Cryptosystem 7.7: *Full Domain Hash*

Let k be a positive integer; let $\mathcal{F}$ be a family of trapdoor one-way permutations such that $f : \{0,1\}^k \rightarrow \{0,1\}^k$ for all $f \in \mathcal{F}$; and let $G : \{0,1\}^* \rightarrow \{0,1\}^k$ be a "random" function. Let $\mathcal{P} = \{0,1\}^*$ and $\mathcal{A} = \{0,1\}^k$, and define

$$\mathcal{K} = \{(f, f^{-1}, G) : f \in \mathcal{F}\}.$$

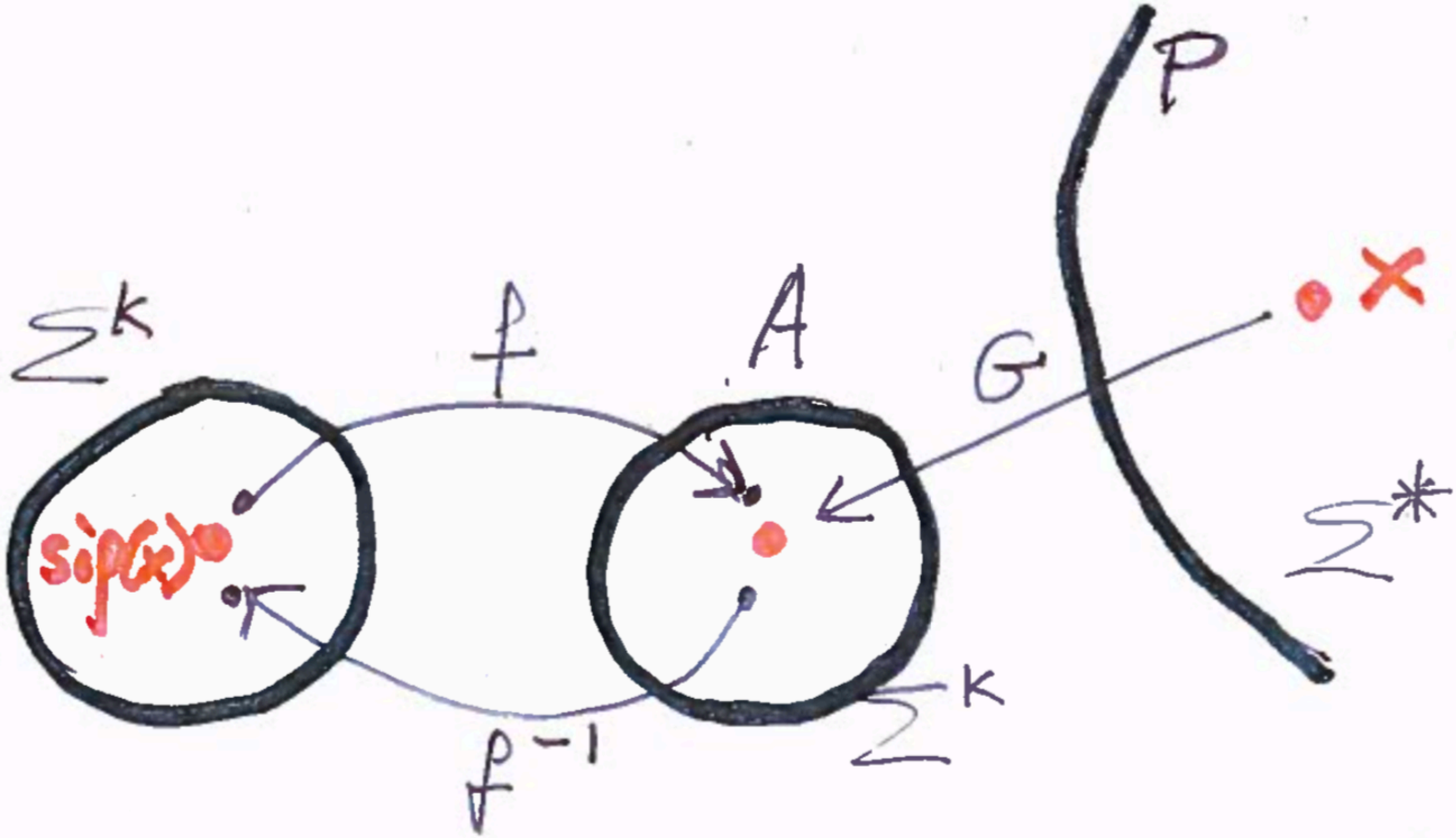
Given a key $K = (f, f^{-1}, G)$, f^{-1} is the private key and (f, G) is the public key.

For $K = (f, f^{-1}, G)$ and $x \in \{0,1\}^*$, define

$$\mathrm{sig}_K(x) = f^{-1}(G(x)).$$

A signature $y = (y_1, \ldots, y_k) \in \{0,1\}^k$ on the message x is verified as follows:

$$\mathrm{ver}_K(x, y) = \mathrm{true} \Leftrightarrow f(y) = G(x).$$


Σ^k
f
A
P
G
x
Σ*
Σ^k
f^{-1}

```
Algorithm 7.2: FDH-INVERT(z_0, q_h)

 external f
 procedure SIMG(x)
  if j > q_h
    then return ("failure")
    else if j = j_0
    then z ← z_0
    else let z ∈ {0,1}^k be chosen at random
  j ← j + 1
  return (z)

 main
  choose j_0 ∈ {1, ..., q_h} at random
  j ← 1
  insert the code for FDH-FORGE(f) here
  if FDH-FORGE(f) = (x, y)
            ⎧ if f(y) = z_0
    then    ⎨   then return (y)
            ⎩   else return ("failure")
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THEOREM 7.2 *Suppose there exists an algorithm* FDH-FORGE *that will output an existential forgery for Full Domain Hash with probability* $\epsilon > 2^{-k}$, *using a key-only attack. Then there exists an algorithm* FDH-INVERT *that will find inverses of random elements* $z_0 \in \{0,1\}^k$ *with probability at least* $(\epsilon - 2^{-k})/q_h$.