Formal Definition of NFA

Based on slides of Aaron Deever

Recall: NFA

- Non-determinism
  - When machine is in a given state and reads a symbol:
    - The machine may have a choice of where to move to next.
    - There may be states where, after reading a given symbol, the machine has nowhere to go
    - Applying the transition function will give, not necessarily 1 state, but 0 or more states.
    - A DFA is just a special case of an NFA

Formal Definition of NFA

- A Non-Deterministic Finite Automata is a 5-tuple $(Q, \Sigma, \delta, q_o, F)$ where
  - Q is a finite set (of states)
  - Σ is a finite alphabet of symbols
  - $q_o \in Q$ is the start state
  - $F \subseteq Q$ is the set of final (accepting) states
  - δ is a function from $Q \times \Sigma\varepsilon$ to P(Q) (transition function)

- $\Sigma\varepsilon = \Sigma \cup \{\varepsilon\}$
- *P(Q) = power set of Q = set of all subsets of Q.*

Formal Definition of NFA

- Transition function
  - δ is a function from Q x $\Sigma\varepsilon$ to P(Q)
  - δ (q, a) = subset of Q (possibly empty)
  - Examples:
    - $\delta(q_3, 0) = \{q_0\}$ usual case, but set notation
    - $\delta(q_0, 1) = \{q_1, q_2\}$ multiple arrows out with common symbol
    - $\delta(q_4, 1) = \varnothing$ no arrow out with that symbol
    - $\delta(q_0, \epsilon) = \{q_1, q_2\}$ two epsilon branches out of that state

Language Accepted by a NFA

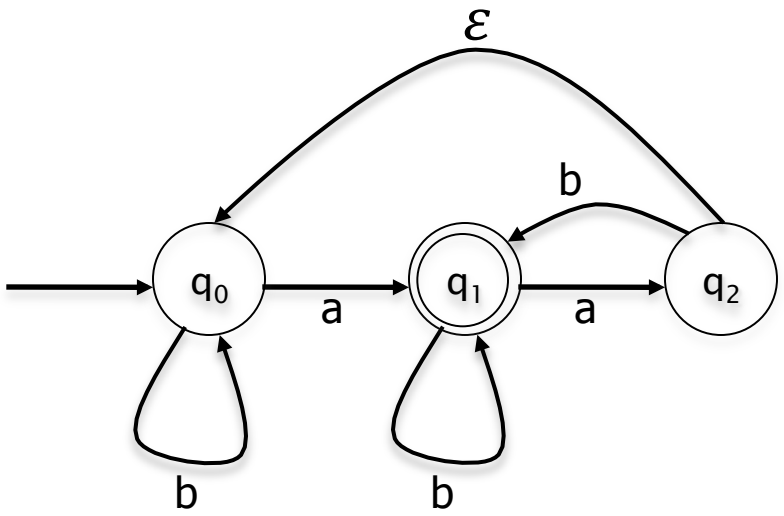
- Let M = $(Q, \Sigma, \delta, q_o, F)$ be an NFA
- And let w be a string over the alphabet Σ

- We say M accepts w if we can write $w = w_1 w_2 \ldots w_m$ where each $w_i \in \Sigma\varepsilon$ and
  - A sequence of states $r_0 r_1 \ldots r_m$ exists with the conditions:
    - $r_0 = q_0$
    - $r_{i+1} \in \delta(r_i, w_{i+1})$ for i = 0, …, m−1
    - $r_m \in F$

  - Note that there may be more than one way to write w that satisfies the conditions
  - Note that w_i can be ϵ

Non-Deterministic Finite Automata

- Are the following strings accepted by the NFA below:
  - aab
  - aaba



Non-Deterministic Finite Automata

- Language accepted by M
  - $L(M) = \{ w \in \Sigma^* \mid w \text{ is accepted by } M \}$
- If L is a language over Σ, L is accepted by M if and only if $L = L(M)$.
  - For all $w \in L$, w is accepted by M.
  - For all $w \notin L$, w is rejected by M.

Formal NFA Example

- Suppose we have NFA N = $(Q, \Sigma, \delta, q_o, F)$
  - How can we convert this NFA into an equivalent NFA, N', having only 1 accept state?
  - What is the formal definition of N' ?

DFA / NFA Equivalence

- Surprisingly enough
  - Adding non-determinism to our DFA does NOT give it any additional language accepting power.
  - DFAs and NFAs are equivalent
    - Every language that is accepted by an NFA is also accepted by a DFA and vice versa

Thanks for your attention!