Bitcoin Signature

or

ECDSA on secp256k1

[or

ElGamal with SHA on EC]

S. Radziszowski spr@cs.rit.edu

Nov 7, 2017

[Nov 28, 2017]

ECDSA – secp256k1

EC – elliptic curve   cubic

F – field, $f(x,y)$ – polynomial

f – cubic in x, qudratic in y

E – set of (x,y), so $f(x,y)=0$

DSA – digital signature algorithm

as in NIST-DSS FIPS

1994, EC added in 2000

sec   Standards for Efficient Crypto

Certicom 2005, 2010

p256 $F = \mathbb{Z}_p$ for special 256-bit

prime p, $p \cong 2^{256}$

k   Koblitz

almost so, but OK

1   index (there is no 2, 3, ...)

Threads of this talk

① Signatures

② EC

③ special Bitcoin curve

④ security, no time ...

many sources:
textbooks
wiki
bitcoin developer guide
those missed will be listed
in the next version of slides

ECDSA

first try

Cryptosystem 7.5: *Elliptic Curve Digital Signature Algorithm*

Let p be a prime or a power of two, and let E be an elliptic curve defined over $\mathbb{F}_p$. Let A be a point on E having prime order q, such that the Discrete Logarithm problem in $\langle A \rangle$ is infeasible. Let $\mathcal{P} = \{0,1\}^*$, $\mathcal{A} = \mathbb{Z}_q^* \times \mathbb{Z}_q^*$, and define

$$\mathcal{K} = \{(p, q, E, A, m, B) : B = mA\},$$

where $0 \leq m \leq q-1$. The values p, q, E, A and B are the public key, and m is the private key.

For $K = (p, q, E, A, m, B)$, and for a (secret) random number k, $1 \leq k \leq q-1$, define

$$\mathrm{sig}_K(x, k) = (r, s),$$

where

$$\begin{aligned} kA &= (u, v) \\ r &= u \bmod q, \quad \text{and} \\ s &= k^{-1}(\text{SHA-1}(x) + mr) \bmod q. \end{aligned}$$

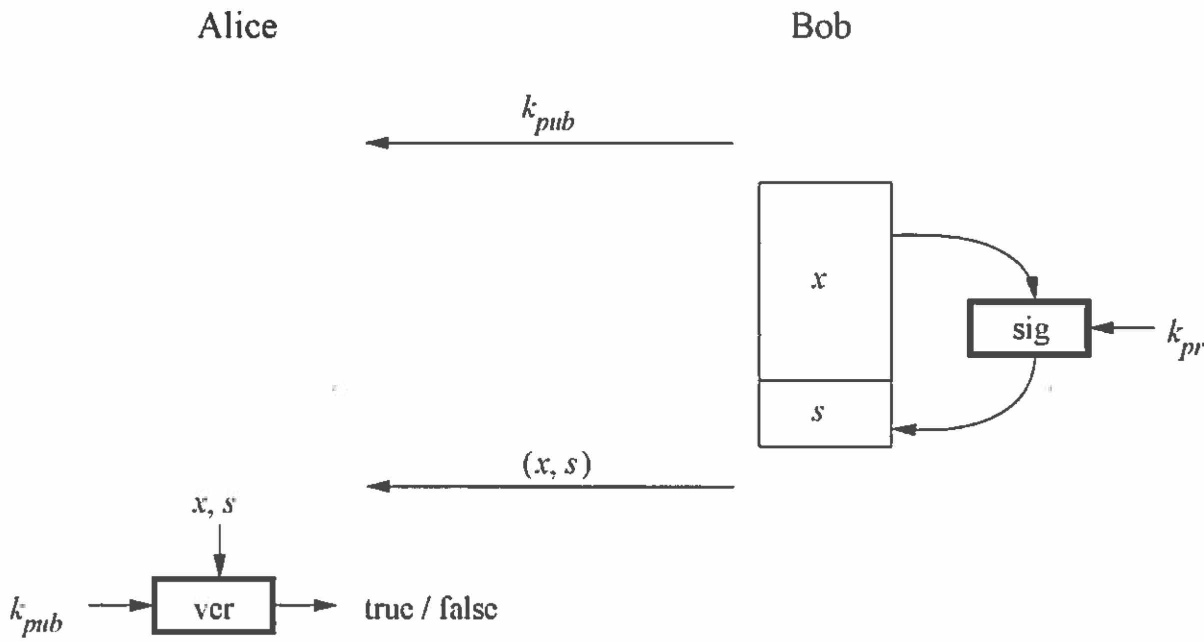
(If either $r = 0$ or $s = 0$, a new random value of k should be chosen.)

For $x \in \{0,1\}^*$ and $r, s \in \mathbb{Z}_q^*$, verification is done by performing the following computations:

$$\begin{aligned} w &= s^{-1} \bmod q \\ i &= w\,\text{SHA-1}(x) \bmod q \\ j &= wr \bmod q \\ (u, v) &= iA + jB \\ \mathrm{ver}_K(x, (r, s)) = \text{true} &\Leftrightarrow u \bmod q = r. \end{aligned}$$

... incomprehensible

Basic Principle of Digital Signatures



Stinson

Definition 7.1: A *signature scheme* is a five-tuple $(\mathcal{P}, \mathcal{A}, \mathcal{K}, \mathcal{S}, \mathcal{V})$, where the following conditions are satisfied:

1. $\mathcal{P}$ is a finite set of possible *messages*
2. $\mathcal{A}$ is a finite set of possible *signatures*
3. $\mathcal{K}$, the *keyspace*, is a finite set of possible *keys*
4. For each $K \in \mathcal{K}$, there is a *signing algorithm* $\mathsf{sig}_K \in \mathcal{S}$ and a corresponding *verification algorithm* $\mathsf{ver}_K \in \mathcal{V}$. Each $\mathsf{sig}_K : \mathcal{P} \to \mathcal{A}$ and $\mathsf{ver}_K : \mathcal{P} \times \mathcal{A} \to \{true, false\}$ are functions such that the following equation is satisfied for every message $x \in \mathcal{P}$ and for every signature $y \in \mathcal{A}$:
$$\mathsf{ver}(x, y) = \begin{cases} true & \text{if } y = \mathsf{sig}(x) \\ false & \text{if } y \neq \mathsf{sig}(x). \end{cases}$$

A pair (x, y) with $x \in \mathcal{P}$ and $y \in \mathcal{A}$ is called a *signed message*.

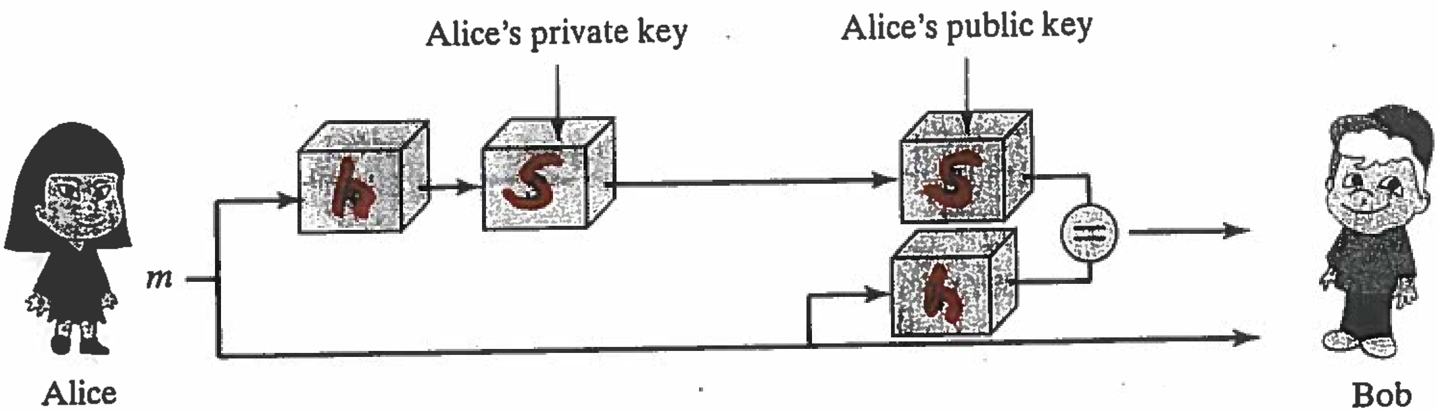
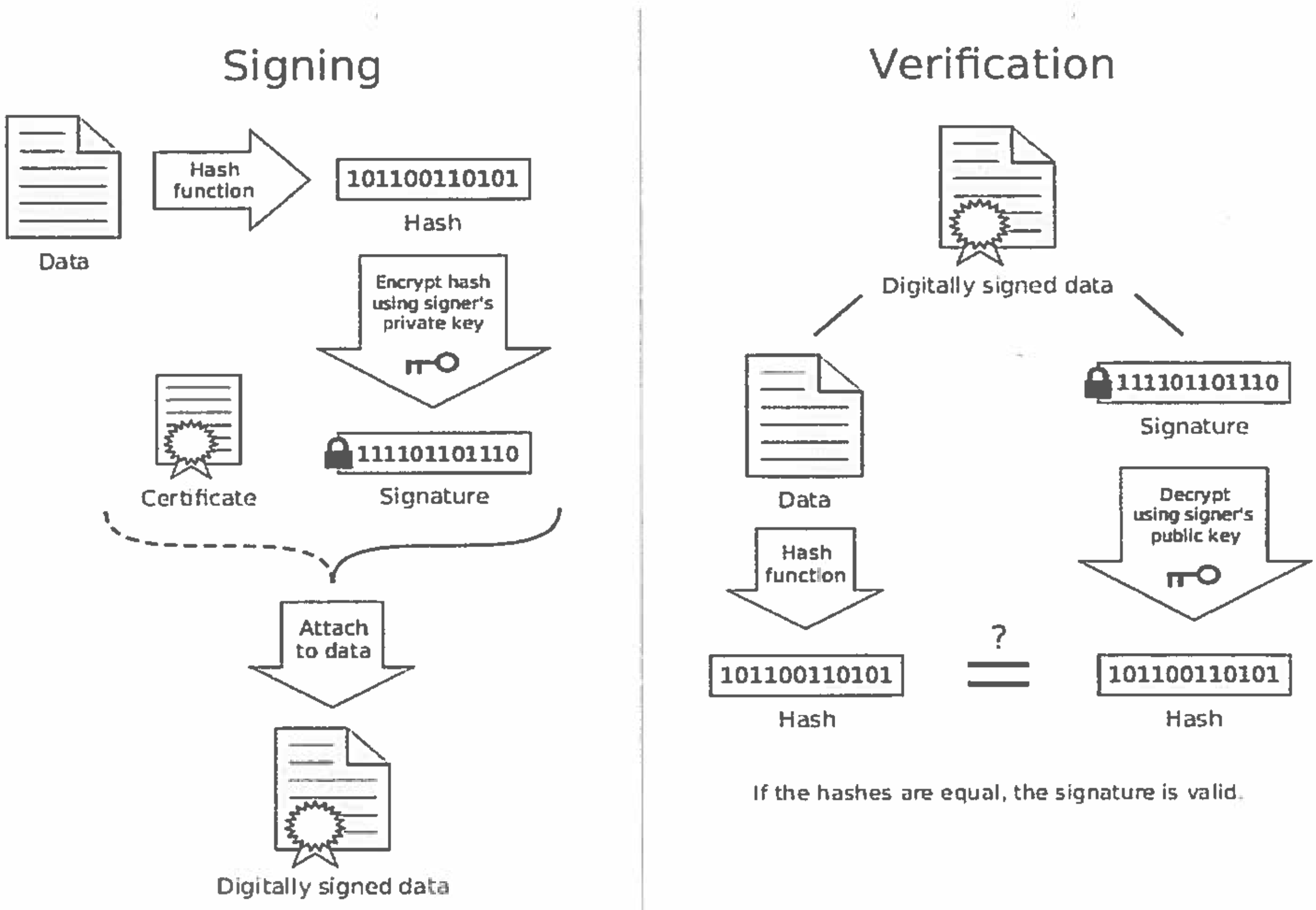


Figure 9.15: Using a digital signature

Public-key System in Use

signature by hash and public-key encryption



[Wikipedia]

1977 Rivest-Shamir-Adleman

Cryptosystem 7.1: *RSA Signature Scheme*

Let $n = pq$, where p and q are primes. Let $\mathcal{P} = \mathcal{A} = \mathbb{Z}_n$, and define

$$\mathcal{K} = \{(n, p, q, a, b) : n = pq, p, q \text{ prime}, ab \equiv 1 \pmod{\phi(n)}\}.$$

The values n and b are the public key, and the values p, q, a are the private key.

For $K = (n, p, q, a, b)$, define

$$\mathrm{sig}_K(x) = x^a \bmod n$$

and

$$\mathrm{ver}_K(x, y) = \text{true} \Leftrightarrow x \equiv y^b \pmod{n}$$

$(x, y \in \mathbb{Z}_n)$.

slow

generating n is expensive and cannot be shared by different users

1985

Cryptosystem 7.2: *ElGamal Signature Scheme*

Let p be a prime such that the discrete log problem in $\mathbb{Z}_p$ is intractable, and let $\alpha \in \mathbb{Z}_p{}^*$ be a primitive element. Let $\mathcal{P} = \mathbb{Z}_p{}^*$, $\mathcal{A} = \mathbb{Z}_p{}^* \times \mathbb{Z}_{p-1}$, and define

$$\mathcal{K} = \{(p, \alpha, a, \beta) : \beta \equiv \alpha^a \pmod{p}\}.$$

The values p, α and β are the public key, and a is the private key.

For $K = (p, \alpha, a, \beta)$, and for a (secret) random number $k \in \mathbb{Z}_{p-1}{}^*$, define

$$\mathrm{sig}_K(x, k) = (\gamma, \delta),$$

where

$$\gamma = \alpha^k \bmod p$$

and

$$\delta = (x - a\gamma)k^{-1} \bmod (p-1).$$

For $x, \gamma \in \mathbb{Z}_p{}^*$ and $\delta \in \mathbb{Z}_{p-1}$, define

$$\mathrm{ver}_K(x, (\gamma, \delta)) = \mathrm{true} \Leftrightarrow \beta^\gamma \gamma^\delta \equiv \alpha^x \pmod{p}.$$

$$x = k\delta + a\gamma$$

$$\alpha^x = \alpha^{a\gamma} \cdot \alpha^{k\delta} = \beta^\gamma \cdot \gamma^\delta$$

1. can be forged for special x
2. keep k secret

Cryptosystem 7.3: *Schnorr Signature Scheme*

Let p be a prime such that the discrete log problem in $\mathbb{Z}_p{}^*$ is intractable, and let q be a prime that divides $p-1$. Let $\alpha \in \mathbb{Z}_p{}^*$ be a qth root of 1 modulo p. Let $\mathcal{P} = \{0,1\}^*$, $\mathcal{A} = \mathbb{Z}_q \times \mathbb{Z}_q$, and define

$$\mathcal{K} = \{(p, q, \alpha, a, \beta) : \beta \equiv \alpha^a \pmod{p}\},$$

where $0 \leq a \leq q-1$. The values p, q, α and β are the public key, and a is the private key. Finally, let $h : \{0,1\}^* \rightarrow \mathbb{Z}_q$ be a secure hash function.

For $K = (p, q, \alpha, a, \beta)$, and for a (secret) random number k, $1 \leq k \leq q-1$, define

$$\mathsf{sig}_K(x, k) = (\gamma, \delta),$$

where

$$\gamma = h(x \parallel \alpha^k)$$

and

$$\delta = k + a\gamma \bmod q.$$

For $x \in \{0,1\}^*$ and $\gamma, \delta \in \mathbb{Z}_q$, verification is done by performing the following computations:

$$\mathsf{ver}_K(x, (\gamma, \delta)) = \mathsf{true} \Leftrightarrow h(x \parallel \alpha^\delta \beta^{-\gamma}) = \gamma.$$

1991 + NIST

Cryptosystem 7.4: *Digital Signature Algorithm*

Let p be a L-bit prime such that the discrete log problem in $\mathbb{Z}_p$ is intractable, where $L \equiv 0 \pmod{64}$ and $512 \leq L \leq 1024$, and let q be a 160-bit prime that divides $p-1$. Let $\alpha \in \mathbb{Z}_p{}^*$ be a qth root of 1 modulo p. Let $\mathcal{P} = \{0,1\}^*$, $\mathcal{A} = \mathbb{Z}_q{}^* \times \mathbb{Z}_q{}^*$, and define

$$\mathcal{K} = \{(p, q, \alpha, a, \beta) : \beta \equiv \alpha^a \pmod{p}\},$$

where $0 \leq a \leq q-1$. The values p, q, α and β are the public key, and a is the private key.

For $K = (p, q, \alpha, a, \beta)$, and for a (secret) random number k, $1 \leq k \leq q-1$, define

$$\mathrm{sig}_K(x, k) = (\gamma, \delta),$$

where

$$\gamma = (\alpha^k \bmod p) \bmod q \quad \text{and}$$
$$\delta = (\text{SHA-1}(x) + a\gamma)k^{-1} \bmod q.$$

(If $\gamma = 0$ or $\delta = 0$, a new random value of k should be chosen.)

For $x \in \{0,1\}^*$ and $\gamma, \delta \in \mathbb{Z}_q{}^*$, verification is done by performing the following computations:

$$e_1 = \text{SHA-1}(x)\,\delta^{-1} \bmod q$$
$$e_2 = \gamma\,\delta^{-1} \bmod q$$
$$\mathrm{ver}_K(x, (\gamma, \delta)) = \text{true} \Leftrightarrow (\alpha^{e_1}\beta^{e_2} \bmod p) \bmod q = \gamma.$$

October 2001
Nist recom. $p \cong 2^{1024}$

DSA

Key generation

Key generation has two phases. The first phase is a choice of *algorithm parameters* which may be shared between different users of the system, while the second phase computes public and private keys for a single user.

Parameter generation

2017

- Choose an approved cryptographic hash function H. In the original DSS, H was always SHA-1, but the stronger SHA-2 hash functions are approved for use in the current DSS.[5][9] The hash output may be truncated to the size of a key pair.
- Decide on a key length L and N. This is the primary measure of the cryptographic strength of the key. The original DSS constrained L to be a multiple of 64 between 512 and 1,024 (inclusive). NIST 800-57 recommends lengths of 2,048 (or 3,072) for keys with security lifetimes extending beyond 2010 (or 2030), using correspondingly longer N.[10] FIPS 186-3 specifies L and N length pairs of (1,024, 160), (2,048, 224), (2,048, 256), and (3,072, 256).[4] N must be less than or equal to the output length of the hash H.
- Choose an N-bit prime q.
- Choose an L-bit prime p such that $p - 1$ is a multiple of q.
- Choose g, a number whose multiplicative order modulo p is q. This may be done by setting $g = h^{(p-1)/q} \bmod p$ for some arbitrary h ($1 < h < p - 1$), and trying again with a different h if the result comes out as 1. Most choices of h will lead to a usable g; commonly $h = 2$ is used.

The algorithm parameters (p, q, g) may be shared between different users of the system.

Per-user keys

Given a set of parameters, the second phase computes private and public keys for a single user:

- Choose a secret key x by some random method, where $0 < x < q$.
- Calculate the public key $y = g^x \bmod p$.

There exist efficient algorithms for computing the modular exponentiations $h^{(p-1)/q} \bmod p$ and $g^x \bmod p$, such as exponentiation by squaring.

ECDSA second try

Cryptosystem 7.5: *Elliptic Curve Digital Signature Algorithm*

Let p be a prime or a power of two, and let E be an elliptic curve defined over $\mathbb{F}_p$. Let A be a point on E having prime order q, such that the Discrete Logarithm problem in $\langle A \rangle$ is infeasible. Let $\mathcal{P} = \{0,1\}^*$, $\mathcal{A} = \mathbb{Z}_q{}^* \times \mathbb{Z}_q{}^*$, and define

$$\mathcal{K} = \{(p, q, E, A, m, B) : B = mA\},$$

where $0 \leq m \leq q-1$. The values p, q, E, A and B are the public key, and m is the private key.

For $K = (p, q, E, A, m, B)$, and for a (secret) random number k, $1 \leq k \leq q-1$, define

$$\mathrm{sig}_K(x, k) = (r, s),$$

where

$$kA = (u, v)$$

$$r = u \bmod q, \quad \text{and}$$

$$s = k^{-1}(\text{SHA-1}(x) + mr) \bmod q.$$

(If either $r = 0$ or $s = 0$, a new random value of k should be chosen.)

For $x \in \{0,1\}^*$ and $r, s \in \mathbb{Z}_q{}^*$, verification is done by performing the following computations:

$$w = s^{-1} \bmod q$$

$$i = w\, \text{SHA-1}(x) \bmod q$$

$$j = wr \bmod q$$

$$(u, v) = iA + jB$$

$$\mathrm{ver}_K(x, (r, s)) = \text{true} \Leftrightarrow u \bmod q = r.$$

looks like DSA, but all messed up

The Generalized Discrete Logarithm Problem

- Given is a finite cyclic group G with the group operation $\circ$ and cardinality n.
- We consider a primitive element $\alpha \in G$ and another element $\beta \in G$.
- The discrete logarithm problem is finding the integer x, where $1 \leq x \leq n$, such that:

$$\beta = \underbrace{\alpha \circ \alpha \circ \alpha \circ \ldots \circ \alpha}_{x \text{ times}} = \alpha^x$$

or, in additive notation

x int, $\alpha, \beta \in G$

$$\beta = \alpha + \alpha + \ldots + \alpha$$
$$= x\alpha$$

$x, \alpha \longrightarrow x\alpha, \beta$ easy

$\alpha, \beta \longrightarrow x$ infeasible to compute

The discrete logarithm problem in $\mathbb{Z}_p$

Problem Instance	$I = (p, \alpha, \beta)$, where p is prime, $\alpha \in \mathbb{Z}_p$ is a primitive element, and $\beta \in \mathbb{Z}_p{}^*$.
Objective	Find the unique integer a, $0 \le a \le p-2$, such that $$\alpha^a \equiv \beta \pmod{p}.$$ We will denote this integer a by $\log_\alpha \beta$.

ECDL analog

$I = (E, P, Q)$

E elliptic curve

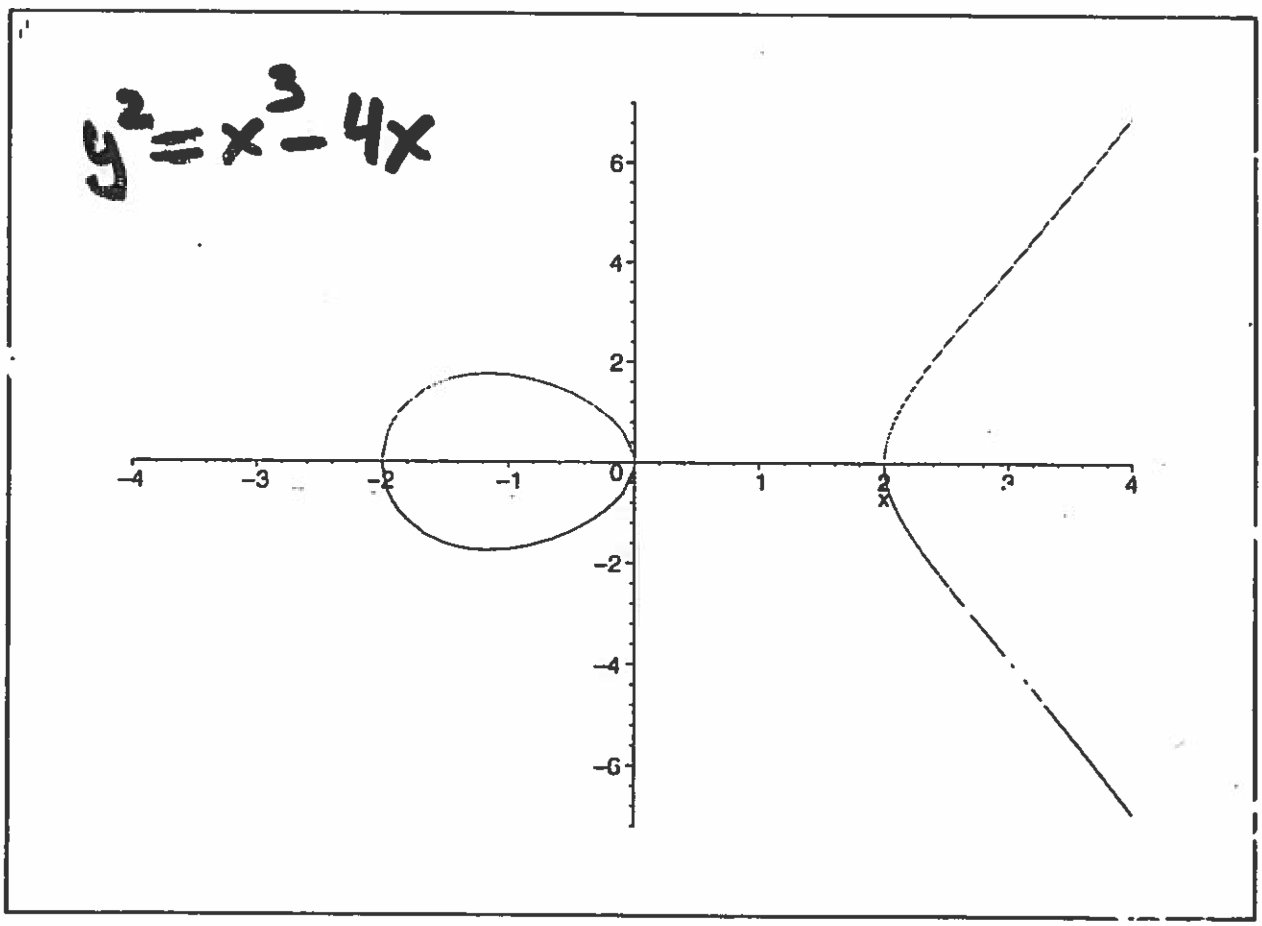
$P, Q \in E$, points

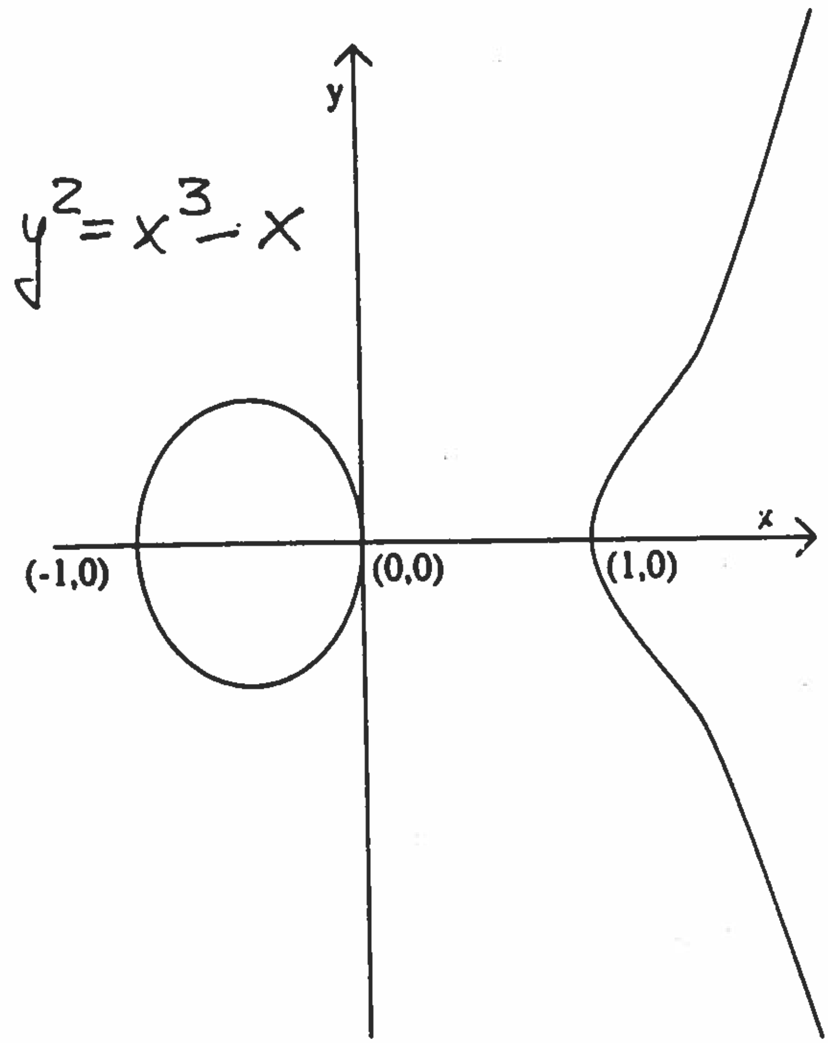
Find k such that $Q = kP$

k integer

Elliptic Curves over the Reals

> **Definition 6.3:** Let $a, b \in \mathbb{R}$ be constants such that $4a^3 + 27b^2 \neq 0$. A *non-singular elliptic curve* is the set E of solutions $(x, y) \in \mathbb{R} \times \mathbb{R}$ to the equation
> $$y^2 = x^3 + ax + b, \tag{6.4}$$
> together with a special point $\mathcal{O}$ called the *point at infinity*.




y
$y^2 = x^3 - x$
x
(-1,0)
(0,0)
(1,0)

Computations on Elliptic Curves (ctd.)

- In cryptography, we are interested in elliptic curves module a prime p:

Definition: Elliptic Curves over prime fields

The elliptic curve over Z_p, $p>3$ is the set of all pairs $(x,y) \in Z_p$ which fulfill

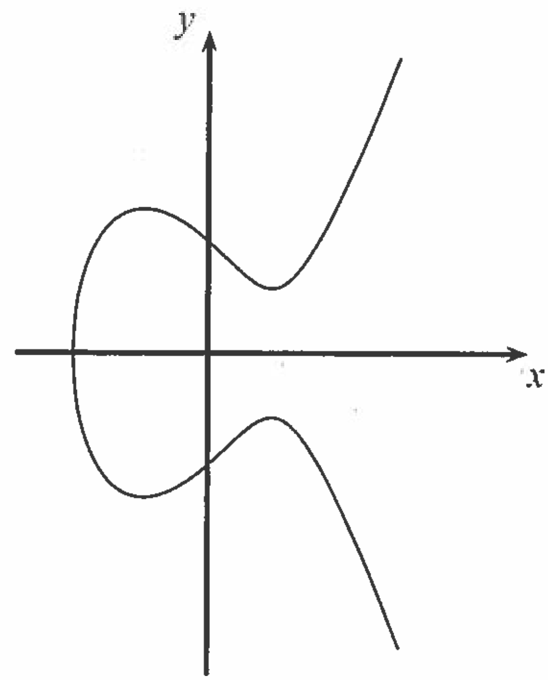
$$y^2 = x^3 + ax + b \bmod p$$

together with an imaginary point of infinity θ, where $a,b \in Z_p$ and the condition

$$4a^3+27b^2 \neq 0 \bmod p.$$

- Note that $Z_p = \{0,1,\dots, p-1\}$ is a set of integers with modulo p arithmetic

∈



$$
\begin{tikzcd}
4a^3 + 27b^2 \arrow[r, "=0"] \arrow[d, "\neq 0"] & \text{singular EC} \\
\text{6.4 has 3 different roots in } \mathbb{C} &
\end{tikzcd}
$$

Defining $P+Q$

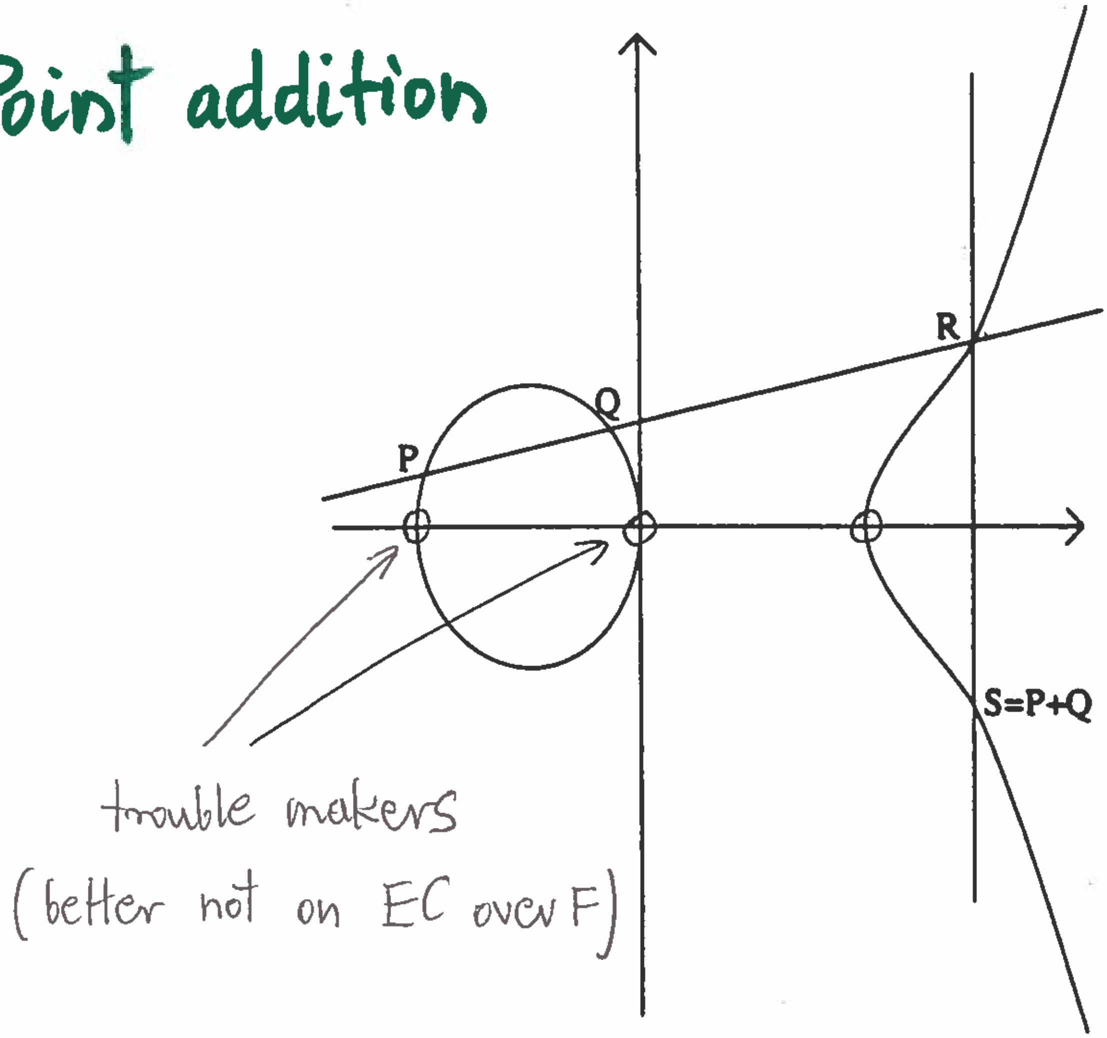
Suppose $P, Q \in E$, where $P = (x_1, y_1)$ and $Q = (x_2, y_2)$. We consider three cases:

1. $x_1 \neq x_2$
2. $x_1 = x_2$ and $y_1 = -y_2$
3. $x_1 = x_2$ and $y_1 = y_2$

In case 1, we define L to be the line through P and Q. L intersects E in the two points P and Q, and it is easy to see that L will intersect E in one further point, which we call R'. If we reflect R' in the x-axis, then we get a point which we name R. We define $P + Q = R$.

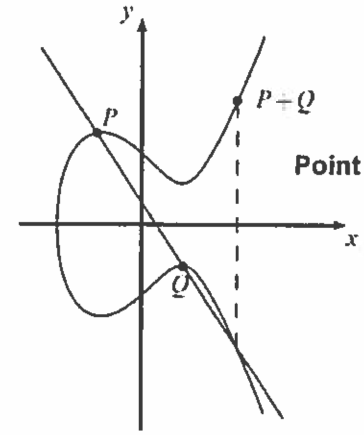
O – infinity, $P + O = O + P = P$

Point addition



Computations on Elliptic Curves (ctd.)

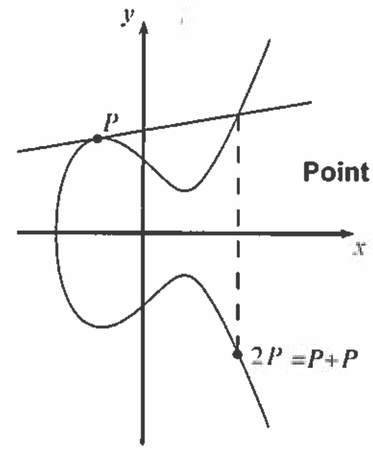
- Generating a *group of points* on elliptic curves based on point addition operation $P+Q = R$, *i.e.*, $(x_P, y_P)+(x_Q, y_Q) = (x_R, y_R)$
- Geometric Interpretation of point addition operation
  - *Draw straight line through P and Q; if P=Q use tangent line instead*
  - *Mirror third intersection point of drawn line with the elliptic curve along the x-axis*
- Elliptic Curve Point Addition and Doubling Formulas



$x_3 = s^2 - x_1 - x_2 \bmod p$ and $y_3 = s(x_1 - x_3) - y_1 \bmod p$

where

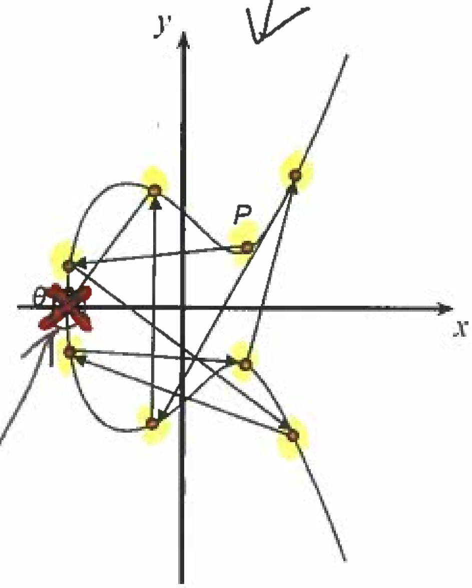
$$s = \begin{cases} \dfrac{y_2 - y_1}{x_2 - x_1} \bmod p & \text{; if } P \neq Q \text{ (point addition)} \\ \dfrac{3x_1^2 + a}{2y_1} \bmod p & \text{; if } P = Q \text{ (point doubling)} \end{cases}$$



Computations on Elliptic Curves (ctd.) $y^2 = x^3 + 2x + 2$

- The points on an elliptic curve and the point at infinity θ form cyclic subgroups

$2P = (5,1)+(5,1) = (6,3)$
$3P = 2P+P = (10,6)$
$4P = (3,1)$
$5P = (9,16)$
$6P = (16,13)$
$7P = (0,6)$
$8P = (13,7)$
$9P = (7,6)$
$10P = (7,11)$
$11P = (13,10)$
$12P = (0,11)$ $\neq$
$13P = (16,4)$
$14P = (9,1)$
$15P = (3,16)$
$16P = (10,11)$
$17P = (6,14)$
$18P = (5,16)$
$19P = \theta$



This elliptic curve has order $\#E = |E| = 19$ since it contains 19 points in its cyclic group.

$P = (5,1)$

better draw it at ∞

Z_{11}

Example 6.7 Let E be the elliptic curve $y^2 = x^3 + x + 6$ over $\mathbb{Z}_{11}$.

$11 \equiv 3 \bmod 4$

$$\pm z^{(11+1)/4} \bmod 11 = \pm z^3 \bmod 11. = \sqrt{z} \bmod 11$$

in action in secp256k1

x	$x^3 + x + 6 \bmod 11$	quadratic residue?	y
0	6	no	
1	8	no	
2	5	yes	4, 7
3	3	yes	5, 6
4	8	no	
5	4	yes	2, 9
6	8	no	
7	4	yes	2, 9
8	9	yes	3, 8
9	7	no	
10	4	yes	2, 9

$$\begin{array}{lll} \alpha = (2,7) & 2\alpha = (5,2) & 3\alpha = (8,3) \\ 4\alpha = (10,2) & 5\alpha = (3,6) & 6\alpha = (7,9) \\ 7\alpha = (7,2) & 8\alpha = (3,5) & 9\alpha = (10,9) \\ 10\alpha = (8,8) & 11\alpha = (5,9) & 12\alpha = (2,4) \end{array}$$

$\text{POINTCOMPRESS}(P) = (x, y \bmod 2)$, where $P = (x, y) \in E$.

$\text{POINTCOMPRESS} : E \backslash \{\mathcal{O}\} \to \mathbb{Z}_p \times \mathbb{Z}_2$,

Algorithm 6.4: $\text{POINTDECOMPRESS}(x, i)$

$z \leftarrow x^3 + ax + b \bmod p$
if z is a quadratic non-residue modulo p
  then return ("failure")
  else $\begin{cases} y \leftarrow \sqrt{z} \bmod p \\ \textbf{if } y \equiv i \pmod 2 \\ \quad \textbf{then return } (x, y) \\ \quad \textbf{else return } (x, p - y) \end{cases}$

HP: US patent 6252960 B1 1998
expires in 2018

130+ crypto and EC patents:
NSA, Certicom, RSA Security, HP, Harris

ECDSA

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For $x \in \{0,1\}^*$ and $r, s \in \mathbb{Z}_q{}^*$, verification is done by performing the following computations:

$$w = s^{-1} \bmod q$$

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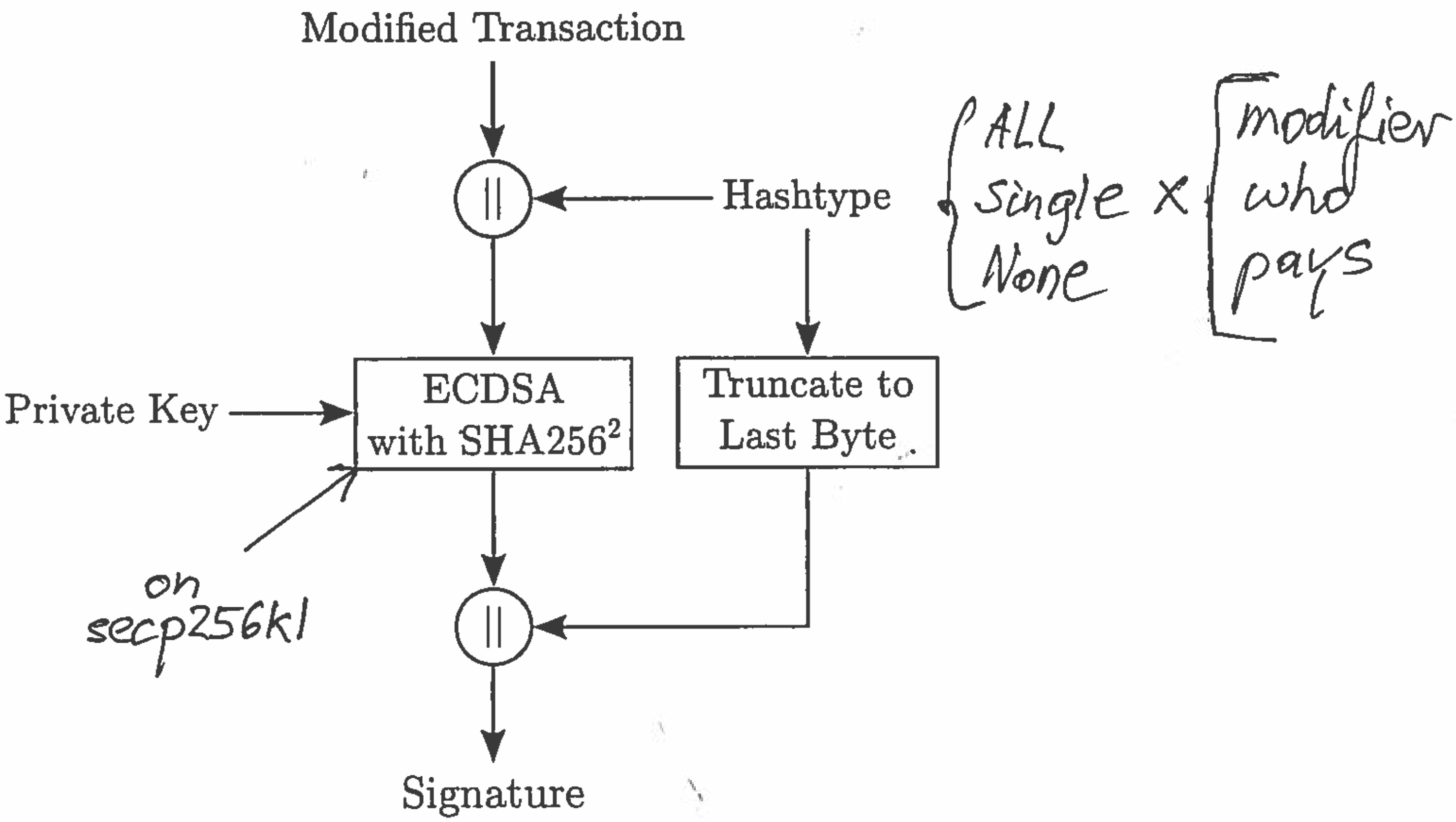
$$j = wr \bmod q$$

$$(u, v) = iA + jB$$

$$\mathrm{ver}_K(x, (r, s)) = \text{true} \Leftrightarrow u \bmod q = r.$$


Modified Transaction
Hashtype
ALL
Single
None
x
modifier
who
pays
Private Key
ECDSA
with SHA256²
Truncate to
Last Byte
on
secp256k1
Signature

Properties of Elliptic Curves

Hasse bound

$$p + 1 - 2\sqrt{p} \leq \#E \leq p + 1 + 2\sqrt{p}.$$

Schoof algorithm
$O(\log^8 p)$ - bit ops

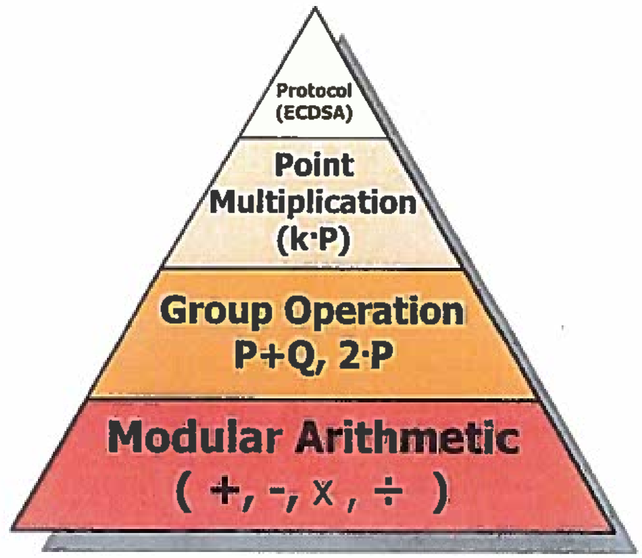
THEOREM 6.1 *Let E be an elliptic curve defined over $\mathbb{Z}_p$, where p is prime and $p > 3$. Then there exist positive integers n_1 and n_2 such that $(E, +)$ is isomorphic to $\mathbb{Z}_{n_1} \times \mathbb{Z}_{n_2}$. Further, $n_2 \mid n_1$ and $n_2 \mid (p-1)$.*

cyclic subgroup of E
of order 2^{160} is "safe"

$n_2 = 1$ iff E cyclic

Implementations in Hardware and Software

- Elliptic curve computations usually regarded as consisting of four layers:
  - Basic modular arithmetic operations are computationally most expensive
  - Group operation implements point doubling and point addition
  - Point multiplication can be implemented using the Double-and-Add method
  - Upper layer protocols like ECDH and ECDSA



- Most efforts should go in optimizations of the modular arithmetic operations, such as
  - Modular addition and subtraction
  - Modular multiplication
  - Modular inversion

two NIST
Koblitz curves
in binary Galois fields

K163

```
p(t) = t^163 + t^7 + t^6 + t^3 + 1
     = 800000000000000000000000000000000000000C9
a    = 1
G_x  = 2fe13c0537bbc11acaa07d793de4e6d5e5c94eee8
G_y  = 289070fb05d38ff58321f2e800536d538ccdaa3d9
n    = 5846006549323611672814741753598448348329118574063
h    = 2
```

K233

```
p(t) = t^233 + t^74 + 1
     = 20000000000000000000000000000000000000004000000000000000001
a    = 0
G_x  = 17232ba853a7e731af129f22ff4149563a419c26bf50a4c9d6eefad6126
G_y  = 1db537dece819b7f70f555a67c427a8cd9bf18aeb9b56e0c11056fae6a3
n    = 3450873173395281893717377931138512760570940988862252126328087024741
h    = 4
```

Standards for Efficient Cryptography

SEC 2: Recommended Elliptic Curve Domain Parameters

Certicom Research

Contact: Daniel R. L. Brown (dbrown@certicom.com)

January 27, 2010

Version 2.0

bits

index

Secp256k1

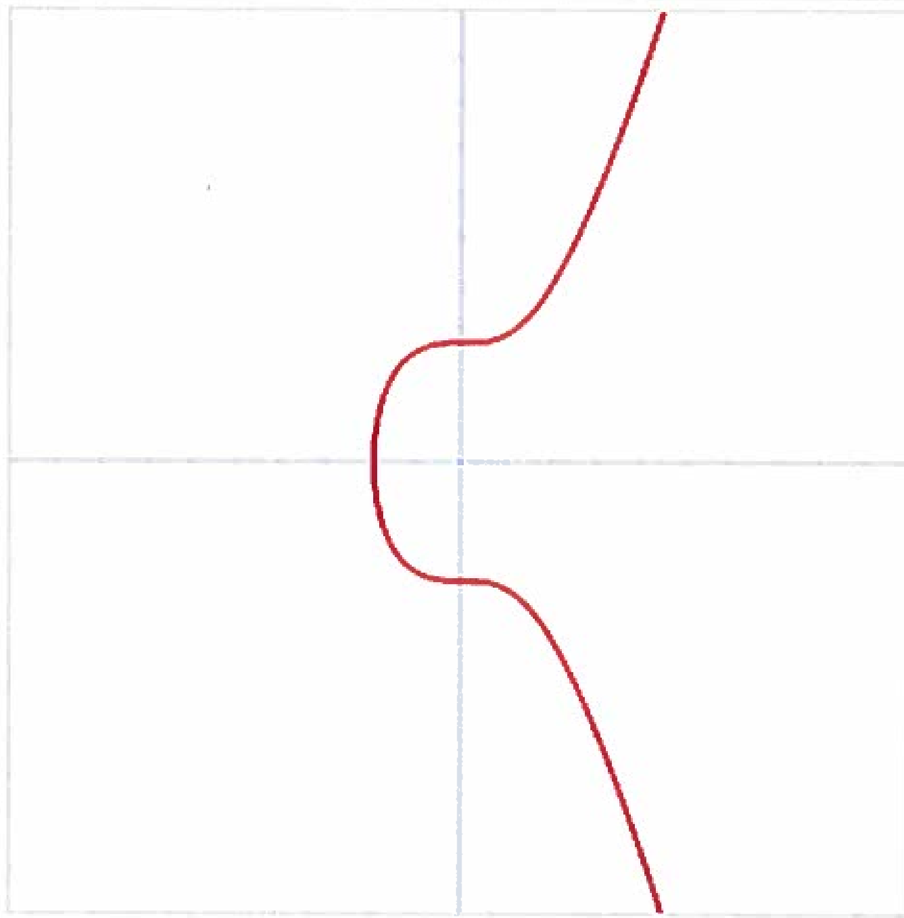
prime

Köblitz
(r verifiably random)

From Bitcoin Wiki

secp256k1 refers to the parameters of the ECDSA curve used in Bitcoin, and is defined in *Standards for Efficient Cryptography (SEC)* (Certicom Research, http://www.secg.org/sec2-v2.pdf).

→ NIST



This is a graph of secp256k1's elliptic curve $y^2 = x^3 + 7$ over the real numbers. Note that because secp256k1 is actually defined over the field Z_p, its graph will in reality look like random scattered points, not anything like this.

Parameters	Section	Strength	Size	RSA/DSA	Koblitz or random
secp192k1	2.2.1	96	192	1536	k
secp192r1	2.2.2	96	192	1536	r
secp224k1	2.3.1	112	224	2048	k
secp224r1	2.3.2	112	224	2048	r
secp256k1	2.4.1	128	256	3072	k
secp256r1	2.4.2	128	256	3072	r
secp384r1	2.5.1	192	384	7680	r
secp521r1	2.6.1	256	521	15360	r

Table 1: Properties of Recommended Elliptic Curve Domain Parameters over $\mathbb{F}_p$

2.4.1 Recommended Parameters secp256k1

(the same as in 2000)

The elliptic curve domain parameters over $\mathbb{F}_p$ associated with a Koblitz curve secp256k1 are specified by the sextuple $T = (p, a, b, G, n, h)$ where the finite field $\mathbb{F}_p$ is defined by:

p = FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFE FFFFFC2F

$= 2^{256} - 2^{32} - 2^9 - 2^8 - 2^7 - 2^6 - 2^4 - 1$

The curve E: $y^2 = x^3 + ax + b$ over $\mathbb{F}_p$ is defined by:

a = 00000000 00000000 00000000 00000000 00000000 00000000 00000000 00000000

b = 00000000 00000000 00000000 00000000 00000000 00000000 00000000 00000007

The base point G in compressed form is:

G = 02 79BE667E F9DCBBAC 55A06295 CE870B07 029BFCDB 2DCE28D9 59F2815B 16F81798

and in uncompressed form is:

G = 04 79BE667E F9DCBBAC 55A06295 CE870B07 029BFCDB 2DCE28D9 59F2815B 16F81798 483ADA77 26A3C465 5DA4FBFC 0E1108A8 FD17B448 A6855419 9C47D08F FB10D4B8

Finally the order n of G and the cofactor are:

n = FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFE BAAEDCE6 AF48A03B BFD25E8C D0364141

h = 01

My questions

1. What is current #0s in POW?

   // around 70

2. Why not single SHA256?

   // like HMAC

3. Domains for private and public keys are close but not the same.

   uncompressed private keys make no sense

   p and |<G>| are distinct and have different roles

4. Is nonce k in ElGamal "child private key"?

5. Docs say

   private_key = SHA256(minikey)

   see 3 above, looks incorrect

POW-based mining

	BTC	reward	
2009	0	50	
2012	10M	25	
2017	17M	12.5	10 ExHa per second $10^{19} \simeq 2^{64}$
2018		6.25	mining revolution
soon			
ever	21M		limit

Provably Secure Signature Schemes

One-time Signatures

Winternitz OTS
used in IOTA

Cryptosystem 7.6: *Lamport Signature Scheme*

Let k be a positive integer and let $\mathcal{P} = \{0,1\}^k$. Suppose $f : Y \rightarrow Z$ is a one-way function, and let $\mathcal{A} = Y^k$. Let $y_{i,j} \in Y$ be chosen at random, $1 \leq i \leq k$, $j = 0, 1$, and let $z_{i,j} = f(y_{i,j})$, $1 \leq i \leq k$, $j = 0, 1$. The key K consists of the $2k$ y's and the $2k$ z's. The y's are the private key while the z's are the public key.

For $K = (y_{i,j}, z_{i,j} : 1 \leq i \leq k, j = 0, 1)$, define

$$\mathsf{sig}_K(x_1, \ldots, x_k) = (y_{1,x_1}, \ldots, y_{k,x_k}).$$

A signature $(a_1, \ldots, a_k)$ on the message $(x_1, \ldots, x_k)$ is verified as follows:

$$\mathsf{ver}_K((x_1, \ldots, x_k), (a_1, \ldots, a_k)) = \text{true} \Leftrightarrow f(a_i) = z_{i,x_i}, 1 \leq i \leq k.$$

Example 7.6 7879 is prime and 3 is a primitive element in $\mathbb{Z}_{7879}{}^*$. Define

$$f(x) = 3^x \bmod 7879.$$

Suppose $k = 3$, and Alice chooses the six (secret) random numbers

$$\begin{array}{lcl}
y_{1,0} = 5831 & & z_{1,0} = 2009 \\
y_{1,1} = 735 & & z_{1,1} = 3810 \\
y_{2,0} = 803 & \overset{f}{\Longrightarrow} & z_{2,0} = 4672 \\
y_{2,1} = 2467 & & z_{2,1} = 4721 \\
y_{3,0} = 4285 & & z_{3,0} = 268 \\
y_{3,1} = 6449. & & z_{3,1} = 5731.
\end{array}$$

These z's are published. Now, suppose Alice wants to sign the message

$$x = (1, 1, 0).$$

The signature for x is

$$(y_{1,1}, y_{2,1}, y_{3,0}) = (735, 2467, 4285).$$

To verify this signature, it suffices to compute the following:

$$\begin{aligned}
3^{735} \bmod 7879 &= 3810 \\
3^{2467} \bmod 7879 &= 4721 \\
3^{4285} \bmod 7879 &= 268.
\end{aligned}$$

Hence, the signature is verified.