Four-Color Ramsey Number $R_4(3)$

$51 \leq R(3,3,3,3) \leq 62$

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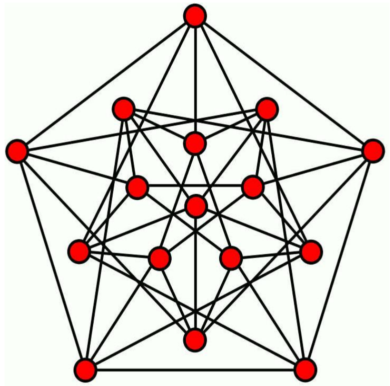
Ramsey numbers $R_k(r)$

Definition. The k-color Ramsey number $R_k(r)$ is equal to n iff n is the smallest integer for which any k-edge-coloring of K_n contains a monochromatic K_r.

- $R_2(3) = 6$, folklore ($\leq$ 1947)
- $R_3(3) = R(3,3,3) = 17$, Greenwood-Gleason (1955)
- $L = \lim_{r\to\infty} R_r(3)^{\frac{1}{r}} > 3.2$, Ageron+ (2022)

- Each color in both critical 3-coloring for $R_3(3)$ is the Clebsch $(3,6;16)$-graph on $GF(2^4)$: $(x,y) \in E$ iff $x - y = \alpha^3$

[Wikipedia]
Alfred Clebsch (1833-1872)

- The smallest open case is $R_4(3)$

Four-Color Ramsey Number $R_4(3)$

year	reference	lower	upper
1955	Greenwood, Gleason	42	66
1967	false rumors	[66]	
1971	Golomb, Baumert	46	
1973	Whitehead	50	65
1973	Chung, Porter	51	
1974	Folkman		65
1995	Sánchez-Flores		64
1995	Kramer (no computer)		62
2004	Fettes-Kramer-R (computer)		62

History of bounds on $R_4(3)$ [from FKR'2004]

- The heuristics don't come close to 50. Why?
- Describe and understand the structure of most $(3,3,3,3;50)$-colorings
- Conjecture: $R_4(3) = 51$

Four colors - $R_4(3)$

color degree sequences for $(3, 3, 3, 3; \geq 60)$-colorings

n	$N(v)$ color degrees	
65	[16, 16, 16, 16]	Whitehead, Folkman 1973-4
64	[16, 16, 16, 15]	Sánchez-Flores 1995
63	[16, 16, 16, 14]	
	[16, 16, 15, 15]	
62	[16, 16, 16, 13]	Kramer 1995+
	[16, 16, 15, 14]	–
	[16, 15, 15, 15]	Fettes-Kramer-R 2004
61	[16, 16, 16, 12]	
	[16, 16, 15, 13]	
	[16, 16, 14, 14]	
	[16, 15, 15, 14]	
	[15, 15, 15, 15]	
60	[16, 16, 16, 11]	looks doable
	[16, 16, 15, 12]	
	[16, 16, 14, 13]	
	[16, 15, 15, 13]	
	[16, 15, 14, 14]	
	[15, 15, 15, 14]	

- Improve on the bound $R_4(3) \leq 62$
- ≤ 60 is very likely achievable by extending the work in FKR'2004