Exploring srg(45,22,10,11) and implications for the Ramsey number R(5,5)

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Dedication

We sincerely dedicate our work to the life of Natasha Parker, without whom this project would not exist. While we never had the honor of meeting Natasha, we feel, through our peers and mentors, the profound impact she had on those around her. May her memory be a blessing.

We are deeply grateful to Natasha Parker's family, whose generosity and support made our attendance at this conference possible.

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Two-Color Ramsey Numbers

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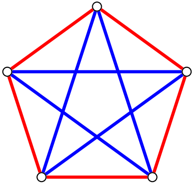
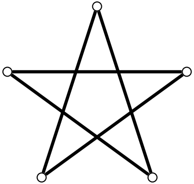


2-coloring of K_5



simple graph (blue kept as edges)

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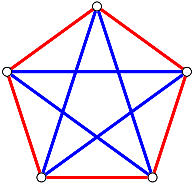
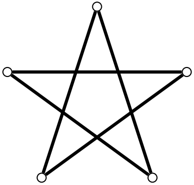


no blue K_3 and no red K_3



no K_3 and no I_3

Therefore this is an $R(3,3)$-good graph on 5 vertices, so it **witnesses** $R(3,3) > 5$ (in fact $R(3,3) = 6$).

Strongly Regular Graphs

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Regularity

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A k-regular graph on v vertices is **strongly regular** if every pair has

- exactly λ common neighbors when adjacent,
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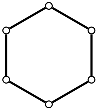
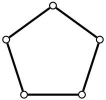


Regular, not SRG:

C_6 (2-regular; μ varies)



Strongly regular:

$C_5 = \mathrm{srg}(5, 2, 0, 1)$

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Motivating the Search

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A Compelling Pattern

The classic diagonal Ramsey numbers are witnessed by strongly regular graphs on exactly $R(k,k) - 1$ vertices:

- $R(3,3) = 6$ is witnessed by $\mathrm{srg}(5,2,0,1)$.
- $R(4,4) = 18$ is witnessed by $\mathrm{srg}(17,8,3,4)$.

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The Target: $v=45$

Within $43 \le R(5,5) \le 46$, the only viable parameter set that could be $R(5,5)$-good is:

$$\mathrm{srg}(45,22,10,11).$$

Identifying such a graph would constructively prove $R(5,5)=46$.

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Goal: find an $R(5,5)$-good graph in $\mathrm{srg}(45,22,10,11)$

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Constraint in CNF		**DIMACS (plain text)**

$(x_1 \vee \overline{x}_2 \vee x_3) \;\wedge\; (\overline{x}_1 \vee x_4)$	$\Rightarrow$	`p cnf 4 2`
		`1 -2 3 0`
		`-1 4 0`

Encoding idea: translate "graph properties" into Boolean variables and CNF clauses (constraints).

Harnessing SAT Solvers

SAT Solver Example

$$(x_1 \vee x_2 \vee x_3) \wedge (\overline{x}_1 \vee \overline{x}_3) \wedge (x_2)$$

SAT solver output: **satisfiable!**

$x_1 = 0,\ x_2 = 1, x_3 = 0$

$$(x_1 \vee x_2 \vee x_3) \wedge (\overline{x}_2) \wedge (x_2)$$

SAT solver output: **unsatisfiable!**

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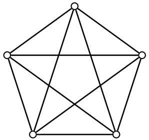
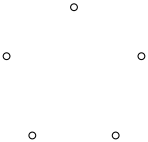


K_5: all pairs are edges



I_5: no pairs are edges

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$$\bigvee_{i,j \in S,\ i<j} e_{ij}.$$

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Prevent an independent set I_5 in S

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Prevent a clique K_5 in S

At least one non-edge must appear inside S:

$$\bigvee_{i,j \in S,\ i<j} \neg e_{ij}.$$

Since $v = 45$, repeat this pair of clauses $\binom{45}{5}$ times.

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Encoding "exactly 22" for vertex i

Require **at least 22 incident edges** and **at least 22 non-edges**:

$$\mathtt{k}\ 22\quad e_{i1}\ \ e_{i2}\ \ \cdots\ \ e_{i,44}$$
$$\mathtt{k}\ 22\quad \neg e_{i1}\ \ \neg e_{i2}\ \ \cdots\ \ \neg e_{i,44}$$

("≥ 22 true" and "≥ 22 false" $\Rightarrow$ exactly 22 true.)

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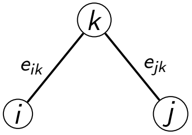
$$v_{ijk} \iff (e_{ik} \wedge e_{jk}),$$

so $v_{ijk} = 1$ exactly when k is adjacent to *both* i and j.



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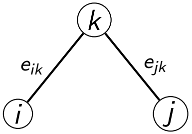

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Key point: Fix a pair $i < j$. There are **43** remaining vertices $k \neq i, j$. Each v_{ijk} flags whether k is a *common neighbor* of i and j, so

$$\sum_{k \neq i,j} v_{ijk} \quad \text{is the number of common neighbors of } i \text{ and } j.$$

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Strong regularity in $\mathrm{srg}(45, 22, 10, 11)$ means:

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The key trick: one equation handles both cases

We enforce

$$e_{ij} + \sum_{k \neq i,j} v_{ijk} = 11.$$

- If $e_{ij} = 1$ (adjacent), the sum of v_{ijk} must be 10.
- If $e_{ij} = 0$ (non-adjacent), the sum of v_{ijk} must be 11.

That matches $\lambda = 10$ and $\mu = 11$ exactly.

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KNF form

We write exactly 11 of the variables e_{ij} plus the 43 values v_{ijk}:

$$\mathtt{k}\ 11 \quad e_{ij} \quad v_{ij,k_1} \quad v_{ij,k_2} \quad \cdots \quad v_{ij,k_{43}}$$

$$\mathtt{k}\ 33 \quad \neg e_{ij} \quad \neg v_{ij,k_1} \quad \neg v_{ij,k_2} \quad \cdots \quad \neg v_{ij,k_{43}}$$

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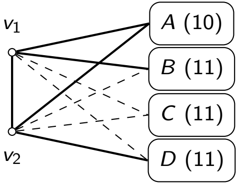

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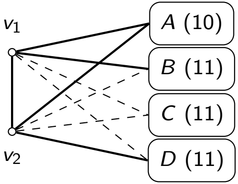
Anchor edge. Set $e_{1,2} = 1$ and distinguish $v_1 \sim v_2$.

The remaining vertices split into four **clusters** forced by $(k, \lambda) = (22, 10)$:

$$\begin{aligned} A &: \quad \text{adjacent to both } v_1, v_2 & (10) \\ B &: \quad \text{adjacent to } v_1 \text{ not } v_2 & (11) \\ D &: \quad \text{adjacent to } v_2 \text{ not } v_1 & (11) \\ C &: \quad \text{adjacent to neither } v_1 \text{ nor } v_2 & (11) \end{aligned}$$



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Canonical Form

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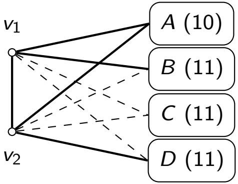
The benefit: A is constrained

Every vertex in A sees both v_1 and v_2. Any triangle inside A would form a K_5 with $\{v_1, v_2\}$.



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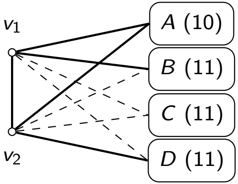

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Enumerating A

So the subgraph induced by A must be $R(3, 5)$-**good**. There are exactly **313 non-isomorphic** such graphs on 10 vertices, so we split into **313 cases**.

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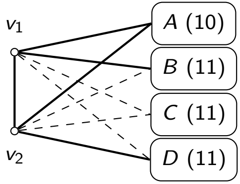

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What remains free

Beyond the forced adjacencies in the diagram (and the chosen case for A), all other edges **within and between** the clusters A, B, C, D are free variables.

Symmetry Breaking

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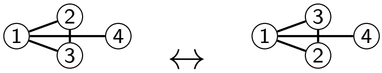
What is symmetry?

A symmetry is a relabeling of vertices that preserves all edges/non-edges. To a SAT solver, that relabeling changes variable names (like e_{ij}), so it can look like a new case.



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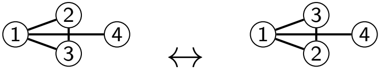

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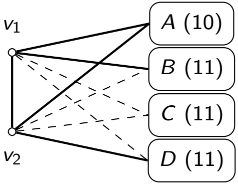
Where it appears here. After fixing the canonical form, the vertices *within* each cluster B, C, and D are still interchangeable: permuting their labels produces an isomorphic instance.



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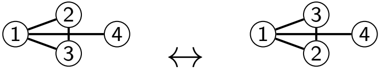

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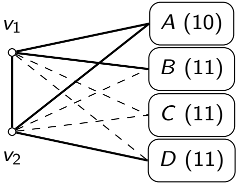

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That is up to $(11!)^3 \approx 6 \times 10^{22}$ redundant labelings of the same structure.

Symmetry Breaking

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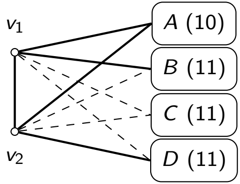
Adjacency vectors

With A fixed, each vertex $x \in B \cup C \cup D$ has a 10-bit adjacency vector to A:

$$\sigma_A(x) := (e_{x,a_1}, \ldots, e_{x,a_{10}}) \in \{0,1\}^{10}.$$

These vectors let us compare vertices by structure (not by name).



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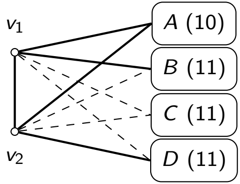

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Lex-ordering within each cluster

Within each class (say $B = \{b_1, \ldots, b_{11}\}$), we enforce a canonical order:

$$\sigma_A(b_1) \le_{\text{lex}} \sigma_A(b_2) \le_{\text{lex}} \cdots \le_{\text{lex}} \sigma_A(b_{11})$$

This removes the 11! relabelings inside each of B, C, D (up to a $(11!)^3$ reduction).

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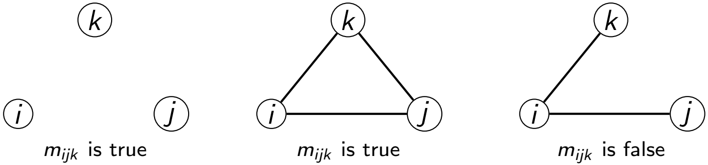

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Enforcing auxiliary variables in encoding

Encoding each m_{ijk} takes two clauses in our CNF formula

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- $m_{ijk} \lor \neg e_{ij} \lor \neg e_{ik} \lor \neg e_{jk}$

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Logically equivalent to:

- $\neg e_{ij} \land \neg e_{ik} \land \neg e_{jk} \to m_{ijk}$
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- $m_{ijk} \vee e_{ij} \vee e_{ik} \vee e_{jk}$
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Logically equivalent to:

- $\neg e_{ij} \wedge \neg e_{ik} \wedge \neg e_{jk} \rightarrow m_{ijk}$
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We only encode the forward implication. This doesn't affect correctness of SAT encoding, and empirically runs faster than encoding bi-implication.

Improved R(5,5)-goodness

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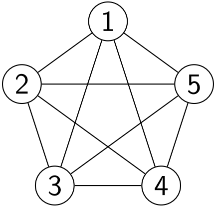
Example clause using auxiliary variables:

$$\neg m_{134} \vee \neg m_{245} \vee \neg m_{135} \vee \neg m_{124} \vee \neg m_{235}$$



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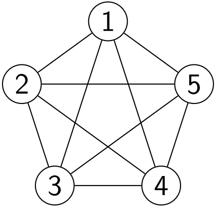

5 triangles cover all 10 edges, and are connected by overlapping edges. Ensures clause is unsatisfiable iff I_5 or K_5 exists

Theoretical and Empirical Improvements

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Old encoding scheme:

- 1 clause of 10 variables to check for I_5
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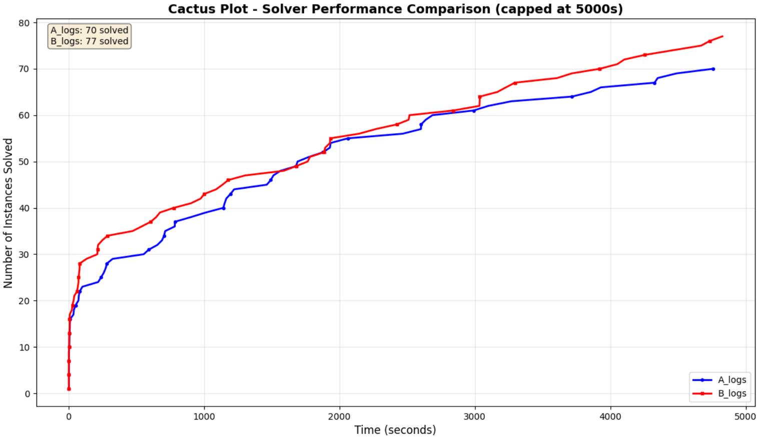
Number of clauses and variables per clause are both cut in half

Theoretical and Empirical Improvements



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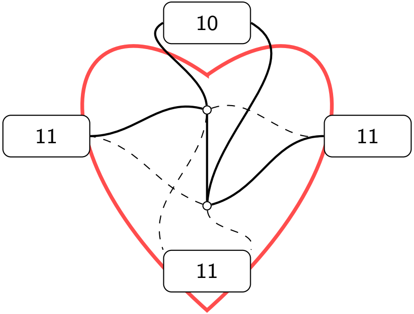
Future Directions

More scaffolding: impose additional canonical "skeleton" structure on the free clusters B, C, D—extra constraints that preserve existence but cut isomorphic search.

Acknowledgments

We thank *Professor John Mackey* and *Zachary Battleman* for their mentorship and guidance throughout this project. We are grateful for their time, feedback, and continued support.



solid = edge   dashed = non-edge