Context Free Languages

Regular Languages are CFLs

- We know that some CFLs are not regular languages

- However, all regular languages are context-free.
  - Converting a DFA into a CFG:
    - Make variable R_i for each state q_i of the DFA
    - Add rule $R_i \rightarrow aR_j$ if $\delta(q_i,a) = q_j$ is a transition in the DFA
    - Add the rule $R_i \rightarrow \epsilon$ if q_i is an accepting state in the DFA
    - Make R_0 the start variable where q_0 is the start state of the DFA

Regular Languages are CFLs

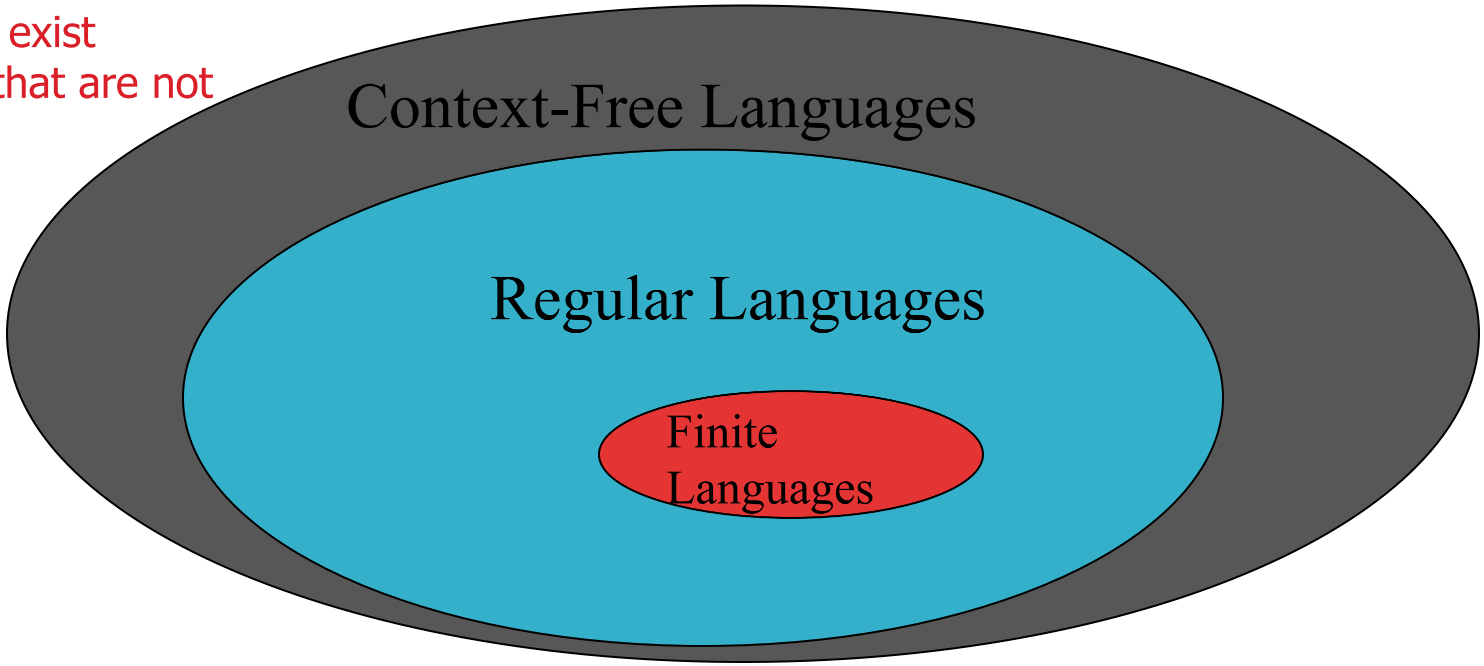
- Another argument using PDAs
  - A DFA *is* a PDA, just a basic one in which we don’t pay any attention to the stack
    - Any language recognized by a PDA is a CFL

Language Space So Far

Is there something beyond CFL?

Regular languages are contained in CFLs

Yes – CFL pumping lemma will be used to show there exist languages that are not CFLs



Is there something beyond regular?

Yes – pumping lemma used to show languages are non-regular

Closure Properties

- We have already seen that CFLs are closed under:
  - Union
  - Concatenation
  - Kleene Star
- Regular Languages are also closed under
  - Intersection
  - Complementation
  - Difference
- What about Context-Free Languages?

Closure Properties

- CFLs are <u>not</u> closed under intersection
- CFL are <u>not</u> closed under difference.
- CFL are <u>not</u> closed under complement.
  - These will be easier to demonstrate after the pumping lemma for CFLs is used to give us some example non-CFL languages.

An Example non-CFL Language

- $L = \{a^i b^i c^i \mid i \geq 0\}$ is NOT a CFL
  - We'll prove using the pumping lemma soon

- What about the complement of L?

  - $L' = \Sigma^* - L = (\Sigma^* - a^*b^*c^*) \cup \{a^i b^j c^k \mid i>j\}$
    $\cup \{a^i b^j c^k \mid i<j\}$
    $\cup \{a^i b^j c^k \mid j>k\}$
    $\cup \{a^i b^j c^k \mid j<k\}$

THIS IS A CFL

An Example non-CFL Language

- $L = \{ww \mid w \in \{a,b\}^*\}$ is NOT a CFL
  - Can be shown using the pumping lemma

- What about the complement of L?

  - Turns out this IS a CFL
    - Union of all odd length strings, and
    - All even length strings for which you can guarantee there is some index where the two halves differ
      - Can be shown to be a CFL (often we do this for homework)

An Example non-CFL Language

- $L = \{a^k \mid k \text{ is a perfect square}\}$ is NOT a CFL
  - Can be shown using the pumping lemma
- What about the complement of L?
  - $L' = \{a^k \mid k \text{ is NOT a perfect square}\}$
    - THIS IS ALSO NOT A CFL

An Example non-CFL Language

- $L_1 = \{a^i b^j c^k \mid i < j, i < k\}$ is NOT a CFL
  - Can be shown using the pumping lemma
- Also $L_2 = \{a^i b^j c^k \mid i < j, i \geq k\}$ is NOT a CFL