# CSCI-761 Homework 6

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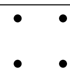
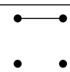
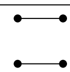
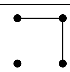
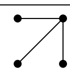
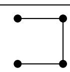
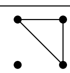
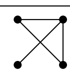
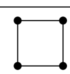
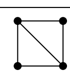
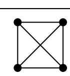
## Problem 1

In the Deck($G$) column, an integer $n$ represents the graph on 3 vertices with $n$ edges.

| Number | $G$ | Deck($G$) | $\exists$vrn($G$) | $\forall$vrn($G$) |
|---|---|---|---|---|
| 0 |  | $(0, 0, 0, 0)$ | 3 | 3 |
| 1 |  | $(1, 1, 0, 0)$ | 3 | 4 |
| 2 |  | $(1, 1, 1, 1)$ | 4 | 4 |
| 3 |  | $(2, 1, 1, 0)$ | 3 | 4 |
| 4 |  | $(2, 2, 2, 0)$ | 3 | 4 |
| 5 |  | $(2, 2, 1, 1)$ | 4 | 4 |
| 6 |  | $(3, 1, 1, 1)$ | 3 | 4 |
| 7 |  | $(3, 2, 2, 1)$ | 3 | 4 |
| 8 |  | $(2, 2, 2, 2)$ | 4 | 4 |
| 9 |  | $(3, 3, 2, 2)$ | 3 | 4 |
| 10 |  | $(3, 3, 3, 3)$ | 3 | 3 |

## Problem 2

Let's prove that $\exists\text{vrn}(G) = 3$ for the graph $K_{1,3}$ (#4). This graph's deck has 3 $P_3$s and one $\overline{K_3}$. If we choose a hand of two $P_3$ cards and one $\overline{K_3}$ card then the only possible graph containing that hand in its deck is $K_{1,3}$, so $\exists\text{vrn}(G) = 3$.

## Problem 3

The only graphs with $\exists\text{vrn}(G) > 3$ are $\overline{C_4}$ (#2), $P_4$ (#5), and $C_4$ (#8).

For #2, the deck is all $\overline{P_3}$s. #6's deck contains three $\overline{P_3}$s, so no choice of cards from #2's deck will differentiate it from #6. #8 is similar, as it is the complement of #2, its deck is all $P_3$s, and it is confused with #4. Thus for both we need to take all four cards.

#5's deck has two $P_3$s and two $\overline{P_3}$s. If we take two of one and one of the other it will be confused with #3 or #7, so again we must take all four.

## Problem 4

The only graphs with $\forall\text{vrn}(G) = 3$ are the complete graph $K_4$ (#10) and its complement (#0). Since #0's deck is all $\overline{K_3}$s, there is only one possible hand of three and no other graph's deck includes it. The same logic applies to #10, whose deck is all $K_3$s. Thus, both have $\forall\text{vrn} = 3$.

## Problem 5

Let's consider $G = K_{1,3}$ (#4) again and prove that $\forall\text{vrn}(G) = 4$. One possible 3-card hand from $G$'s deck is all three $P_3$, which is also included in the deck of #8. Thus, not all choices of 3 cards work. We can see from the table that no other graph shares the entire deck with $G$, so $\forall\text{vrn}(G) = 4$.