CSCI-761 Combinatorial Computing

Assignment 6

due April 8, 2021

Name: ____

1. **Pruning Backtracking**

   CNF-SAT is a problem of finding satisfying assignments to a Boolean formula α in conjunctive normal form, i.e. where α is given by a set of clauses, each of which is a disjunction of literals, each of which is a variable or a negated variable. Suppose you are solving instances of CNF-SAT by backtracking. Describe informally a bounding function (in the spirit of bounding functions used in the maximum clique search algorithms considered in class) which could be used to prune the search space for satisfying assignments to instances of CNF-SAT.

   To follow the struggles of many researchers trying to do it better and better already for decades, explore `http://www.satcompetition.org`, which is a website of worldwide competitions for designing state-of-art SAT-solvers.

2. **Ramsey/Folkman numbers**

   (a) Which of the following are true?
   Give yes/no answer and a brief justification for each.

   - $C_5 \rightarrow (3,3)^e$

   - $C_5 \rightarrow (3,3)^v$

   - $C_5 \rightarrow (2,2)^v$

   - $C_5 \rightarrow (2,2,2)^v$

   - $K_4 \rightarrow (3,3)^e$

- $K_5 \to (3,3)^e$

- $K_6 \to (3,3)^e$

- $K_5 \to (3,3)^v$

- $K_5 \to (2,2,2)^v$

- $K_5 \to (2,2,2,2)^v$

- $K_5 \to (2,2,2,2,2)^v$

(b) Is it easier to prove a lower or upper bound for a Ramsey number?
Is it easier to prove a lower or upper bound for a Folkman number?
Explain your answers.

(c) Prove that $k > R(s,t)$ implies $F_e(s,t;k) = R(s,t)$.
Hint: It is not hard, just combine the definitions.

(d) Prove that $K_3 + C_5 \to (3, 3)^e$.

3. **Progress on Folkman problems**

   In 2014, Christopher Wood wrote a comprehensive survey of the area, at `https://www.cs.rit.edu/~spr/COURSES/CCOMP/cawfolk.pdf`. In 2018, Aleksandar Bikov defended his PhD dissertation titled "Computation and Bounding of Folkman Numbers", available at `https://arxiv.org/abs/1806.09601` (both are also linked from the item 14 on the course page). List four results which are in the latter but not in the former.

   (a)

   (b)

   (c)

   (d)

4. **Making Graphs**

Make nauty canonical labelings of the following five graphs. In each case describe the steps how you made them. In each case state the main combinatorial properties they have.

(a) The Petersen Graph

(b) The Grötzsch Graph

(c) The Clebsch Graph

(d) The Paley Graph P_{17}

(e) The Schläfli Graph