# Vertex Ramsey arrowing and $\chi$

V

## Theorem (Nenov 2001, Lin 1972, others)

$$m = 1 + \sum_{i=1}^{r} (a_i - 1)$$

If $G \longrightarrow (a_1, \cdots, a_r)^v$ then $\chi(G) \geq m$

## Proof

For contradiction, suppose $\chi(G) \leq \sum_{i=1}^{r} (a_i - 1)$.

We will $r$-partition $V(G)$ so that color $i$ (part $i$) doesn't span $K_{a_i}$ in $G$

$c: V \longrightarrow \{1, \dots, \chi(G)\}$, an optimal coloring

$C_i = \{v \in V : c(v) = i\}$ is ind-set for color $i$

$V(G)$ partition

part 1: colors $1 \dots a_1 - 1$, $\bigcup_{i=1}^{a_1-1} C_i$

part 2: colors $a_1, \dots, a_2 + a_1 - 2$, $\bigcup_{i=a_1}^{a_2+a_1-2} C_i$

...

part $r$: last $a_r - 1$ colors, $\bigcup_{\text{last } i}^{\chi} C_i$

Claim: No part $i$ can span $K_{a_i}$

$\square$

Example: $G \to (3,3)^v \Rightarrow \chi(G) \geq 5$, or 4-chromatic $G$ can be $\Delta$-free 2-partitioned