CSCI.761 Assignment 03

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1. List all the elements of automorphism groups of two $(3, 4; 8)$-graphs (Graph 2 and Graph 3 on the page). Make two listings of permutations for each: as mappings and in cycle notation (thus, 4 listings in total).

   Automorphisms of Graph 2:

   Mapping:

   0 1 2 3 4 5 6 7
   0 1 3 2 4 6 5 7
   1 0 2 3 7 5 6 3
   1 0 3 2 7 6 5 4
   2 3 0 1 5 4 7 6
   3 2 0 1 6 4 7 5
   2 3 1 0 5 7 4 6
   3 2 1 0 6 7 4 5

   Cycle notation:

   (1) (2 3) (5 6)
   (0 1) (4 7)
   (0 1) (2 3) (4 7) (5 6)
   (0 2) (1 3) (4 5) (6 7)
   (0 3 1 2) (4 6 7 5)
   (0 2 1 3) (4 5 7 6)
   (0 3) (1 2) (4 6) (5 7)

   Automorphisms of Graph 3:

   Mapping:

   0 1 2 3 4 5 6 7
   0 2 1 3 5 4 7 6
   1 0 4 6 2 7 3 5
   1 4 0 6 7 2 5 3
   2 5 0 7 6 1 4 3
   4 7 1 5 3 0 2 6
   5 6 2 4 3 0 1 7
   7 3 4 2 6 1 0 5
   6 3 5 1 7 2 0 4
   3 6 7 0 5 4 1 2
   3 7 6 0 4 5 2 1
   6 5 3 1 2 7 4 0
   7 4 3 2 1 6 5 0
   5 2 6 4 0 3 7 1
   4 1 7 5 0 3 6 2
   2 0 5 7 1 6 3 4

   Cycle notation:

   (1)
   (1 2)(4 5)(6 7)
   (0 1)(2 4)(3 6)(5 7)
   (0 1 4 7 3 6 5 2)

(0 2) (1 5) (3 7) (4 6)
(0 4 3 5) (1 7 6 2)
(0 5) (1 6) (3 4)
(0 7 5 1 3 2 4 6)
(0 6) (1 3) (2 5) (4 7)
(0 3) (1 6) (2 7) (4 5)
(0 3) (1 7) (2 6)
(0 6 4 2 3 1 5 7)
(0 7) (1 4) (2 3) (6 5)
(0 5 3 4) (1 2 6 7)
(0 4) (2 7) (3 5)
(0 2 5 1) (3 7 4 6)

2. For classical two-color Ramsey numbers $R(s,t)$, it holds that $R(s,t) \leq R(s,t-1) + R(s-1,t)$ for all $r, s \geq 3$. Furthermore, this inequality is strict if both $R(s,t-1)$ and $R(s-1,t)$ are even. Using this, $R(s,t) = R(t,s)$, and $R(s,2) = s$, derive the best upper bounds you can obtain for $R(s,t)$, for all $3 \leq s \leq t \leq 10$. Present the bounds in a table. Mark the entries for which you used the clause "if both $R(s,t-1)$ and $R(s-1,t)$ are even".

s\t	3	4	5	6	7	8	9	10
3	6	9	14	19	26	33	42	51
4		18	31	50	75	108	149	200
5			62	111	186	293	442	641
6				222	407	700	1141	1782
7					814	1513	2654	4435
8						3026	5679	10114
9							11358	21471
10								42942