Ex13]* G. Exoo, Indiana State University, *personal communication* (1998). Constructions available at http://ginger.indstate.edu/ge/RAMSEY.

[Ex14]* G. Exoo, Indiana State University, *New Lower Bounds for Table III,* (2000). Constructions available at http://ginger.indstate.edu/ge/RAMSEY.

[Ex15]* G. Exoo, Indiana State University, *personal communication* (2002-2004). Constructions available at http://ginger.indstate.edu/ge/RAMSEY.

[Ex16]* G. Exoo, Indiana State University, *personal communication* (2005-2006). Constructions available at http://ginger.indstate.edu/ge/RAMSEY.

[Ex17]* G. Exoo, Indiana State University, *personal communication* (2010-2011). Constructions available at http://ginger.indstate.edu/ge/RAMSEY.

[Ex18]* G. Exoo, Indiana State University, *personal communication* (2012-2013). Constructions available at http://ginger.indstate.edu/ge/RAMSEY.

[Ex19]* G. Exoo, On the Ramsey Number $R(4,6)$, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P66, **19**(1) (2012), 5 pages.

[Ex20]* G. Exoo, On Some Small Classical Ramsey Numbers, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #P68, **20**(1) (2013), 6 pages.

[Ex21]* G. Exoo, Ramsey Colorings from p-Groups, *in preparation*, (2013).

[EHM1] G. Exoo, H. Harborth and I. Mengersen, The Ramsey Number of K_4 versus K_5-e, *Ars Combinatoria*, **25A** (1988) 277-286.

[EHM2] G. Exoo, H. Harborth and I. Mengersen, On Ramsey Number of $K_{2,n}$, in *Graph Theory, Combinatorics, Algorithms, and Applications* (Y. Alavi, F.R.K. Chung, R.L. Graham and D.F. Hsu eds.), SIAM Philadelphia, (1989) 207-211.

[ExRe]* G. Exoo and D.F. Reynolds, Ramsey Numbers Based on C_5-Decompositions, *Discrete Mathematics*, **71** (1988) 119-127.

[-] G. Exoo, see also [CEHMS, XXER].

F

[FLPS] R.J. Faudree, S.L. Lawrence, T.D. Parsons and R.H. Schelp, Path-Cycle Ramsey Numbers, *Discrete Mathematics*, **10** (1974) 269-277.

[FM]** R.J. Faudree and B.D. McKay, A Conjecture of Erdős and the Ramsey Number $r(W_6)$, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **13** (1993) 23-31.

[FRS1] R.J. Faudree, C.C. Rousseau and R.H. Schelp, All Triangle-Graph Ramsey Numbers for Connected Graphs of Order Six, *Journal of Graph Theory*, **4** (1980) 293-300.

[FRS2] R.J. Faudree, C.C. Rousseau and R.H. Schelp, Studies Related to the Ramsey Number $r(K_5-e)$, in *Graph Theory and Its Applications to Algorithms and Computer Science*, (Y. Alavi et al. eds.), John Wiley and Sons, New York, (1985) 251-271.

[FRS3] R.J. Faudree, C.C. Rousseau and R.H. Schelp, Generalizations of the Tree-Complete Graph Ramsey Number, in *Graphs and Applications*, (F. Harary and J.S. Maybee eds.), John Wiley and Sons, New York, (1985) 117-126.

[FRS4] R.J. Faudree, C.C. Rousseau and R.H. Schelp, Small Order Graph-Tree Ramsey Numbers, *Discrete Mathematics*, **72** (1988) 119-127.

[FRS5] R.J. Faudree, C.C. Rousseau and R.H. Schelp, A Good Idea in Ramsey Theory, in *Graph Theory, Combinatorics, Algorithms, and Applications* (San Francisco, CA 1989), SIAM Philadelphia, PA (1991) 180-189.

[FRS6] R.J. Faudree, C.C. Rousseau and R.H. Schelp, Problems in Graph Theory from Memphis, in *The Mathematics of Paul Erdős II*, R.L. Graham et al. (eds.), Springer, New York, (2013) 95-118.

[FRS7] R.J. Faudree, C.C. Rousseau and J. Sheehan, More from the Good Book, in *Proceedings of the Ninth Southeastern Conference on Combinatorics, Graph Theory, and Computing*, Utilitas Mathematica Publ., *Congressus Numerantium*, **XXI** (1978) 289-299.

[FRS8] R.J. Faudree, C.C. Rousseau and J. Sheehan, Strongly Regular Graphs and Finite Ramsey Theory, *Linear Algebra and its Applications*, **46** (1982) 221-241.

[FRS9] R.J. Faudree, C.C. Rousseau and J. Sheehan, Cycle-Book Ramsey Numbers, *Ars Combinatoria*, **31** (1991) 239-248.

[FS1] R.J. Faudree and R.H. Schelp, All Ramsey Numbers for Cycles in Graphs, *Discrete Mathematics*, **8** (1974) 313-329.

[FS2] R.J. Faudree and R.H. Schelp, Path Ramsey Numbers in Multicolorings, *Journal of Combinatorial Theory*, Series B, **19** (1975) 150-160.

[FS3] R.J. Faudree and R.H. Schelp, Ramsey Numbers for All Linear Forests, *Discrete Mathematics*, **16** (1976) 149-155.

[FS4] R.J. Faudree and R.H. Schelp, Some Problems in Ramsey Theory, in *Theory and Applications of Graphs*, (conference proceedings, Kalamazoo, MI 1976), Lecture Notes in Mathematics **642**,

Springer, Berlin, (1978) 500-515.

[FSR] R.J. Faudree, R.H. Schelp and C.C. Rousseau, Generalizations of a Ramsey Result of Chvátal, in *Proceedings of the Fourth International Conference on the Theory and Applications of Graphs*, (Kalamazoo, MI 1980), John Wiley & Sons, (1981) 351-361.

[FSS1] R.J. Faudree, R.H. Schelp and M. Simonovits, On Some Ramsey Type Problems Connected with Paths, Cycles and Trees, *Ars Combinatoria*, **29A** (1990) 97-106.

[FSS2] R.J. Faudree, A. Schelten and I. Schiermeyer, The Ramsey Number $r(C_7, C_7, C_7)$, *Discussiones Mathematicae Graph Theory*, **23** (2003) 141-158.

[FS] R.J. Faudree and M. Simonovits, Ramsey Problems and Their Connection to Turán-Type Extremal Problems, *Journal of Graph Theory*, **16** (1992) 25-50.

[-] R.J. Faudree, see also [BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRSGJ, BEFS, BF, BFRS, EFRS1, EFRS2, EFRS3, EFRS4, EFRS5, EFRS6, EFRS7, EFRS8, EFRS9].

[FKR]** S. Fettes, R.L. Kramer and S.P. Radziszowski, An Upper Bound of 62 on the Classical Ramsey Number $R(3,3,3,3)$, *Ars Combinatoria*, **72** (2004) 41-63.

[Fid1]* R. Fidytek, Two- and Three-Color Ramsey Numbers for Paths and Cycles, *manuscript* (2010).

[Fid2]* R. Fidytek, Ramsey Graphs $R(K_n, K_m - e)$, http://fidytek.inf.ug.edu.pl/ramsey (2010).

[-] R. Fidytek, see also [DzFi1, DzFi2].

[FiŁu1] A. Figaj and T. Łuczak, The Ramsey Number for a Triple of Long Even Cycles, *Journal of Combinatorial Theory*, Series B, **97** (2007) 584-596.

[FiŁu2] A. Figaj and T. Łuczak, The Ramsey Numbers for a Triple of Long Cycles, *preprint*, arXiv, http://front.math.ucdavis.edu/0709.0048 (2007).

[FizGM] G. Fiz Pontiveros, S. Griffiths and R. Morris, The Triangle-Free Process and $R(3,k)$, *preprint*, arXiv, http://arxiv.org/abs/1302.6279 (2013).

[FizGMSS] G. Fiz Pontiveros, S. Griffiths, R. Morris, D. Saxton and J. Skokan, The Ramsey Number of the Clique and the Hypercube, *preprint*, arXiv, http://arxiv.org/abs/1306.0461 (2013).

[Fol] J. Folkman, Notes on the Ramsey Number $N(3,3,3,3)$, *Journal of Combinatorial Theory*, Series A, **16** (1974) 371-379.

[-] N. Fountoulakis, see [CooFKO1, CooFKO2, KüCFO].

[FoxSu1] J. Fox and B. Sudakov, Density Theorems for Bipartite Graphs and Related Ramsey-type Results, *Combinatorica*, **29** (2009) 153-196.

[FoxSu2] J. Fox and B. Sudakov, Two Remarks on the Burr-Erdős Conjecture, *European Journal of Combinatorics*, **30** (2009) 1630-1645.

[-] J. Fox, see also [ConFS1, ConFS2, ConFS3, ConFS4, ConFS5, ConFS6].

[FraWi] P. Frankl and R.M. Wilson, Intersection Theorems with Geometric Consequences, *Combinatorica*, **1** (1981) 357-368.

[Fra1] K. Fraughnaugh Jones, Independence in Graphs with Maximum Degree Four, *Journal of Combinatorial Theory*, Series B, **37** (1984) 254-269.

[Fra2] K. Fraughnaugh Jones, Size and Independence in Triangle-Free Graphs with Maximum Degree Three, *Journal of Graph Theory*, **14** (1990) 525-535.

[FrLo] K. Fraughnaugh and S.C. Locke, Finding Independent Sets in Triangle-Free Graphs, *SIAM Journal of Discrete Mathematics*, **9** (1996) 674-681.

[Fre] H. Fredricksen, Schur Numbers and the Ramsey Numbers $N(3,3,...,3;2)$, *Journal of Combinatorial Theory*, Series A, **27** (1979) 376-377.

[FreSw]* H. Fredricksen and M.M. Sweet, Symmetric Sum-Free Partitions and Lower Bounds for Schur Numbers, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #R32, **7** (2000), 9 pages.

[Fuj1]* H. Fujita, Ramsey Numbers and Ramsey Graphs, website, http://opal.inf.kyushu-u.ac.jp/~fujita/ramsey.html.

[Fuj2] S. Fujita, Generalized Ramsey Numbers for Graphs with Three Disjoint Cycles versus a Complete Graph, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #P14, **19**(2) (2012), 11 pages.

[Für] Z. Füredi, Large Chromatic Number and Ramsey Graphs, *preprint*, arXiv, http://arxiv.org/abs/1103.3917 (2011).

[-] Z. Füredi, see also [AxFM].

Ga - Gr

[-] P. Garcia-Vázquez, see [BoCGR].

[Gas] W. Gasarch, Applications of Ramsey Theory to Computer Science, collection of pointers to papers, http://www.cs.umd.edu/~gasarch/ramsey/ramsey.html (2009, 2011).

[GauST] S. Gautam, A.K. Srivastava and A. Tripathi, On Multicolour Noncomplete Ramsey Graphs of Star Graphs, *Discrete Applied Mathematics*, **156** (2008) 2423-2428.

[Gerb]* R. Gerbicz, New Lower Bounds for Two Color and Multicolor Ramsey Numbers, *preprint*, arXiv, http://arxiv.org/abs/1004.4374 (2010).

[GeGy] L. Gerencsér and A. Gyárfás, On Ramsey-Type Problems, *Annales Universitatis Scientiarum Budapestinensis, Eötvös Sect. Math.*, **10** (1967) 167-170.

[Gi1] G. Giraud, Une généralisation des nombres et de l'inégalité de Schur, *C.R. Acad. Sc. Paris*, Séries A-B, **266** (1968) A437-A440.

[Gi2] G. Giraud, Minoration de certains nombres de Ramsey binaires par les nombres de Schur généralisés, *C.R. Acad. Sc. Paris*, Séries A-B, **266** (1968) A481-A483.

[Gi3] G. Giraud, Nouvelles majorations des nombres de Ramsey binaires-bicolores, *C.R. Acad. Sc. Paris*, Séries A-B, **268** (1969) A5-A7.

[Gi4] G. Giraud, Majoration du nombre de Ramsey ternaire-bicolore en (4,4), *C.R. Acad. Sc. Paris*, Séries A-B, **269** (1969) A620-A622.

[Gi5] G. Giraud, Une minoration du nombre de quadrangles unicolores et son application à la majoration des nombres de Ramsey binaires-bicolores, *C.R. Acad. Sc. Paris*, Séries A-B, **276** (1973) A1173-A1175.

[Gi6] G. Giraud, Sur le problème de Goodman pour les quadrangles et la majoration des nombres de Ramsey, *Journal of Combinatorial Theory*, Series B, **27** (1979) 237-253.

[-] A.M. Gleason, see [GG].

[GoK] W. Goddard and D.J. Kleitman, An Upper Bound for the Ramsey Numbers $r(K_3, G)$, *Discrete Mathematics*, **125** (1994) 177-182.

[GoR1]** J. Goedgebeur and S.P. Radziszowski, New Computational Upper Bounds for Ramsey Numbers $R(3,k)$, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #P30, **20**(1) (2013), 28 pages.

[GoR2]** J. Goedgebeur and S.P. Radziszowski, The Ramsey Number $R(3, K_{10}-e)$ and Computational Bounds for $R(3,G)$, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #P19, **20**(4) (2013), 25 pages.

[-] J. Goedgebeur, see also [BrCGM, BrGS].

[GoMC] A. Gonçalves and E.L. Monte Carmelo, Some Geometric Structures and Bounds for Ramsey Numbers, *Discrete Mathematics*, **280** (2004) 29-38.

[GoJa1] R.J. Gould and M.S. Jacobson, Bounds for the Ramsey Number of a Disconnected Graph versus Any Graph, *Journal of Graph Theory*, **6** (1982) 413-417.

[GoJa2] R.J. Gould and M.S. Jacobson, On the Ramsey Number of Trees versus Graphs with Large Clique Number, *Journal of Graph Theory*, **7** (1983) 71-78.

[-] R.J. Gould, see also [BEFRSGJ, ChGP].

[GrNe] R.L. Graham and J. Nešetřil, Ramsey Theory and Paul Erdős (Recent Results from a Historical Perspective), *Bolyai Society Mathematical Studies*, **11**, Budapest (2002) 339-365.

[GrRö] R.L. Graham and V. Rödl, Numbers in Ramsey Theory, in *Surveys in Combinatorics*, (ed. C. Whitehead), Cambridge University Press, 1987, 111-153.

[GRR1] R.L. Graham, V. Rödl and A. Ruciński, On Graphs with Linear Ramsey Numbers, *Journal of Graph Theory*, **35** (2000) 176-192.

[GRR2] R.L. Graham, V. Rödl and A. Ruciński, On Bipartite Graphs with Linear Ramsey Numbers, Paul Erdős and his mathematics, *Combinatorica*, **21** (2001) 199-209.

[GRS] R.L. Graham, B.L. Rothschild and J.H. Spencer, *Ramsey Theory*, John Wiley & Sons, second edition 1990.

[-] R.L. Graham, see also [ChGra1, ChGra2, EG].

[GrY] J.E. Graver and J. Yackel, Some Graph Theoretic Results Associated with Ramsey's Theorem, *Journal of Combinatorial Theory*, **4** (1968) 125-175.

[GG] R.E. Greenwood and A.M. Gleason, Combinatorial Relations and Chromatic Graphs, *Canadian Journal of Mathematics*, **7** (1955) 1-7.

[GH] U. Grenda and H. Harborth, The Ramsey Number $r(K_3, K_7 - e)$, *Journal of Combinatorics, Information & System Sciences*, **7** (1982) 166-169.

[-] S. Griffiths, see [FizGM, FizGMSS].

[Gri] J.R. Griggs, An Upper Bound on the Ramsey Numbers $R(3,k)$, *Journal of Combinatorial Theory*, Series A, **35** (1983) 145-153.

[GR]** C. Grinstead and S. Roberts, On the Ramsey Numbers $R(3,8)$ and $R(3,9)$, *Journal of Combinatorial Theory*, Series B, **33** (1982) 27-51.

[-] C. Grinstead, see also [ChGri].

[Grol1] V. Grolmusz, Superpolynomial Size Set-Systems with Restricted Intersections mod 6 and Explicit Ramsey Graphs, *Combinatorica*, **20** (2000) 73-88.

[Grol2] V. Grolmusz, Low Rank Co-Diagonal Matrices and Ramsey Graphs, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #R15, **7** (2000), 7 pages.

[Grol3] V. Grolmusz, Set-Systems with Restricted Multiple Intersections, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #R8, **9** (2002), 10 pages.

[Gros1] J.W. Grossman, Some Ramsey Numbers of Unicyclic Graphs, *Ars Combinatoria*, **8** (1979) 59-63.

[Gros2] J.W. Grossman, The Ramsey Numbers of the Union of Two Stars, *Utilitas Mathematica*, **16** (1979) 271-279.

[GHK] J.W. Grossman, F. Harary and M. Klawe, Generalized Ramsey Theory for Graphs, X: Double Stars, *Discrete Mathematics*, **28** (1979) 247-254.

[-] J.W. Grossman, see also [BG].

Gu - Gy

[GuLi] Gu Hua and Li Yusheng, On Ramsey Number of $K_{2,t+1}$ vs $K_{1,n}$, *Journal of Nanjing University Mathematical Biquarterly*, **19** (2002) 150-153.

[GuSL] Gu Hua, Song Hongxue and Liu Xiangyang, Ramsey Numbers $r(K_{1,4}, G)$ for All Three-Partite Graphs G of Order Six, *Journal of Southeast University*, (English Edition), **20** (2004) 378-380.

[-] Gu Hua, see also [SonGQ].

[GuoV] Guo Yubao and L. Volkmann, Tree-Ramsey Numbers, *Australasian Journal of Combinatorics,* **11** (1995) 169-175.

[-] L. Gupta, see [GGS].

[GGS] S.K. Gupta, L. Gupta and A. Sudan, On Ramsey Numbers for Fan-Fan Graphs, *Journal of Combinatorics, Information & System Sciences,* **22** (1997) 85-93.

[GyLSS] A. Gyárfás, J. Lehel, G.N. Sárközy and R.H. Schelp, Monochromatic Hamiltonian Berge-Cycles in Colored Complete Uniform Hypergraphs, *Journal of Combinatorial Theory*, Series B, **98** (2008) 342-358.

[GyRa] A. Gyárfás and G. Raeisi, The Ramsey Number of Loose Triangles and Quadrangles in Hypergraphs, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P30, **19(**2) (2012), 9 pages.

[GyRSS] A. Gyárfás, M. Ruszinkó, G.N. Sárközy and E. Szemerédi, Three-color Ramsey Numbers for Paths, *Combinatorica,* **27** (2007) 35-69. *Corrigendum* in **28** (2008) 499-502.

[GySá1] A. Gyárfás and G.N. Sárközy, The 3-Colour Ramsey Number of a 3-Uniform Berge Cycle, *Combinatorics, Probability and Computing*, **20** (2011) 53-71.

[GySá2] A. Gyárfás and G.N. Sárközy, Star versus Two Stripes Ramsey Numbers and a Conjecture of Schelp, *Combinatorics, Probability and Computing*, **21** (2012) 179-186.

[GySS1] A. Gyárfás, G.N. Sárközy and E. Szemerédi, The Ramsey Number of Diamond-Matchings and Loose Cycles in Hypergraphs, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #R126, **15** (2008), 14 pages.

[GySS2] A. Gyárfás, G.N. Sárközy and E. Szemerédi, Monochromatic Hamiltonian 3-Tight Berge Cycles in 2-Colored 4-Uniform Hypergraphs, *Journal of Graph Theory,* **63** (2010) 288-299.

[GySeT] A. Gyárfás, A. Sebő and N. Trotignon, The Chromatic Gap and Its Extremes, *Journal of Combinatorial Theory,* Series B, **102** (2012) 1155-1178.

[GyTu] A. Gyárfás and Z. Tuza, An Upper Bound on the Ramsey Number of Trees, *Discrete Mathematics,* **66** (1987) 309-310.

[-] A. Gyárfás, see also [AxGLM, GeGy].

H

[Häg] R. Häggkvist, On the Path-Complete Bipartite Ramsey Number, *Discrete Mathematics,* **75** (1989) 243-245.

[Han]* D. Hanson, Sum-Free Sets and Ramsey Numbers, *Discrete Mathematics,* **14** (1976) 57-61.

[-] D. Hanson, see also [AbbH].

[Hans] M. Hansson, On Generalized Ramsey Numbers for Two Sets of Cycles, *Självständiga Arbeten i Matematik,* Matematiska Institutionen, Stockholms Universitet, **28** (2012).

[Har1] F. Harary, Recent Results on Generalized Ramsey Theory for Graphs, in *Graph Theory and Applications,* (Y. Alavi et al. eds.) Springer, Berlin (1972) 125-138.

[Har2] F. Harary, Generalized Ramsey Theory I to XIII: Achievement and Avoidance Numbers, in *Proceedings of the Fourth International Conference on the Theory and Applications of Graphs,* (Kalamazoo, MI 1980), John Wiley & Sons, (1981) 373-390.

[-] F. Harary, see also [CH1, CH2, CH3, GHK].

[HaKr1]** H. Harborth and S. Krause, Ramsey Numbers for Circulant Colorings, *Congressus Numerantium,* **161** (2003) 139-150.

[HaKr2]** H. Harborth and S. Krause, Distance Ramsey Numbers, *Utilitas Mathematica,* **70** (2006) 197-200.

[HaMe1] H. Harborth and I. Mengersen, An Upper Bound for the Ramsey Number $r(K_5-e)$, *Journal of Graph Theory,* **9** (1985) 483-485.

[HaMe2] H. Harborth and I. Mengersen, All Ramsey Numbers for Five Vertices and Seven or Eight Edges, *Discrete Mathematics*, **73** (1988/89) 91-98.

[HaMe3] H. Harborth and I. Mengersen, The Ramsey Number of $K_{3,3}$, in *Combinatorics, Graph Theory, and Applications*, Vol. **2** (Y. Alavi, G. Chartrand, O.R. Oellermann and J. Schwenk eds.), John Wiley & Sons, (1991) 639-644.

[-] H. Harborth, see also [BoH, CEHMS, EHM1, EHM2, GH].

[HaMe4] M. Harborth and I. Mengersen, Some Ramsey Numbers for Complete Bipartite Graphs, *Australasian Journal of Combinatorics*, **13** (1996) 119-128.

[-] T. Harmuth, see [BBH1, BBH2].

[Has] Hasmawati, The Ramsey Numbers for Disjoint Union of Stars, *Journal of the Indonesian Mathematical Society*, **16** (2010) 133-138.

[HaABS] Hasmawati, H. Assiyatun, E.T. Baskoro and A.N.M. Salman, Ramsey Numbers on a Union of Identical Stars versus a Small Cycle, in Computational Geometry and Graph Theory, Kyoto CGGT 2007, *LNCS* 4535, Springer, Berlin (2008) 85-89.

[HaBA1] Hasmawati, E.T. Baskoro and H. Assiyatun, Star-Wheel Ramsey Numbers, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **55** (2005) 123-128.

[HaBA2] Hasmawati, E.T. Baskoro and H. Assiyatun, The Ramsey Numbers for Disjoint Unions of Graphs, *Discrete Mathematics*, **308** (2008) 2046-2049.

[-] Hasmawati, see also [BaHA].

[HaŁP1+] P.E. Haxell, T. Łuczak, Y. Peng, V. Rödl, A. Ruciński, M. Simonovits and J. Skokan, The Ramsey Number for Hypergraph Cycles I, *Journal of Combinatorial Theory*, Series A, **113** (2006) 67-83.

[HaŁP2+] P.E. Haxell, T. Łuczak, Y. Peng, V. Rödl, A. Ruciński and J. Skokan, The Ramsey Number for 3-Uniform Tight Hypergraph Cycles, *Combinatorics, Probability and Computing*, **18** (2009) 165-203.

[HaŁT] P.E. Haxell, T. Łuczak and P.W. Tingley, Ramsey Numbers for Trees of Small Maximum Degree, *Combinatorica*, **22** (2002) 287-320.

[HeLD]* Changxiang He, Yusheng Li and Lin Dong, Three-Color Ramsey Numbers of K_n Dropping an Edge, *Graphs and Combinatorics*, **28** (2012) 663-669.

[Hein] K. Heinrich, Proper Colourings of K_{15}, *Journal of the Australian Mathematical Society*, Series A, **24** (1977) 465-495.

[He1] G.R.T. Hendry, Diagonal Ramsey Numbers for Graphs with Seven Edges, *Utilitas Mathematica*, **32** (1987) 11-34.

[He2] G.R.T. Hendry, Ramsey Numbers for Graphs with Five Vertices, *Journal of Graph Theory*, **13** (1989) 245-248.

[He3] G.R.T. Hendry, The Ramsey Numbers $r(K_2+\overline{K}_3, K_4)$ and $r(K_1+C_4, K_4)$, *Utilitas Mathematica*, **35** (1989) 40-54, addendum in **36** (1989) 25-32.

[He4] G.R.T. Hendry, Critical Colorings for Clancy's Ramsey Numbers, *Utilitas Mathematica*, **41** (1992) 181-203.

[He5] G.R.T. Hendry, Small Ramsey Numbers II. Critical Colorings for $r(C_5+e, K_5)$, *Quaestiones Mathematica*, **17** (1994) 249-258.

[-] G.R.T. Hendry, see also [YH].

[HiIr]* R. Hill and R.W. Irving, On Group Partitions Associated with Lower Bounds for Symmetric Ramsey Numbers, *European Journal of Combinatorics*, **3** (1982) 35-50.

[Hir] J. Hirschfeld, A Lower Bound for Ramsey's Theorem, *Discrete Mathematics*, **32** (1980) 89-91.

[Ho] Pak Tung Ho, On Ramsey Unsaturated and Saturated Graphs, *Australasian Journal of Combinatorics*, **46** (2010) 13-18.

[HoMe] M. Hoeth and I. Mengersen, Ramsey Numbers for Graphs of Order Four versus Connected Graphs of Order Six, *Utilitas Mathematica*, **57** (2000) 3-19.

[HoIs] J. Hook and G. Isaak, Star-Critical Ramsey Numbers, *Discrete Applied Mathematics*, **159** (2011) 328-334.

[HuSo] Huang Da Ming and Song En Min, Properties and Lower Bounds of the Third Order Ramsey Numbers (in Chinese), *Mathematica Applicata*, **9** (1996) 105-107.

[Hua1] Huang Guotai, Some Generalized Ramsey Numbers (in Chinese), *Mathematica Applicata*, **1** (1988) 97-101.

[Hua2] Huang Guotai, An Unsolved Problem of Gould and Jacobson (in Chinese), *Mathematica Applicata*, **9** (1996) 234-236.

[-] Huang Jian, see [HWSYZH].

[-] Huang Wenke, see [DuHu].

[HWSYZH] (also abbreviated by HW+) Huang Yi Ru, Wang Yuandi, Sheng Wancheng, Yang Jiansheng, Zhang Ke Min and Huang Jian, New Upper Bound Formulas with Parameters for Ramsey Numbers, *Discrete Mathematics*, **307** (2007) 760-763.

[HZ1] Huang Yi Ru and Zhang Ke Min, An New Upper Bound Formula for Two Color Classical Ramsey Numbers, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **28** (1998) 347-350.

[HZ2] Huang Yi Ru and Zhang Ke Min, New Upper Bounds for Ramsey Numbers, *European Journal of Combinatorics*, **19** (1998) 391-394.

[-] Huang Yi Ru, see also [BolJY+, YHZ1, YHZ2].

I

[Ir] R.W. Irving, Generalised Ramsey Numbers for Small Graphs, *Discrete Mathematics*, **9** (1974) 251-264.

[-] R.W. Irving, see also [HiIr].

[-] G. Isaak, see [HoIs].

[Isb1] J.R. Isbell, $N(4,4;3) \geq 13$, *Journal of Combinatorial Theory*, **6** (1969) 210.

[Isb2] J.R. Isbell, $N(5,4;3) \geq 24$, *Journal of Combinatorial Theory*, Series A, **34** (1983) 379-380.

[Ishi] Y. Ishigami, Linear Ramsey Numbers for Bounded-Degree Hypergraphs, *Electronic Notes in Discrete Mathematics*, **29** (2007) 47-51.

J

[Jac] M.S. Jacobson, On the Ramsey Number for Stars and a Complete Graph, *Ars Combinatoria*, **17** (1984) 167-172.

[-] M.S. Jacobson, see also [BEFRSGJ, GoJa1, GoJa2].

[JaAl1] M.M.M. Jaradat and B.M.N. Alzaleq, The Cycle-Complete Graph Ramsey Number $r(C_8, K_8)$, *SUT Journal of Mathematics*, **43** (2007) 85-98.

[JaAl2] M.M.M. Jaradat and B.M.N. Alzaleq, Cycle-Complete Graph Ramsey Numbers $r(C_4, K_9)$, $r(C_5, K_8) \leq 33$, *International Journal of Mathematical Combinatorics*, **1** (2009) 42-45.

[JaBa] M.M.M. Jaradat and A.M.M. Baniabedalruhman, The Cycle-Complete Graph Ramsey Number $r(C_8, K_7)$, *International Journal of Pure and Applied Mathematics*, **41** (2007) 667-677.

[-] M.M.M. Jaradat, see also [BatJA].

[JR1] C.J. Jayawardene and C.C. Rousseau, An Upper Bound for the Ramsey Number of a Quadrilateral versus a Complete Graph on Seven Vertices, *Congressus Numerantium*, **130** (1998) 175-188.

[JR2] C.J. Jayawardene and C.C. Rousseau, Ramsey Numbers $r(C_6, G)$ for All Graphs G of Order Less than Six, *Congressus Numerantium*, **136** (1999) 147-159.

[JR3] C.J. Jayawardene and C.C. Rousseau, The Ramsey Numbers for a Quadrilateral vs. All Graphs on Six Vertices, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **35** (2000) 71-87. Erratum in **51** (2004) 221.

[JR4] C.J. Jayawardene and C.C. Rousseau, Ramsey Numbers $r(C_5, G)$ for All Graphs G of Order Six, *Ars Combinatoria,* **57** (2000) 163-173.

[JR5] C.J. Jayawardene and C.C. Rousseau, The Ramsey Number for a Cycle of Length Five vs. a Complete Graph of Order Six, *Journal of Graph Theory,* **35** (2000) 99-108.

[-] C.J. Jayawardene, see also [BolJY+, RoJa1, RoJa2].

[-] Jiang Baoqi, see [SunYJLS].

[JiSa] Tao Jiang and M. Salerno, Ramsey Numbers of Some Bipartite Graphs versus Complete Graphs, *Graphs and Combinatorics,* **27** (2011) 121-128.

[Jin]** Jin Xia, Ramsey Numbers Involving a Triangle: Theory & Applications, *Technical Report RIT-TR-93-019,* MS thesis, Department of Computer Science, Rochester Institute of Technology, 1993.

[-] Jin Xia, see also [RaJi].

[JoPe] K. Johst and Y. Person, On the Multicolor Ramsey Number of a Graph with m Edges, *preprint,* arXiv, http://arxiv.org/abs/1311.5471 (2013).

[JGT] *Journal of Graph Theory,* special volume on Ramsey theory, **7**, Number 1, (1983).

K

[Ka1] J.G. Kalbfleisch, Construction of Special Edge-Chromatic Graphs, *Canadian Mathematical Bulletin,* **8** (1965) 575-584.

[Ka2]* J.G. Kalbfleisch, Chromatic Graphs and Ramsey's Theorem, *Ph.D. thesis,* University of Waterloo, January 1966.

[Ka3] J.G. Kalbfleisch, On Robillard's Bounds for Ramsey Numbers, *Canadian Mathematical Bulletin,* **14** (1971) 437-440.

[KaSt] J.G. Kalbfleisch and R.G. Stanton, On the Maximal Triangle-Free Edge-Chromatic Graphs in Three Colors, *Journal of Combinatorial Theory,* **5** (1968) 9-20.

[KáRos] G. Károlyi and V. Rosta, Generalized and Geometric Ramsey Numbers for Cycles, *Theoretical Computer Science,* **263** (2001) 87-98.

[-] P. Keevash, see [BohK1, BohK2].

[KerRo] M. Kerber and C. Rowan, CommonLisp program for computing upper bounds on classical Ramsey numbers, http://www.cs.bham.ac.uk/~mmk/demos/ramsey-upper-limit.lisp (2009).

[Kéry] G. Kéry, On a Theorem of Ramsey (in Hungarian), *Matematikai Lapok,* **15** (1964) 204-224.

[Kim] J.H. Kim, The Ramsey Number $R(3, t)$ Has Order of Magnitude $t^2/\log t$, *Random Structures and Algorithms,* **7** (1995) 173-207.

[KlaM1] K. Klamroth and I. Mengersen, Ramsey Numbers of K_3 versus (p, q)-Graphs, *Ars Combinatoria,* **43** (1996) 107-120.

[KlaM2] K. Klamroth and I. Mengersen, The Ramsey Number of $r(K_{1,3}, C_4, K_4)$, *Utilitas Mathematica,* **52** (1997) 65-81.

[-] K. Klamroth, see also [ArKM].

[-] M. Klawe, see [GHK].

[-] D.J. Kleitman, see [GoK].

[KoSS1] Y. Kohayakawa, M. Simonovits and J. Skokan, The 3-colored Ramsey Number of Odd Cycles, *Electronic Notes in Discrete Mathematics,* **19** (2005) 397-402.

[KoSS2] Y. Kohayakawa, M. Simonovits and J. Skokan, The 3-colored Ramsey Number of Odd Cycles, to appear in *Journal of Combinatorial Theory,* Series B (2013).

[Köh] W. Köhler, On a Conjecture by Grossman, *Ars Combinatoria,* **23** (1987) 103-106.

[-] J. Komlós, see [CsKo, AjKS, AjKSS].

[Kor] A. Korolova, Ramsey Numbers of Stars versus Wheels of Similar Sizes, *Discrete Mathematics,* **292** (2005) 107-117.

[KosMV1] A. Kostochka, D. Mubayi and J. Verstraëte, On Independent Sets in Hypergraphs, *preprint,* arXiv, http://arxiv.org/abs/1106.3098 (2011).

[KosMV2] A. Kostochka, D. Mubayi and J. Verstraëte, Hypergraph Ramsey Numbers: Triangles versus Cliques, to appear in *Journal of Combinatorial Theory,* Series B (2013).

[KosPR] A. Kostochka, P. Pudlák and V. Rödl, Some Constructive Bounds on Ramsey Numbers, *Journal of Combinatorial Theory,* Series B, **100** (2010) 439-445.

[KoRö1] A.V. Kostochka and V. Rödl, On Graphs with Small Ramsey Numbers, *Journal of Graph Theory,* **37** (2001) 198-204.

[KoRö2] A.V. Kostochka and V. Rödl, On Graphs with Small Ramsey Numbers, II, *Combinatorica,* **24** (2004) 389-401.

[KoRö3] A.V. Kostochka and V. Rödl, On Ramsey Numbers of Uniform Hypergraphs with Given Maximum Degree, *Journal of Combinatorial Theory,* Series A, **113** (2006) 1555-1564.

[KoSu] A.V. Kostochka and B. Sudakov, On Ramsey Numbers of Sparse Graphs, *Combinatorics, Probability and Computing,* **12** (2003) 627-641.

[-] R.L. Kramer, see [FKR].

[KrRod] I. Krasikov and Y. Roditty, On Some Ramsey Numbers of Unicyclic Graphs, *Bulletin of the Institute of Combinatorics and its Applications,* **33** (2001) 29-34.

[-] S. Krause, see [HaKr1, HaKr2].

[KLR]* D.L. Kreher, Li Wei and S.P. Radziszowski, Lower Bounds for Multi-Colored Ramsey Numbers From Group Orbits, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **4** (1988) 87-95.

[-] D.L. Kreher, see also [RK1, RK2, RK3, RK4].

[Kriv] M. Krivelevich, Bounding Ramsey Numbers through Large Deviation Inequalities, *Random Structures and Algorithms,* **7** (1995) 145-155.

[-] M. Krivelevich, see also [AlBK, AlKS].

[KroMe] M. Krone and I. Mengersen, The Ramsey Numbers $r(K_5-2K_2, 2K_3)$, $r(K_5-e, 2K_3)$ and $r(K_5, 2K_3)$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **81** (2012) 257-260.

[-] M. Kubale, see [DzKP].

[KüCFO] D. Kühn, O. Cooley, N. Fountoulakis and D. Osthus, Ramsey Numbers of Sparse Hypergraphs, *Electronic Notes in Discrete Mathematics,* **29** (2007) 29-33.

[-] D. Kühn, see also [CooFKO1, CooFKO2].

La - Li

[-] A. Lange, see [LivLR].

[-] P.C.B. Lam, see [ShiuLL].

[La1] S.L. Lawrence, Cycle-Star Ramsey Numbers, *Notices of the American Mathematical Society,* **20** (1973) Abstract A-420.

[La2] S.L. Lawrence, Bipartite Ramsey Theory, *Notices of the American Mathematical Society,* **20** (1973) Abstract A-562.

[-] S.L. Lawrence, see also [FLPS].

[LayMa] C. Laywine and J.P. Mayberry, A Simple Construction Giving the Two Non-isomorphic Triangle-Free 3-Colored K_{16}'s, *Journal of Combinatorial Theory,* Series B, **45** (1988) 120-124.

[LaMu] F. Lazebnik and D. Mubayi, New Lower Bounds for Ramsey Numbers of Graphs and Hypergraphs, *Advances in Applied Mathematics*, **28** (2002) 544-559.

[LaWo1] F. Lazebnik and A. Woldar, New Lower Bounds on the Multicolor Ramsey Numbers $r_k(C_4)$, *Journal of Combinatorial Theory*, Series B, **79** (2000) 172-176.

[LaWo2] F. Lazebnik and A. Woldar, General Properties of Some Families of Graphs Defined by Systems of Equations, *Journal of Graph Theory*, **38** (2001) 65-86.

[Lef] H. Lefmann, Ramsey Numbers for Monotone Paths and Cycles, *Ars Combinatoria*, **35** (1993) 271-279.

[-] H. Lefmann, see also [DLR].

[-] J. Lehel, see [BaLS, GyLSS].

[Les]* A. Lesser, Theoretical and Computational Aspects of Ramsey Theory, *Examensarbeten i Matematik*, Matematiska Institutionen, Stockholms Universitet, **3** (2001).

[-] D. Leven, see [BlLR].

[-] Li Bingxi, see [SunYWLX, SunYXL].

[LiWa1] Li Da Yong and Wang Zhi Jian, The Ramsey Number $r(mC_4, nC_4)$ (in Chinese), *Journal of Shanghai Tiedao University*, **20** (1999) 66-70, 83.

[LiWa2] Li Da Yong and Wang Zhi Jian, The Ramsey Numbers $r(mC_4, nC_5)$, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **45** (2003) 245-252.

[-] Li Guiqing, see [SLLL, SLZL].

[-] Li Jinwen, see [ZLLS].

[LSLW]* Li Qiao, Su Wenlong, Luo Haipeng and Wu Kang, Lower Bounds for Some Two-Color Ramsey Numbers, *manuscript* (2011).

[-] Li Qiao, see also [SLL, SLLL].

[-] Li Wei, see [KLR].

[Li1] Li Yusheng, Some Ramsey Numbers of Graphs with Bridge, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **25** (1997) 225-229.

[Li2] Li Yusheng, The Shannon Capacity of a Communication Channel, Graph Ramsey Number and a Conjecture of Erdős, *Chinese Science Bulletin*, **46** (2001) 2025-2028.

[Li3] Yusheng Li, Ramsey Numbers of a Cycle, *Taiwanese Journal of Mathematics*, **12** (2008) 1007-1013.

[Li4] Yusheng Li, The Multi-Color Ramsey Number of an Odd Cycle, *Journal of Graph Theory*, **62** (2009) 324-328.

[LiLih] Yusheng Li and Ko-Wei Lih, Multi-Color Ramsey Numbers of Even Cycles, *European Journal of Combinatorics*, **30** (2009) 114-118.

[LiR1] Li Yusheng and C.C. Rousseau, On Book-Complete Graph Ramsey Numbers, *Journal of Combinatorial Theory*, Series B, **68** (1996) 36-44.

[LiR2] Li Yusheng and C.C. Rousseau, Fan-Complete Graph Ramsey Numbers, *Journal of Graph Theory*, **23** (1996) 413-420.

[LiR3] Li Yusheng and C.C. Rousseau, On the Ramsey Number $r(H + \bar{K}_n, K_n)$, *Discrete Mathematics*, **170** (1997) 265-267.

[LiR4] Li Yusheng and C.C. Rousseau, A Ramsey Goodness Result for Graphs with Many Pendant Edges, *Ars Combinatoria*, **49** (1998) 315-318.

[LiRS] Li Yusheng, C.C. Rousseau and L. Soltés, Ramsey Linear Families and Generalized Subdivided Graphs, *Discrete Mathematics*, **170** (1997) 269-275.

[LiRZ1] Li Yusheng, C.C. Rousseau and Zang Wenan, Asymptotic Upper Bounds for Ramsey Functions, *Graphs and Combinatorics*, **17** (2001) 123-128.

[LiRZ2] Li Yusheng, C.C. Rousseau and Zang Wenan, An Upper Bound on Ramsey Numbers, *Applied Mathematics Letters*, **17** (2004) 663-665.

[LiShen] Yusheng Li and Jian Shen, Bounds for Ramsey Numbers of Complete Graphs Dropping an Edge, *European Journal of Combinatorics,* **29** (2008) 88-94.

[LiTZ] Li Yusheng, Tang Xueqing and Zang Wenan, Ramsey Functions Involving $K_{m,n}$ with n Large, *Discrete Mathematics,* **300** (2005) 120-128.

[LiZa1] Li Yusheng and Zang Wenan, Ramsey Numbers Involving Large Dense Graphs and Bipartite Turán Numbers, *Journal of Combinatorial Theory,* Series B, **87** (2003) 280-288.

[LiZa2] Li Yusheng and Zang Wenan, The Independence Number of Graphs with a Forbidden Cycle and Ramsey Numbers, *Journal of Combinatorial Optimization,* **7** (2003) 353-359.

[-] Li Yusheng, see also [BaiLi, BaLX, CaLRZ, Doli, DoLL1, DoLL2, GuLi, HeLD, LinLi1, LinLi2, LinLD, Shiu LL, SonLi, SunLi, WaLi].

[-] Li Zhenchong, see [LSL, LuLL].

[-] Meilian Liang, see [LuLL].

[LiaWXS]* Wenzhong Liang, Kang Wu, Xiaodong Xu and Wenlong Su, New Lower Bounds for Seven Classical Ramsey Numbers, *in preparation,* (2011).

[LiaWXCS]* Liang Wenzhong, Wu Kang, Xu Chengzhang, Chen Hong and Su Wenlong, Using the Two Stage Automorphism of Paley to Calculate the Lower Bound of Ramsey, *Journal of Inner Mongolia Normal University,* **41** (2012) 591-596.

[-] Liang Wenzhong, see also [ChWXSL].

[-] Ko-Wei Lih, see [LiLih].

[LinLi1] Qizhong Lin and Yusheng Li, On Ramsey Numbers of Fans, *Discrete Applied Mathematics,* **157** (2009) 191-194.

[LinLi2] Qizhong Lin and Yusheng Li, Ramsey Numbers of K_3 and $K_{n,n}$, *Applied Mathematics Letters,* **25** (2012) 380-384.

[LinLD] Qizhong Lin, Yusheng Li and Lin Dong, Ramsey Goodness and Generalized Stars, *European Journal of Combinatorics,* **31** (2010) 1228-1234.

[-] Qizhong Lin, see also [DoLL1, DoLL2].

[-] Lin Xiaohui, see [SunYJLS, SunYLZ1, SunYLZ2].

[Lind] B. Lindström, Undecided Ramsey-Numbers for Paths, *Discrete Mathematics,* **43** (1983) 111-112.

[Ling] A.C.H. Ling, Some Applications of Combinatorial Designs to Extremal Graph Theory, *Ars Combinatoria,* **67** (2003) 221-229.

[-] Andy Liu, see [AbbL].

[-] Hong Liu, see [AxGLM].

[-] Liu Linzhong, see [ZLLS].

[-] Liu Shu Yan, see [SonBL].

[-] Liu Xiangyang, see [GuSL].

[-] Liu Yanwu, see [SonYL].

[LivLR]** I. Livinsky, A. Lange and S.P. Radziszowski, Computation of the Ramsey Numbers $R(C_4, K_9)$ and $R(C_4, K_{10})$, *manuscript* (2013).

Lo - Lu

[Loc] S.C. Locke, Bipartite Density and the Independence Ratio, *Journal of Graph Theory,* **10** (1986) 47-53.

[-] S.C. Locke, see also [FrLo].

[Lor] P.J. Lorimer, The Ramsey Numbers for Stripes and One Complete Graph, *Journal of Graph Theory,* **8** (1984) 177-184.

[LorMu] P.J. Lorimer and P.R. Mullins, Ramsey Numbers for Quadrangles and Triangles, *Journal of Combinatorial Theory,* Series B, **23** (1977) 262-265.

[LorSe] P.J. Lorimer and R.J. Segedin, Ramsey Numbers for Multiple Copies of Complete Graphs, *Journal of Graph Theory,* **2** (1978) 89-91.

[LorSo] P.J. Lorimer and W. Solomon, The Ramsey Numbers for Stripes and Complete Graphs 1, *Discrete Mathematics,* **104** (1992) 91-97. Corrigendum in *Discrete Mathematics,* **131** (1994) 395.

[-] P.J. Lorimer, see also [CocL1, CocL2].

[Lortz] R. Lortz, A Note on the Ramsey Number of $K_{2,2}$ versus $K_{3,n}$, *Discrete Mathematics,* **306** (2006) 2976-2982.

[LoM1] R. Lortz and I. Mengersen, On the Ramsey Numbers $r(K_{2,n-1}, K_{2,n})$ and $r(K_{2,n}, K_{2,n})$, *Utilitas Mathematica,* **61** (2002) 87-95.

[LoM2] R. Lortz and I. Mengersen, Bounds on Ramsey Numbers of Certain Complete Bipartite Graphs, *Results in Mathematics,* **41** (2002) 140-149.

[LoM3]* R. Lortz and I. Mengersen, Off-Diagonal and Asymptotic Results on the Ramsey Number $r(K_{2,m}, K_{2,n})$, *Journal of Graph Theory,* **43** (2003) 252-268.

[LoM4]* R. Lortz and I. Mengersen, Further Ramsey Numbers for Small Complete Bipartite Graphs, *Ars Combinatoria,* **79** (2006) 195-203.

[LoM5] R. Lortz and I. Mengersen, Ramsey Numbers for Small Graphs versus Small Disconnected Graphs, *Australasian Journal of Combinatorics,* **51** (2011) 89-108.

[Łuc] T. Łuczak, $R(C_n, C_n, C_n) \le (4+o(1))n$, *Journal of Combinatorial Theory,* Series B, **75** (1999) 174-187.

[ŁucSS] T. Łuczak, M. Simonovits and J. Skokan, On the Multi-Colored Ramsey Numbers of Cycles, *Journal of Graph Theory,* **69** (2012) 169-175.

[-] T. Łuczak, see also [FiŁu1, FiŁu2, HaŁP1+, HaŁP2+, HaŁT].

[LSL]* Luo Haipeng, Su Wenlong and Li Zhenchong, The Properties of Self-Complementary Graphs and New Lower Bounds for Diagonal Ramsey Numbers, *Australasian Journal of Combinatorics,* **25** (2002) 103-116.

[LSS1]* Luo Haipeng, Su Wenlong and Shen Yun-Qiu, New Lower Bounds of Ten Classical Ramsey Numbers, *Australasian Journal of Combinatorics,* **24** (2001) 81-90.

[LSS2]* Luo Haipeng, Su Wenlong and Shen Yun-Qiu, New Lower Bounds for Two Multicolor Classical Ramsey Numbers, *Radovi Matematički,* **13** (2004) 15-21.

[-] Luo Haipeng, see also [LSLW, SL, SLL, SLLL, SLZL, WSLX1, WSLX2].

[LuLL]* Liang Luo, Meilian Liang and Zhenchong Li, Computation of Ramsey Numbers $R(C_m, W_n)$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **81** (2012) 145-149.

M

[Mac]* J. Mackey, Combinatorial Remedies, *Ph.D. thesis,* Department of Mathematics, University of Hawaii, 1994.

[MaOm] L. Maherani and G.R. Omidi, Around a Conjecture of Erdős on Graph Ramsey Numbers, *preprint,* arXiv, http://arxiv.org/abs/1211.6287 (2012).

[MaORS1] L. Maherani, G.R. Omidi, G. Raeisi and M. Shahsiah, The Ramsey Number of Loose Paths in 3-Uniform Hypergraphs, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P12, **20**(1) (2013), 8 pages.

[MaORS2] L. Maherani, G.R. Omidi, G. Raeisi and M. Shahsiah, On Three-Color Ramsey Number of Paths, *preprint,* arXiv, http://arxiv.org/abs/1207.3771 (2012).

[Mat]* R. Mathon, Lower Bounds for Ramsey Numbers and Association Schemes, *Journal of Combinatorial Theory,* Series B, **42** (1987) 122-127.

[-] J.P. Mayberry, see [LayMa].

[McS] C. McDiarmid and A. Steger, Tidier Examples for Lower Bounds on Diagonal Ramsey Numbers, *Journal of Combinatorial Theory*, Series A, **74** (1996) 147-152.

[McK]** B.D. McKay, Australian National University, *personal communication* (2003+). Graphs available at http://cs.anu.edu.au/people/bdm/data/ramsey.html.

[MPR]** B.D. McKay, K. Piwakowski and S.P. Radziszowski, Ramsey Numbers for Triangles versus Almost-Complete Graphs, *Ars Combinatoria*, **73** (2004) 205-214.

[MR1]** B.D. McKay and S.P. Radziszowski, The First Classical Ramsey Number for Hypergraphs is Computed, *Proceedings of the Second Annual ACM-SIAM Symposium on Discrete Algorithms*, SODA'91, San Francisco, (1991) 304-308.

[MR2]* B.D. McKay and S.P. Radziszowski, A New Upper Bound for the Ramsey Number $R(5,5)$, *Australasian Journal of Combinatorics*, **5** (1992) 13-20.

[MR3]** B.D. McKay and S.P. Radziszowski, Linear Programming in Some Ramsey Problems, *Journal of Combinatorial Theory*, Series B, **61** (1994) 125-132.

[MR4]** B.D. McKay and S.P. Radziszowski, $R(4,5) = 25$, *Journal of Graph Theory*, **19** (1995) 309-322.

[MR5]** B.D. McKay and S.P. Radziszowski, Subgraph Counting Identities and Ramsey Numbers, *Journal of Combinatorial Theory*, Series B, **69** (1997) 193-209.

[MZ]** B.D. McKay and Zhang Ke Min, The Value of the Ramsey Number $R(3,8)$, *Journal of Graph Theory*, **16** (1992) 99-105.

[-] B.D. McKay, see also [FM].

[McN]** J. McNamara, SUNY Brockport, *personal communication* (1995).

[McR]** J. McNamara and S.P. Radziszowski, The Ramsey Numbers $R(K_4-e, K_6-e)$ and $R(K_4-e, K_7-e)$, *Congressus Numerantium*, **81** (1991) 89-96.

[-] H. Mélot, see [BrCGM].

[MeO] I. Mengersen and J. Oeckermann, Matching-Star Ramsey Sets, *Discrete Applied Mathematics*, **95** (1999) 417-424.

[-] I. Mengersen, see also [ArKM, CEHMS, EHM1, EHM2, HoMe, HaMe1, HaMe2, HaMe3, HaMe4, KlaM1, KlaM2, KroMe, LoM1, LoM2, LoM3, LoM4, LoM5].

[-] Zhengke Miao, see [ChenCMN].

[-] M. Miller, see [BaSNM].

[MiPal] T.K. Mishra and S.P. Pal, Lower Bounds for Ramsey Numbers for Complete Bipartite and 3-Uniform Tripartite Subgraphs, WALCOM 2013, *LNCS* 7748, Springer, Berlin (2013) 257-264.

[MiSa] H. Mizuno and I. Sato, Ramsey Numbers for Unions of Some Cycles, *Discrete Mathematics*, **69** (1988) 283-294.

[MoCa] E.L. Monte Carmelo, Configurations in Projective Planes and Quadrilateral-Star Ramsey Numbers, *Discrete Mathematics*, **308** (2008) 3986-3991.

[-] E.L. Monte Carmelo, see also [GoMC].

[-] L.P. Montejano, see [ChaMR].

[-] R. Morris, see [FizGM, FizGMSS].

[MoSST] G.O. Mota, G.N. Sárközy, M. Schacht and A. Taraz, Ramsey Numbers for Bipartite Graphs with Small Bandwidth, *manuscript* (2013).

[-] D. Mubayi, see [AxFM, AxGLM, KosMV1, KosMV2, LaMu].

[-] P.R. Mullins, see [LorMu].

[-] S. Musdalifah, see [SuAM, SuAAM].

N

[-] S.M. Nababan, see [BaSNM].

[NaORS] B. Nagle, S. Olsen, V. Rödl and M. Schacht, On the Ramsey Number of Sparse 3-Graphs, *Graphs and Combinatorics*, **24** (2008) 205-228.

[Neš] J. Nešetřil, Ramsey Theory, chapter 25 in *Handbook of Combinatorics*, ed. R.L. Graham, M. Grötschel and L. Lovász, The MIT-Press, Vol. II, 1996, 1331-1403.

[NeOs] J. Nešetřil and P. Ossona de Mendez, Fraternal Augmentations, Arrangeability and Linear Ramsey Numbers, *European Journal of Combinatorics*, **30** (2009) 1696-1703.

[-] J. Nešetřil, see also [GrNe].

[-] C.T. Ng, see [ChenCMN, ChenCNZ, CheCZN].

[Nik] V. Nikiforov, The Cycle-Complete Graph Ramsey Numbers, *Combinatorics, Probability and Computing*, **14** (2005) 349-370.

[NiRo1] V. Nikiforov and C.C. Rousseau, Large Generalized Books Are p-Good, *Journal of Combinatorial Theory*, Series B, **92** (2004) 85-97.

[NiRo2] V. Nikiforov and C.C. Rousseau, Book Ramsey Numbers I, *Random Structures and Algorithms*, **27** (2005) 379-400.

[NiRo3] V. Nikiforov and C.C. Rousseau, A Note on Ramsey Numbers for Books, *Journal of Graph Theory*, **49** (2005) 168-176.

[NiRo4] V. Nikiforov and C.C. Rousseau, Ramsey Goodness and Beyond, *Combinatorica*, **29** (2009) 227-262.

[NiRS] V. Nikiforov, C.C. Rousseau and R.H. Schelp, Book Ramsey Numbers and Quasi-Randomness, *Combinatorics, Probability and Computing*, **14** (2005) 851-860.

[-] A. Nowik, see [DzNS].

O

[-] J. Oeckermann, see [MeO].

[-] S. Olsen, see [NaORS].

[OmRa1] G.R. Omidi and G. Raeisi, On Multicolor Ramsey Number of Paths versus Cycles, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #P24, **18**(1) (2011), 16 pages.

[OmRa2] G.R. Omidi and G. Raeisi, A Note on the Ramsey Number of Stars - Complete Graphs, European Journal of Combinatorics, **32** (2011) 598-599.

[OmRa3] G.R. Omidi and G. Raeisi, Ramsey Numbers for Multiple Copies of Hypergraphs, *preprint*, arXiv, http://arxiv.org/abs/1303.0474 (2013).

[OmSh] G.R. Omidi and M. Shahsiah, Ramsey Numbers of 3-Uniform Loose Paths and Loose Cycles, *preprint*, arXiv, http://arxiv.org/abs/1211.5800 (2012).

[-] G.R. Omidi, see also [MaORS1, MaORS2].

[-] P. Ossona de Mendez, see [NeOs].

[-] D. Osthus, see [CooFKO1, CooFKO2, KüCFO].

P

[-] S.P. Pal, see [MiPal].

[-] Linqiang Pan, see [ShaXBP, ShaXSP].

[Par1] T.D. Parsons, The Ramsey Numbers $r(P_m, K_n)$, *Discrete Mathematics*, **6** (1973) 159-162.

[Par2] T.D. Parsons, Path-Star Ramsey Numbers, *Journal of Combinatorial Theory*, Series B, **17** (1974) 51-58.

[Par3] T.D. Parsons, Ramsey Graphs and Block Designs, I, *Transactions of the American Mathematical Society*, **209** (1975) 33-44.

[Par4] T.D. Parsons, Ramsey Graphs and Block Designs, *Journal of Combinatorial Theory*, Series A, **20** (1976) 12-19.

[Par5] T.D. Parsons, Graphs from Projective Planes, *Aequationes Mathematica*, **14** (1976) 167-189.

[Par6] T.D. Parsons, Ramsey Graph Theory, in *Selected Topics in Graph Theory*, (L.W. Beineke and R.J. Wilson eds.), Academic Press, (1978) 361-384.

[-] T.D. Parsons, see also [FLPS].

[Peng] Xing Peng, The Ramsey Number of Generalized Loose Paths in Uniform Hypergraphs, *preprint*, arXiv, http://arxiv.org/abs/1305.0294 (2013).

[-] Yuejian Peng, see [HaŁP1+, HaŁP2+].

[-] Y. Person, see [JoPe].

[-] O. Pikhurko, see [BePi].

[Piw1]* K. Piwakowski, Applying Tabu Search to Determine New Ramsey Graphs, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #R6, **3**(1) (1996), 4 pages.

[Piw2]** K. Piwakowski, A New Upper Bound for $R_3(K_4-e)$, *Congressus Numerantium*, **128** (1997) 135-141.

[PR1]** K. Piwakowski and S.P. Radziszowski, $30 \le R(3,3,4) \le 31$, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **27** (1998) 135-141.

[PR2]** K. Piwakowski and S.P. Radziszowski, Towards the Exact Value of the Ramsey Number $R(3,3,4)$, *Congressus Numerantium*, **148** (2001) 161-167.

[-] K. Piwakowski, see also [MPR, DzKP].

[Pokr] A. Pokrovskiy, Calculating Ramsey Numbers by Partitioning Coloured Graphs, *preprint*, arXiv, http://arxiv.org/abs/1309.3952 (2013).

[PoRRS] J. Polcyn, V. Rödl, A. Ruciński and E. Szemerédi, Short Paths in Quasi-Random Triple Systems with Sparse Underlying Graphs, *Journal of Combinatorial Theory*, Series B, **96** (2006) 584-607.

[-] A.D. Polimeni, see [ChGP, CRSPS].

[-] J.R. Portillo, see [BoPo].

[-] L.M. Pretorius, see [SwPr].

[-] P. Pudlák, see [AlPu, CPR, KosPR].

Q

[-] Qian Xinjin, see [SonGQ].

R

[Ra1]** S.P. Radziszowski, The Ramsey Numbers $R(K_3, K_8-e)$ and $R(K_3, K_9-e)$, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **8** (1990) 137-145.

[Ra2] S.P. Radziszowski, Small Ramsey Numbers, *Technical Report RIT-TR-93-009*, Department of Computer Science, Rochester Institute of Technology (1993).

[Ra3]** S.P. Radziszowski, On the Ramsey Number $R(K_5-e, K_5-e)$, *Ars Combinatoria*, **36** (1993) 225-232.

[Ra4] S.P. Radziszowski, Ramsey Numbers Involving Cycles, in *Ramsey Theory: Yesterday, Today and Tomorrow* (ed. A. Soifer), Progress in Mathematics 285, Springer-Birkhauser 2011, 41-62.

[RaJi] S.P. Radziszowski and Jin Xia, Paths, Cycles and Wheels in Graphs without Antitriangles, *Australasian Journal of Combinatorics*, **9** (1994) 221-232.

[RK1]* S.P. Radziszowski and D.L. Kreher, Search Algorithm for Ramsey Graphs by Union of Group Orbits, *Journal of Graph Theory*, **12** (1988) 59-72.

[RK2]** S.P. Radziszowski and D.L. Kreher, On $R(3,k)$ Ramsey Graphs: Theoretical and Computational Results, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **4** (1988) 37-52.

[RK3]** S.P. Radziszowski and D.L. Kreher, Upper Bounds for Some Ramsey Numbers $R(3,k)$, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **4** (1988) 207-212.

[RK4] S.P. Radziszowski and D.L. Kreher, Minimum Triangle-Free Graphs, *Ars Combinatoria*, **31** (1991) 65-92.

[RT]* S.P. Radziszowski and Kung-Kuen Tse, A Computational Approach for the Ramsey Numbers $R(C_4,K_n)$, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **42** (2002) 195-207.

[RST]* S.P. Radziszowski, J. Stinehour and Kung-Kuen Tse, Computation of the Ramsey Number $R(W_5,K_5)$, *Bulletin of the Institute of Combinatorics and its Applications*, **47** (2006) 53-57.

[-] S.P. Radziszowski, see also [BaRT, BlLR, CalSR, DyDR, FKR, GoR1, GoR2, KLR, LivLR, MPR, MR1, MR2, MR3, MR4, MR5, McR, PR1, PR2, ShWR, WuSR, WuSZR, XuR1, XuR2, XuR3, XSR1, XSR2, XXER, XXR].

[-] G. Raeisi, see [GyRa, MaORS1, MaORS2, OmRa1, OmRa2, OmRa3].

[-] J.L. Ramirez Alfonsin, see [ChaMR].

[Ram] F.P. Ramsey, On a Problem of Formal Logic, *Proceedings of the London Mathematical Society*, **30** (1930) 264-286.

[RaHo]** Ramsey@Home, A distributed computing project searching for lower bounds for Ramsey numbers, http://www.ramseyathome.com/ramsey (2009).

[-] A. Rao, see [BarRSW].

[Rao]* S. Rao, Applying a Genetic Algorithm to Improve the Lower Bounds of Multi-Color Ramsey Numbers, *MS thesis*, Department of Computer Science, Rochester Institute of Technology, 1997.

[-] G. Resta, see [CPR].

[-] M.P. Revuelta, see [BoCGR].

[-] S.W. Reyner, see [BR].

[-] D.F. Reynolds, see [ExRe].

[Rob1] F.S. Roberts, *Applied Combinatorics*, Prentice-Hall, Englewood Cliffs, 1984.

[-] J.A. Roberts, see [BuRo1, BuRo2].

[-] S. Roberts, see [GR].

[Rob2]* A. Robertson, New Lower Bounds for Some Multicolored Ramsey Numbers, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #R12, **6** (1999), 6 pages.

[Rob3]* A. Robertson, Difference Ramsey Numbers and Issai Numbers, *Advances in Applied Mathematics*, **25** (2000) 153-162.

[Rob4] A. Robertson, New Lower Bounds Formulas for Multicolored Ramsey Numbers, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #R13, **9** (2002), 6 pages.

[-] Y. Roditty, see [KrRod].

[RöTh] V. Rödl and R. Thomas, Arrangeability and Clique Subdivisions, in *The Mathematics of Paul Erdős II*, 236-239, Algorithms and Combinatorics **14**, Springer, Berlin, 1997.

[-] V. Rödl, see also [AlRö, CRST, DLR, GrRö, GRR1, GRR2, HaŁP1+, HaŁP2+, KosPR, KoRö1, KoRö2, KoRö3, NaORS, PoRRS].

[-] L. Rónyai, see [AlRóS].

[Ros1] V. Rosta, On a Ramsey Type Problem of J.A. Bondy and P. Erdős, I & II, *Journal of Combinatorial Theory*, Series B, **15** (1973) 94-120.

[Ros2] V. Rosta, Ramsey Theory Applications, Dynamic Survey in *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #DS13, (2004), 43 pages.

[-] V. Rosta, see also [BuRo3, KáRos].

[-] B.L. Rothschild, see [GRS].

[Rou] C.C. Rousseau, *personal communication*, (2006).

[RoJa1] C.C. Rousseau and C.J. Jayawardene, The Ramsey Number for a Quadrilateral vs. a Complete Graph on Six Vertices, *Congressus Numerantium*, **123** (1997) 97-108.

[RoJa2] C.C. Rousseau and C.J. Jayawardene, Harary's Problem for $K_{2,k}$, *manuscript* (1999).

[RS1] C.C. Rousseau and J. Sheehan, On Ramsey Numbers for Books, *Journal of Graph Theory*, **2** (1978) 77-87.

[RS2] C.C. Rousseau and J. Sheehan, A Class of Ramsey Problems Involving Trees, *Journal of the London Mathematical Society (2)*, **18** (1978) 392-396.

[-] C.C. Rousseau, see also [BolJY+, BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRSG, BFRSJ, CaLRZ, CRSPS, EFRS1, EFRS2, EFRS3, EFRS4, EFRS5, EFRS6, EFRS7, EFRS8, EFRS9, FRS1, FRS2, FRS3, FRS4, FRS5, FRS6, FRS7, FRS8, FRS9, FSR, JR1, JR2, JR3, JR4, JR5, LiR1, LiR2, LiR3, LiR4, LiRS, LiRZ1, LiRZ2, NiRo1, NiRo2, NiRo3, NiRo4, NiRS].

[-] C. Rowan, see [KerRo].

[-] P. Rowlinson, see [YR1, YR2, YR3].

[-] A. Ruciński, see [GRR1, GRR2, HaŁP1+, HaŁP2+, PoRRS].

[-] M. Ruszinkó, see [GyRSS].

Sa - Sh

[-] M. Salerno, see [JiSa].

[SaBr1] A.N.M. Salman and H.J. Broersma, The Ramsey Numbers of Paths versus Kipases, *Electronic Notes in Discrete Mathematics*, **17** (2004) 251-255.

[SaBr2] A.N.M. Salman and H.J. Broersma, Paths-Fan Ramsey Numbers, *Discrete Applied Mathematics*, **154** (2006) 1429-1436.

[SaBr3] A.N.M. Salman and H.J. Broersma, The Ramsey Numbers for Paths versus Wheels, *Discrete Mathematics*, **307** (2007) 975-982.

[SaBr4] A.N.M. Salman and H.J. Broersma, Path-Kipas Ramsey Numbers, *Discrete Applied Mathematics*, **155** (2007) 1878-1884.

[-] A.N.M. Salman, see also [HaABS].

[San] A. Sánchez-Flores, An Improved Bound for Ramsey Number $N(3,3,3,3;2)$, *Discrete Mathematics*, **140** (1995) 281-286.

[Sár] G.N. Sárközy, Monochromatic Cycle Partitions of Edge-Colored Graphs, *Journal of Graph Theory*, **66** (2011) 57-64.

[-] G.N. Sárközy, see also [GyLSS, GyRSS, GySá1, GySá2, GySS1, GySS2, MoSST].

[-] I. Sato, see [MiSa].

[-] D. Saxton, see [FizGMSS].

[-] M. Schacht, see [MoSST, NaORS].

[Scha] M. Schaefer, Graph Ramsey Theory and the Polynomial Hierarchy, *Journal of Computer and System Sciences*, **62** (2001) 290-322.

[-] R.H. Schelp, see [BaLS, BaSS, BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRSGJ, BEFS, BFRS, ChenS, EFRS1, EFRS2, EFRS3, EFRS4, EFRS5, EFRS6, EFRS7, EFRS8, EFRS9, FLPS, FRS1, FRS2, FRS3, FRS4, FRS5, FRS6, FS1, FS2, FS3, FS4, FSR, FSS1, GyLSS, NiRS].

[-] J. Schönheim, see [BiaS].

[SchSch1]* A. Schelten and I. Schiermeyer, Ramsey Numbers $r(K_3, G)$ for Connected Graphs G of Order Seven, *Discrete Applied Mathematics,* **79** (1997) 189-200.

[SchSch2] A. Schelten and I. Schiermeyer, Ramsey Numbers $r(K_3, G)$ for $G \cong K_7 - 2P_2$ and $G \cong K_7 - 3P_2$, *Discrete Mathematics,* **191** (1998) 191-196.

[-] A. Schelten, see also [FSS2].

[Schi1] I. Schiermeyer, All Cycle-Complete Graph Ramsey Numbers $r(C_m, K_6)$, *Journal of Graph Theory,* **44** (2003) 251-260.

[Schi2] I. Schiermeyer, The Cycle-Complete Graph Ramsey Number $r(C_5, K_7)$, *Discussiones Mathematicae Graph Theory,* **25** (2005) 129-139.

[-] I. Schiermeyer, see also [FSS2, SchSch1, SchSch2].

[-] J.C. Schlage-Puchta, see [BrGS].

[Schu] C.-U. Schulte, Ramsey-Zahlen für Bäume und Kreise, *Ph.D. thesis,* Heinrich-Heine-Universität Düsseldorf, (1992).

[-] M.J. Schuster, see [CalSR].

[-] S. Schuster, see [ChaS].

[-] A. Schwenk, see [ChvS].

[Scob] M.W. Scobee, On the Ramsey Number $R(m_1P_3, m_2P_3, m_3P_3)$ and Related Results, ..., *MA thesis,* University of Louisville (1993).

[-] A. Sebő, see [GySeT].

[-] R.J. Segedin, see [LorSe].

[-] M. Shahsiah, see [MaORS1, MaORS2, OmSh].

[-] R. Shaltiel, see [BarRSW].

[Shao]* Zehui Shao, *personal communication* (2008).

[ShaoWX]* Shao Zehui, Wang Zicheng and Xiao Jianhua, Lower Bounds for Ramsey Numbers Based on Simulated Annealing Algorithm (in Chinese), *Computer Engineering and Applications,* **45** (2009) 70-71.

[ShaXBP]* Zehui Shao, Jin Xu, Qiquan Bao and Linqiang Pan, Computation of Some Generalized Ramsey Numbers, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **75** (2010) 217-228.

[ShaXB]* Zehui Shao, Xiaodong Xu and Qiquan Bao, On the Ramsey Numbers $R(C_m, B_n)$, *Ars Combinatoria,* **94** (2010) 265-271.

[ShaXSP]* Zehui Shao, Xiaodong Xu, Xiaolong Shi and Linqiang Pan, Some Three-Color Ramsey Numbers $R(P_4, P_5, C_k)$ and $R(P_4, P_6, C_k)$, *European Journal of Combinatorics,* **30** (2009) 396-403.

[-] Zehui Shao, see also [XSR1, XSR2].

[Shas] A. Shastri, Lower Bounds for Bi-Colored Quaternary Ramsey Numbers, *Discrete Mathematics,* **84** (1990) 213-216.

[She1] J.B. Shearer, A Note on the Independence Number of Triangle-Free Graphs, *Discrete Mathematics,* **46** (1983) 83-87.

[She2]* J.B. Shearer, Lower Bounds for Small Diagonal Ramsey Numbers, *Journal of Combinatorial Theory,* Series A, **42** (1986) 302-304.

[She3] J.B. Shearer, A Note on the Independence Number of Triangle-Free Graphs II, *Journal of Combinatorial Theory,* Series B, **53** (1991) 300-307.

[She4]* J.B. Shearer, Independence Numbers of Paley Graphs (data for primes 1 mod 4 up to 7000), http://www.research.ibm.com/people/s/shearer/indpal.html (1996).

[-] J. Sheehan, see [CRSPS, CEHMS, FRS7, FRS8, FRS9, RS1, RS2].

[-] Jian Shen, see [LiShen].

[-] Shen Yun-Qiu, see [LSS1, LSS2].

[-] Sheng Wancheng, see [HWSYZH].

[ShWR]* D. Shetler, M. Wurtz and S.P. Radziszowski, On Some Multicolor Ramsey Numbers Involving K_3+e and K_4-e, *SIAM Journal on Discrete Mathematics,* **26** (2012) 1256-1264.

[-] Shi Lei, see [SunYJLS].

[Shi1] Lingsheng Shi, Cube Ramsey Numbers Are Polynomial, *Random Structures & Algorithms,* **19** (2001) 99--101.

[Shi2] Lingsheng Shi, Upper Bounds for Ramsey Numbers, *Discrete Mathematics,* **270** (2003) 251-265.

[Shi3] Lingsheng Shi, Linear Ramsey Numbers of Sparse Graphs, *Journal of Graph Theory,* **50** (2005) 175-185.

[Shi4] Lingsheng Shi, The Tail Is Cut for Ramsey Numbers of Cubes, *Discrete Mathematics,* **307** (2007) 290-292.

[Shi5] Lingsheng Shi, Ramsey Numbers of Long Cycles versus Books or Wheels, *European Journal of Combinatorics,* **31** (2010) 828-838.

[ShZ1] Shi Ling Sheng and Zhang Ke Min, An Upper Bound Formula for Ramsey Numbers, *manuscript* (2001).

[ShZ2] Shi Ling Sheng and Zhang Ke Min, A Sequence of Formulas for Ramsey Numbers, *manuscript* (2001).

[-] Xiaolong Shi, see [ShaXSP].

[ShiuLL] Shiu Wai Chee, Peter Che Bor Lam and Li Yusheng, On Some Three-Color Ramsey Numbers, *Graphs and Combinatorics,* **19** (2003) 249-258.

Si - St

[Sid1] A.F. Sidorenko, On Turán Numbers $T(n,5,4)$ and Number of Monochromatic 4-cliques in 2-colored 3-graphs (in Russian), *Voprosy Kibernetiki,* **64** (1980) 117-124.

[Sid2] A.F. Sidorenko, An Upper Bound on the Ramsey Number $R(K_3,G)$ Depending Only on the Size of the Graph G, *Journal of Graph Theory,* **15** (1991) 15-17.

[Sid3] A.F. Sidorenko, The Ramsey Number of an N-Edge Graph versus Triangle Is at Most $2N+1$, *Journal of Combinatorial Theory,* Series B, **58** (1993) 185-196.

[-] M. Simonovits, see [AjKSS, BaSS, FSS1, FS, HaŁP1+, KoSS1, KoSS2, ŁucSS].

[-] J. Skokan, see [AllBS, BenSk, FizGMSS, HaŁP1+, HaŁP2+, KoSS1, KoSS2, ŁucSS].

[-] M.J. Smuga-Otto, see [AbbS].

[Sob] A. Sobczyk, Euclidian Simplices and the Ramsey Number $R(4,4;3)$, *Technical Report #10, Clemson University* (1967).

[Soi1] A. Soifer, *The Mathematical Coloring Book, Mathematics of coloring and the colorful life of its creators,* Springer 2009.

[Soi2] A. Soifer, *Ramsey Theory: Yesterday, Today and Tomorrow,* Progress in Mathematics 285, Springer-Birkhauser 2011.

[-] W. Solomon, see [LorSo].

[-] L. Soltés, see [LiRS].

[Song1] Song En Min, Study of Some Ramsey Numbers (in Chinese), a note (announcement of results without proofs), *Mathematica Applicata,* **4**(2) (1991) 6.

[Song2] Song En Min, New Lower Bound Formulas for the Ramsey Numbers $N(k,k,...,k;2)$ (in Chinese), *Mathematica Applicata,* **6** (1993) suppl., 113-116.

[Song3] Song En Min, An Investigation of Properties of Ramsey Numbers (in Chinese), *Mathematica Applicata,* **7** (1994) 216-221.

[Song4] Song En Min, Properties and New Lower Bounds of the Ramsey Numbers $R(p,q;4)$ (in Chinese), *Journal of Huazhong University of Science and Technology*, **23** (1995) suppl. II, 1-4.

[SonYL] Song En Min, Ye Weiguo and Liu Yanwu, New Lower Bounds for Ramsey Number $R(p,q;4)$, *Discrete Mathematics*, **145** (1995) 343-346.

[-] Song En Min, see also [HuSo, ZLLS].

[Song5] Song Hongxue, Asymptotic Upper Bounds for Wheel-Complete Graph Ramsey Numbers, *Journal of Southeast University* (English Edition), ISSN 1003-7985, **20** (2004) 126-129.

[Song6] Song Hongxue, A Ramsey Goodness Result for Graphs with Large Pendent Trees, *Journal of Mathematical Study (China)*, **42** (2009) 36-39.

[Song7] Song Hong-xue, Asymptotic Upper Bounds for K_2+T_m : Complete Graph Ramsey Numbers, *Journal of Mathematics (China)*, **30** (2010) 797-802.

[Song8] Song Hongxue, Asymptotic Lower Bounds of Ramsey Numbers for r-Uniform Hypergraphs, *Advances in Mathematics, Shuxue Jinzhan*, **40** (2011) 179-186.

[Song9] Hongxue Song, Asymptotic Upper Bounds for $K_{1,m,k}$: Complete Graph Ramsey Numbers, *Ars Combinatoria*, **111** (2013) 137-144.

[SonBL] Song Hong Xue, Bai Lu Feng and Liu Shu Yan, Asymptotic Upper Bounds for the Wheel-Complete Graph Ramsey Numbers (in Chinese), *Acta Mathematica Scientia, Series A*, ISSN 1003-3998, **26** (2006) 741-746.

[SonGQ] Song Hongxue, Gu Hua and Qian Xinjin, On the Ramsey Number of K_3 versus K_2+T_n (in Chinese), *Journal of Liaoning Normal University, Natural Science Edition*, ISSN 1000-1735, **27** (2004) 142-145.

[SonLi] Song Hongxue and Li Yusheng, Asymptotic Lower Bounds of Ramsey Numbers for 4-Uniform Hypergraphs (in Chinese), *Journal of Nanjing University Mathematical Biquarterly*, **26** (2009) 216-224.

[-] Song Hongxue, see also [GuSL].

[Spe1] J.H. Spencer, Ramsey's Theorem - A New Lower Bound, *Journal of Combinatorial Theory*, Series A, **18** (1975) 108-115.

[Spe2] J.H. Spencer, Asymptotic Lower Bounds for Ramsey Functions, *Discrete Mathematics*, **20** (1977) 69-76.

[Spe3] J.H. Spencer, Eighty Years of Ramsey $R(3,k)$ ... and Counting! in *Ramsey Theory: Yesterday, Today and Tomorrow* (ed. A. Soifer), Progress in Mathematics 285, Springer-Birkhauser 2011, 27-39.

[-] J.H. Spencer, see also [BES, GRS].

[-] T.S. Spencer, see [BahS].

[Spe4]* T. Spencer, University of Nebraska at Omaha, *personal communication* (1993), and, Upper Bounds for Ramsey Numbers via Linear Programming, *manuscript* (1994).

[-] A.K. Srivastava, see [GauST].

[Stahl] S. Stahl, On the Ramsey Number $R(F,K_m)$ where F is a Forest, *Canadian Journal of Mathematics*, **27** (1975) 585-589.

[-] R.G. Stanton, see [KaSt].

[Stat] W. Staton, Some Ramsey-type Numbers and the Independence Ratio, *Transactions of the American Mathematical Society*, **256** (1979) 353-370.

[-] A. Steger, see [McS].

[-] J. Stinehour, see [RST].

[Stev] S. Stevens, Ramsey Numbers for Stars versus Complete Multipartite Graphs, *Congressus Numerantium*, **73** (1990) 63-71.

[-] M.J. Stewart, see [CRSPS].

[Stone] J.C. Stone, Utilizing a Cancellation Algorithm to Improve the Bounds of $R(5,5)$, (1996), http://oas.okstate.edu/ojas/jstone.htm. This paper claims incorrectly that $R(5,5)=50$.

Su - Sz

[SL]* Su Wenlong and Luo Haipeng, Prime Order Cyclic Graphs and New Lower Bounds for Three Classical Ramsey Numbers $R(4,n)$ (in Chinese), *Journal of Mathematical Study*, **31**, 4 (1998) 442-446.

[SLL]* Su Wenlong, Luo Haipeng and Li Qiao, New Lower Bounds of Classical Ramsey Numbers $R(4,12)$, $R(5,11)$ and $R(5,12)$, *Chinese Science Bulletin*, **43**, 6 (1998) 528.

[SLLL]* Su Wenlong, Luo Haipeng, Li Guiqing and Li Qiao, Lower Bounds of Ramsey Numbers Based on Cubic Residues, *Discrete Mathematics*, **250** (2002) 197-209.

[SLZL]* Su Wenlong, Luo Haipeng, Zhang Zhengyou and Li Guiqing, New Lower Bounds of Fifteen Classical Ramsey Numbers, *Australasian Journal of Combinatorics*, **19** (1999) 91-99.

[-] Su Wenlong, see also [ChWXSL, LiaWXCS, LiaWXS, LSL, LSLW, LSS1, LSS2, WSLX1, WSLX2, XWCS].

[Sud1] B. Sudakov, A Note on Odd Cycle-Complete Graph Ramsey Numbers, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #N1, **9** (2002), 4 pages.

[Sud2] B. Sudakov, Large K_r-Free Subgraphs in K_s-Free Graphs and Some Other Ramsey-Type Problems, *Random Structures and Algorithms*, **26** (2005) 253-265.

[Sud3] B. Sudakov, Ramsey Numbers and the Size of Graphs, *SIAM Journal on Discrete Mathematics*, **21** (2007) 980-986.

[Sud4] B. Sudakov, A Conjecture of Erdős on Graph Ramsey Numbers, *Advances in Mathematics*, **227** (2011) 601-609

[-] B. Sudakov, see also [AlKS, ConFS1, ConFS2, ConFS3, ConFS4, ConFS5, ConFS6, FoxSu1, FoxSu2, KoSu].

[-] A. Sudan, see [GGS].

[Sudar] I.W. Sudarsana, The Goodness of Long Path with Respect to Multiple Copies of Small Wheel, *Far East Journal of Mathematical Sciences*, **59** (2011) 47-55.

[SuAM] I.W. Sudarsana, Adiwijaya and S. Musdalifah, The Ramsey Number for a Linear Forest versus Two Identical Copies of Complete Graphs, COCOON 2010, *LNCS* 6196, Springer, Berlin (2010) 209-215.

[SuAAM] I.W. Sudarsana, H. Assiyatun, Adiwijaya and S. Musdalifah, The Ramsey Number for a Linear Forest versus Two Identical Copies of Complete Graph, *Discrete Mathematics, Algorithms and Applications*, **2** (2010) 437-444.

[SuBAU1] I.W. Sudarsana, E.T. Baskoro, H. Assiyatun and S. Uttunggadewa, The Ramsey Number of a Certain Forest with Respect to a Small Wheel, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **71** (2009) 257-264.

[SuBAU2] I.W. Sudarsana, E.T. Baskoro, H. Assiyatun and S. Uttunggadewa, The Ramsey Numbers of Linear Forest versus $3K_3 \bigcup 2K_4$, *Journal of the Indonesian Mathematical Society*, **15** (2009) 61-67.

[SuBAU3] I.W. Sudarsana, E.T. Baskoro, H. Assiyatun and S. Uttunggadewa, The Ramsey Numbers for the Union Graph with H-Good Components, *Far East Journal of Mathematical Sciences*, **39** (2010) 29-40.

[Sun]* Sun Yongqi, Research on Ramsey Numbers of Some Graphs (in Chinese), *Ph. D. thesis*, Dalian University of Technology, China, July 2006.

[SunY]* Sun Yongqi and Yang Yuansheng, Study of the Three Color Ramsey Number $R_3(C_8)$ (in Chinese), *Journal of Beijing Jiaotong University*, **35** (2011) 14-17.

[SunYJLS] Sun Yongqi, Yang Yuansheng, Jiang Baoqi, Lin Xiaohui and Shi Lei, On Multicolor Ramsey Numbers for Even Cycles in Graphs, *Ars Combinatoria*, **84** (2007) 333-343.

[SunYLZ1]* Sun Yongqi, Yang Yuansheng, Lin Xiaohui and Zheng Wenping, The Value of the Ramsey Number $R_4(C_4)$, *Utilitas Mathematica,* **73** (2007) 33-44.

[SunYLZ2]* Sun Yongqi, Yang Yuansheng, Lin Xiaohui and Zheng Wenping, On the Three Color Ramsey Numbers $R(C_m, C_4, C_4)$, *Ars Combinatoria,* **84** (2007) 3-11.

[SunYW]* Sun Yongqi, Yang Yuansheng and Wang Zhihai, The Value of the Ramsey Number $R_5(C_6)$, *Utilitas Mathematica,* **76** (2008) 25-31.

[SunYWLX]* Sun Yongqi, Yang Yuansheng, Wang Wei, Li Bingxi and Xu Feng, Study of Three Color Ramsey numbers $R(C_{m_1}, C_{m_2}, C_{m_3})$ (in Chinese), *Journal of Dalian University of Technology,* ISSN 1000-8608, **46** (2006) 428-433.

[SunYXL] Sun Yongqi, Yang Yuansheng, Xu Feng and Li Bingxi, New Lower Bounds on the Multicolor Ramsey Numbers $R_r(C_{2m})$, *Graphs and Combinatorics,* **22** (2006) 283-288.

[-] Sun Yongqi, see also [WuSR, WuSZR, ZhaSW].

[SunLi] Sun Yuqin and Li Yusheng, On an Upper Bound of Ramsey Number $r_k(K_{m,n})$ with Large n, *Heilongjiang Daxue Ziran Kexue Xuebao,* ISSN 1001-7011, **23** (2006) 668-670.

[SunZ] Zhi-Hong Sun, Ramsey Numbers for Trees, *Bulletin of the Australian Mathematical Society,* **86** (2012) 164-176.

[SunWW] Zhi-Hong Sun, Lin-Lin Wang and Yi-Li Wu, Turán's Problem and Ramsey Numbers for Trees, *preprint,* arXiv, http://arxiv.org/abs/1110.2725 (2011).

[Sur] Surahmat, Cycle-Wheel Ramsey Numbers. Some results, open problems and conjectures. *Math Track,* ISSN 1817-3462, 1818-5495, **2** (2006) 56-64.

[SuBa1] Surahmat and E.T. Baskoro, On the Ramsey Number of a Path or a Star versus W_4 or W_5, *Proceedings of the 12-th Australasian Workshop on Combinatorial Algorithms,* Bandung, Indonesia, July 14-17 (2001) 174-179.

[SuBa2] Surahmat and E.T. Baskoro, The Ramsey Number of Linear Forest versus Wheel, paper presented at the *13-th Australasian Workshop on Combinatorial Algorithms,* Fraser Island, Queensland, Australia, July 7-10, 2002.

[SuBB1] Surahmat, E.T. Baskoro and H.J. Broersma, The Ramsey Numbers of Large Star-like Trees versus Large Odd Wheels, *Technical Report* #1621, Faculty of Mathematical Sciences, University of Twente, The Netherlands, (2002).

[SuBB2] Surahmat, E.T. Baskoro and H.J. Broersma, The Ramsey Numbers of Large Cycles versus Small Wheels, *Integers: Electronic Journal of Combinatorial Number Theory,* http://www.integers-ejcnt.org/vol4.html, #A10, **4** (2004), 9 pages.

[SuBB3] Surahmat, E.T. Baskoro and H.J. Broersma, The Ramsey Numbers of Fans versus K_4, *Bulletin of the Institute of Combinatorics and its Applications,* **43** (2005) 96-102.

[SuBB4] Surahmat, E.T. Baskoro and H.J. Broersma, The Ramsey Numbers of Large Star and Large Star-Like Trees versus Odd Wheels, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **65** (2008) 153-162.

[SuBT1] Surahmat, E.T. Baskoro and I. Tomescu, The Ramsey Numbers of Large Cycles versus Wheels, *Discrete Mathematics,* **306** (2006), 3334-3337.

[SuBT2] Surahmat, E.T. Baskoro and I. Tomescu, The Ramsey Numbers of Large Cycles versus Odd Wheels, *Graphs and Combinatorics,* **24** (2008), 53-58.

[SuBTB] Surahmat, E.T. Baskoro, I. Tomescu and H.J. Broersma, On Ramsey Numbers of Cycles with Respect to Generalized Even Wheels, *manuscript* (2006).

[SuBUB] Surahmat, E.T. Baskoro, S. Uttunggadewa and H.J. Broersma, An Upper Bound for the Ramsey Number of a Cycle of Length Four versus Wheels, in *LNCS* 3330, Springer, Berlin (2005) 181-184.

[-] Surahmat, see also [AliSur, BaSu, BaSNM].

[SwPr] C.J. Swanepoel and L.M. Pretorius, Upper Bounds for a Ramsey Theorem for Trees, *Graphs and Combinatorics,* **10** (1994) 377-382.

[-] M.M. Sweet, see [FreSw].

[-] T. Szabó, see [AlRóS].

[Szem] E. Szemerédi, Regular Partitions of Graphs, Problèmes Combinatoires et Théorie des Graphes (Orsay, 1976), Colloques Internationaux du Centre National de la Recherche Scientifique, CNRS Paris, **260** (1978) 399--401.

[-] E. Szemerédi, see also [AjKS, AjKSS, CRST, GyRSS, GySS1, GySS2, PoRRS].

[-] P. Szuca, see [DzNS].

T

[-] Tang Xueqing, see [LiTZ].

[-] A. Taraz, see [MoSST].

[-] R. Thomas, see [RöTh].

[Tho] A. Thomason, An Upper Bound for Some Ramsey Numbers, *Journal of Graph Theory*, **12** (1988) 509-517.

[-] P.W. Tingley, see [HałT].

[-] I. Tomescu, see [AliBT1, AliBT2, SuBT1, SuBT2, SuBTB].

[-] C.A. Tovey, see [CaET].

[-] A. Tripathi, see [GauST].

[Tr] Trivial results.

[-] N. Trotignon, see [GySeT].

[-] W.T. Trotter Jr., see [CRST].

[Tse1]* Kung-Kuen Tse, On the Ramsey Number of the Quadrilateral versus the Book and the Wheel, *Australasian Journal of Combinatorics*, **27** (2003) 163-167.

[Tse2]* Kung-Kuen Tse, A Note on the Ramsey Numbers $R(C_4, B_n)$, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **58** (2006) 97-100.

[Tse3]* Kung-Kuen Tse, A Note on Some Ramsey Numbers $R(C_p, C_q, C_r)$, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **62** (2007) 189-192.

[-] Kung-Kuen Tse, see also [BaRT, RST, RT].

[-] Z. Tuza, see [GyTu].

U

[-] S. Uttunggadewa, see [SuBAU1, SuBAU2, SuBAU3, SuBUB].

V

[-] J. Verstraëte, see [KosMV1, KosMV2].

[-] L. Volkmann, see [GuoV].

W

[Walk] K. Walker, Dichromatic Graphs and Ramsey Numbers, *Journal of Combinatorial Theory*, **5** (1968) 238-243.

[Wall] W.D. Wallis, On a Ramsey Number for Paths, *Journal of Combinatorics, Information & System Sciences*, **6** (1981) 295-296.

[Wan] Wan Honghui, Upper Bounds for Ramsey Numbers $R(3,3,\cdots,3)$ and Schur Numbers, *Journal of Graph Theory*, **26** (1997) 119-122.

[-] Wang Gongben, see [WW, WWY1, WWY2].

[-] Lin-Lin Wang, see [SunWW].

[WW]* Wang Qingxian and Wang Gongben, New Lower Bounds of Ramsey Numbers $r(3,q)$ (in Chinese), *Acta Scientiarum Naturalium, Universitatis Pekinensis*, **25** (1989) 117-121.

[WWY1]* Wang Qingxian, Wang Gongben and Yan Shuda, A Search Algorithm And New Lower Bounds for Ramsey Numbers $r(3,q)$, *manuscript* (1994).

[WWY2]* Wang Qingxian, Wang Gongben and Yan Shuda, The Ramsey Numbers $R(K_3,K_q-e)$ (in Chinese), *Beijing Daxue Xuebao Ziran Kexue Ban*, **34** (1998) 15-20.

[-] Wang Wei, see [SunYWLX, SunYXL].

[WaLi] Ye Wang and Yusheng Li, Lower Bounds for Ramsey Numbers of K_n with a Small Subgraph Removed, *Discrete Applied Mathematics*, **160** (2012) 2063-2068.

[-] Wang Yuandi, see [HWSYZH].

[-] Wang Zhihai, see [SunYW].

[-] Wang Zhi Jian, see [LiWa1, LiWa2].

[-] Wang Zicheng, see [ShaoWX].

[West] D. West, *Introduction to Graph Theory*, second edition, Prentice Hall, 2001.

[Wh] E.G. Whitehead, The Ramsey Number $N(3,3,3,3;2)$, *Discrete Mathematics*, **4** (1973) 389-396.

[-] A. Widgerson, see [BarRSW].

[-] E.R. Williams, see [AbbW].

[-] R.M. Wilson, see [FraWi].

[-] A. Woldar, see [LaWo1, LaWo2].

[WSLX1]* Kang Wu, Wenlong Su, Haipeng Luo and Xiaodong Xu, New Lower Bound for Seven Classical Ramsey Numbers $R(3,q)$, *Applied Mathematics Letters*, **22** (2009) 365-368.

[WSLX2]* Kang Wu, Wenlong Su, Haipeng Luo and Xiaodong Xu, A Generalization of Generalized Paley Graphs and New Lower Bounds for $R(3,q)$, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #N25, **17** (2010), 10 pages.

[-] Wu Kang, see also [ChWXSL, LiaWXCS, LiaWXS, LSLW, XWCS].

[WuSR] Wu Yali, Sun Yongqi and S.P. Radziszowski, Wheel and Star-critical Ramsey Numbers for Quadrilateral, *manuscript* (2013).

[WuSZR] Wu Yali, Sun Yongqi, Zhang Rui and S.P. Radziszowski, Ramsey Numbers of C_4 versus Wheels and Stars, *manuscript* (2013).

[-] Wu Yali, see also [ZhaSW].

[-] Yi-Li Wu, see [SunWW].

[-] M. Wurtz, see [ShWR].

X

[-] Xiao Jianhua, see [ShaoWX].

[XieZ]* Xie Jiguo and Zhang Xiaoxian, A New Lower Bound for Ramsey Number $r(3,13)$ (in Chinese), *Journal of Lanzhou Railway Institute*, **12** (1993) 87-89.

[-] Xie Zheng, see [XX1, XX2, XXER, XXR].

[XWCS]* Chengzhang Xu, Kang Wu, Hong Chen and Wenlong Su, New Lower Bounds for Some Ramsey Numbers Based on Cyclic Graphs, *in preparation*, (2011).

[-] Xu Chengzhang, see also [LiaWXCS].

[-] Jin Xu, see [ShaXBP].

[-] Xu Feng, see [SunYWLX, SunYXL].

[-] Ran Xu, see [ChenCX].

[Xu] Xu Xiaodong, *personal communication*, (2004).

[XuR1] Xiaodong Xu and S.P. Radziszowski, An Improvement to Mathon's Cyclotomic Ramsey Colorings, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #N1, **16**(1) (2009), 5 pages.

[XuR2] Xiaodong Xu and S.P. Radziszowski, $28 \le R(C_4, C_4, C_3, C_3) \le 36$, *Utilitas Mathematica*, **79** (2009) 253-257.

[XuR3] Xiaodong Xu and S.P. Radziszowski, Bounds on Shannon Capacity and Ramsey Numbers from Product of Graphs, *IEEE Transactions on Information Theory*, **59** (2013) 4767-4770.

[XSR1]* Xiaodong Xu, Zehui Shao and S.P. Radziszowski, Bounds on Some Ramsey Numbers Involving Quadrilateral, *Ars Combinatoria*, **90** (2009) 337-344.

[XSR2]* Xiaodong Xu, Zehui Shao and S.P. Radziszowski, More Constructive Lower Bounds on Classical Ramsey Numbers, *SIAM Journal on Discrete Mathematics*, **25** (2011) 394-400.

[XX1]* Xu Xiaodong and Xie Zheng, A Constructive Approach for the Lower Bounds on the Ramsey Numbers $r(k, l)$, *manuscript* (2002).

[XX2] Xu Xiaodong and Xie Zheng, A Constructive Approach for the Lower Bounds on Multicolor Ramsey Numbers, *manuscript* (2002).

[XXER]* Xu Xiaodong, Xie Zheng, G. Exoo and S.P. Radziszowski, Constructive Lower Bounds on Classical Multicolor Ramsey Numbers, *Electronic Journal of Combinatorics*, http://www.combinatorics.org, #R35, **11**(1) (2004), 24 pages.

[XXR] Xu Xiaodong, Xie Zheng and S.P. Radziszowski, A Constructive Approach for the Lower Bounds on the Ramsey Numbers $R(s, t)$, *Journal of Graph Theory*, **47** (2004) 231-239.

[-] Xu Xiaodong, see also [ChWXSL, LiaWXS, ShaXB, ShaXSP, WSLX1, WSLX2].

[-] Xu Zhiqiang, see [BaLX].

Y

[-] J. Yackel, see [GrY].

[-] Yan Shuda, see [WWY1, WWY2].

[YHZ1] Yang Jian Sheng, Huang Yi Ru and Zhang Ke Min, The Value of the Ramsey Number $R(C_n, K_4)$ is $3(n-1)+1$ $(n \ge 4)$, *Australasian Journal of Combinatorics*, **20** (1999) 205-206.

[YHZ2] Yang Jian Sheng, Huang Yi Ru and Zhang Ke Min, $R(C_6, K_5) = 21$ and $R(C_7, K_5) = 25$, *European Journal of Combinatorics*, **22** (2001) 561-567.

[-] Yang Jian Sheng, see also [BolJY+, HWSYZH].

[YY]** Yang Yuansheng, On the Third Ramsey Numbers of Graphs with Six Edges, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **17** (1995) 199-208.

[YH]* Yang Yuansheng and G.R.T. Hendry, The Ramsey Number $r(K_1+C_4, K_5-e)$, *Journal of Graph Theory*, **19** (1995) 13-15.

[YR1]** Yang Yuansheng and P. Rowlinson, On the Third Ramsey Numbers of Graphs with Five Edges, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **11** (1992) 213-222.

[YR2]* Yang Yuansheng and P. Rowlinson, On Graphs without 6-Cycles and Related Ramsey Numbers, *Utilitas Mathematica*, **44** (1993) 192-196.

[YR3]* Yang Yuansheng and P. Rowlinson, The Third Ramsey Numbers for Graphs with at Most Four Edges, *Discrete Mathematics*, **125** (1994) 399-406.

[-] Yang Yuansheng, see also [SunY, SunYJLS, SunYLZ1, SunYLZ2, SunYW, SunYWLX, SunYXL].

[-] Ye Weiguo, see [SonYL].

[Yu1]* Yu Song Nian, A Computer Assisted Number Theoretical Construction of $(3,k)$-Ramsey Graphs, *Annales Universitatis Scientiarum Budapestinensis, Sect. Comput.,* **10** (1989) 35-44.

[Yu2]* Yu Song Nian, Maximal Triangle-Free Circulant Graphs and the Function $K(c)$ (in Chinese), *Journal of Shanghai University, Natural Science,* **2** (1996) 678-682.

Z

[-] Zang Wenan, see [LiRZ1, LiRZ2, LiTZ, LiZa1, LiZa2].

[Zeng] Zeng Wei Bin, Ramsey Numbers for Triangles and Graphs of Order Four with No Isolated Vertex, *Journal of Mathematical Research & Exposition,* **6** (1986) 27-32.

[ZZ1] Zhang Ke Min and Zhang Shu Sheng, Some Tree-Stars Ramsey Numbers, *Proceedings of the Second Asian Mathematical Conference* 1995, 287-291, World Sci. Publishing, River Edge, NJ, 1998.

[ZZ2] Zhang Ke Min and Zhang Shu Sheng, The Ramsey Numbers for Stars and Stripes, *Acta Mathematica Scientia,* **25A** (2005) 1067-1072.

[-] Zhang Ke Min, see also [BolJY+, ChenZZ1, ChenZZ2, ChenZZ3, ChenZZ4, ChenZZ5, ChenZZ6, HWSYZH, HZ1, HZ2, MZ, ShZ1, ShZ2, YHZ1, YHZ2, ZhaCZ1, ZhaCZ2, ZZ3].

[ZhaCC1] Lianmin Zhang, Yaojun Chen and T.C. Edwin Cheng, The Ramsey Numbers for Cycles versus Wheels of Even Order, *European Journal of Combinatorics,* **31** (2010) 254-259.

[ZhaSW]* Zhang Rui, Sun Yongqi and Wu Yali, On the Four Color Ramsey Numbers for Hexagons, *Ars Combinatoria,* **111** (2013) 515-522.

[-] Zhang Rui, see also [WuSZR].

[-] Zhang Shu Sheng, see [ZZ1, ZZ2].

[-] Zhang Xiaoxian, see [XieZ].

[-] Zhang Yuming, see [CaLRZ].

[Zhang1] Zhang Yunqing, On Ramsey Numbers of Short Paths versus Large Wheels, *Ars Combinatoria,* **89** (2008) 11-20.

[Zhang2] Zhang Yunqing, The Ramsey Numbers for Stars of Odd Small Order versus a Wheel of Order Nine, *Nanjing Daxue Xuebao Shuxue Bannian Kan,* ISSN 0469-5097, **25** (2008) 35-40.

[ZhaCC2] Yunqing Zhang, T.C. Edwin Cheng and Yaojun Chen, The Ramsey Numbers for Stars of Odd Order versus a Wheel of Order Nine, *Discrete Mathematics, Algorithms and Applications,* **1** (2009) 413-436.

[ZhaCZ1] Yunqing Zhang, Yaojun Chen and Kemin Zhang, The Ramsey Numbers for Stars of Even Order versus a Wheel of Order Nine, *European Journal of Combinatorics,* **29** (2008) 1744-1754.

[ZhaCZ2] Yunqing Zhang, Yaojun Chen and Kemin Zhang, The Ramsey Numbers for Trees of High Degree versus a Wheel of Order Nine, *manuscript* (2009).

[ZZ3] Yunqing Zhang and Ke Min Zhang, The Ramsey Number $R(C_8,K_8)$, *Discrete Mathematics,* **309** (2009) 1084-1090.

[-] Zhang Yunqing, see also [ChenCNZ, ChenCZ1, ChenZZ1, ChenZZ2, ChenZZ3, ChenZZ4, ChenZZ5, ChenZZ6, CheCZN].

[-] Zhang Zhengyou, see [SLZL].

[ZLLS] Zhang Zhongfu, Liu Linzhong, Li Jinwen and Song En Min, Some Properties of Ramsey Numbers, *Applied Mathematics Letters,* **16** (2003) 1187-1193.

[Zhao] Yi Zhao, Proof of the $(n/2-n/2-n/2)$ Conjecture for Large n, *Electronic Journal of Combinatorics,* http://www.combinatorics.org, #P27, **18**(1) (2011), 61 pages.

[-] Zheng Wenping, see [SunYLZ1, SunYLZ2].

[Zhou1] Zhou Huai Lu, Some Ramsey Numbers for Graphs with Cycles (in Chinese), *Mathematica Applicata*, **6** (1993) 218.

[Zhou2] Zhou Huai Lu, The Ramsey Number of an Odd Cycle with Respect to a Wheel (in Chinese), *Journal of Mathematics, Shuxue Zazhi* (Wuhan), **15** (1995) 119-120.

[Zhou3] Zhou Huai Lu, On Book-Wheel Ramsey Number, *Discrete Mathematics*, **224** (2000) 239-249.