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CSCI 761 - Topics In Advanced Algorithms

Assignment 3

**Part 1:**

Graph 2 Automorphism Groups Mappings:

- 0 1 2 3 4 5 6 7 -> 0 1 2 3 4 5 6 7
- 0 1 2 3 4 5 6 7 -> 0 1 3 2 4 6 5 7
- 0 1 2 3 4 5 6 7 -> 1 0 2 3 7 5 6 4
- 0 1 2 3 4 5 6 7 -> 1 0 3 2 7 6 5 4
- 0 1 2 3 4 5 6 7 -> 2 3 0 1 5 4 7 6
- 0 1 2 3 4 5 6 7 -> 2 3 1 0 5 7 4 6
- 0 1 2 3 4 5 6 7 -> 3 2 0 1 6 4 7 5
- 0 1 2 3 4 5 6 7 -> 3 2 1 0 6 7 4 5

Graph 2 Automorphism Groups Cycle Notation:

- ()
- (2 3)(5 6)
- (0 1)(4 7)
- (0 1)(2 3)(4 7)(5 6)
- (0 2)(1 3)(4 5)(6 7)
- (0 2 1 3)(4 5 7 6)
- (0 3 1 2)(4 6 7 5)
- (0 3)(1 2)(4 6)(5 7)

Graph 3 Automorphism Groups Mappings:

- 0 1 2 3 4 5 6 7 -> 0 1 2 3 4 5 6 7
- 0 1 2 3 4 5 6 7 -> 0 2 1 3 5 4 7 6
- 0 1 2 3 4 5 6 7 -> 1 0 4 6 2 7 3 5
- 0 1 2 3 4 5 6 7 -> 1 4 0 6 7 2 5 3
- 0 1 2 3 4 5 6 7 -> 2 0 5 7 1 6 3 4
- 0 1 2 3 4 5 6 7 -> 2 5 0 7 6 1 4 3
- 0 1 2 3 4 5 6 7 -> 3 6 7 0 5 4 1 2

- 0 1 2 3 4 5 6 7 -> 3 7 6 0 4 5 2 1
- 0 1 2 3 4 5 6 7 -> 4 1 7 5 0 3 6 2
- 0 1 2 3 4 5 6 7 -> 4 7 1 5 3 0 2 6
- 0 1 2 3 4 5 6 7 -> 5 2 6 4 0 3 7 1
- 0 1 2 3 4 5 6 7 -> 5 6 2 4 3 0 1 7
- 0 1 2 3 4 5 6 7 -> 6 3 5 1 7 2 0 4
- 0 1 2 3 4 5 6 7 -> 6 5 3 1 2 7 4 0
- 0 1 2 3 4 5 6 7 -> 7 3 4 2 6 1 0 5
- 0 1 2 3 4 5 6 7 -> 7 4 3 2 1 6 5 0

Graph 3 Automorphism Groups Cycle Notation:
- ()
- (1 2)(4 5)(6 7)
- (0 1)(2 4)(3 6)(5 7)
- (0 1 4 7 3 6 5 2)
- (0 2 5 6 3 7 4 1)
- (0 2)(1 5)(3 7)(4 6)
- (0 3)(1 6)(2 7)(4 5)
- (0 3)(1 7)(2 6)
- (0 4)(2 7)(3 5)
- (0 4 3 5)(1 7 6 2)
- (0 5 3 4)(1 2 6 7)
- (0 5)(1 6)(3 4)
- (0 6)(1 3)(2 5)(4 7)
- (0 6 4 2 3 1 5 7)
- (0 7 5 1 3 2 4 6)
- (0 7)(1 4)(2 3)(5 6)

The entries where I used the clause "if both R(s,t-1) and R(s-1,t) are even" are underlined.

|    | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----|---|---|---|---|---|---|---|----|
| 3  | 6 | <u>9</u> | 14 | <u>19</u> | 26 | <u>33</u> | 42 | <u>51</u> |
| 4  |   | 18 | <u>31</u> | 50 | <u>75</u> | 108 | <u>149</u> | 200 |
| 5  |   |   | 62 | <u>111</u> | 186 | <u>293</u> | 442 | <u>641</u> |
| 6  |   |   |   | 222 | <u>407</u> | 700 | <u>1141</u> | 1782 |
| 7  |   |   |   |   | 814 | <u>1513</u> | 2654 | <u>4435</u> |
| 8  |   |   |   |   |   | 3026 | <u>5679</u> | 10114 |
| 9  |   |   |   |   |   |   | 11358 | <u>21471</u> |
| 10 |   |   |   |   |   |   |   | 42942 |

The upper bounds on R(s,s) implied by R(s,s) <= choose(2s-2,s-1) for 3 <= s <= 10 are:
- S = 3 -> choose(4,2) = 6
- S = 4 -> choose(6,3) = 20
- S = 5 -> choose(8,4) = 70
- S = 6 -> choose(10,5) = 252
- S = 7 -> choose(12,6) = 924
- S = 8 -> choose(14,7) = 3432
- S = 9 -> choose(16,8) = 12870
- S = 10 -> choose(18,9) = 48620

The the upper bounds on R(s,s) for 3 <= s <= 10 are:
- S = 3 -> R(3,3) <= 55
- S = 4 -> R(4,4) <= 209
- S = 5 -> R(5,5) <= 793
- S = 6 -> R(6,6) <= 3011
- S = 7 -> R(7,7) <= 11442
- S = 8 -> R(8,8) <= 43478
- S = 9 -> R(9,9) <= 165217

- S = 10 -> R(10,10) <= 627822

**Part 3:**

The lower bounds on R(s,s) for 3 <= s <=10 are:
- S = 3 -> R(3,3) >= 3
- S = 4 -> R(4,4) >= 5
- S = 5 -> R(5,5) >= 6
- S = 6 -> R(6,6) >= 9
- S = 7 -> R(7,7) >= 12
- S = 8 -> R(8,8) >= 17
- S = 9 -> R(9,9) >= 23
- S = 10 -> R(10,10) >= 33

The best which can be obtained numerically from $2*choose(n,s)/2^{choose(s,2)} < 1$ are:
- S = 3 -> n = 3 -> R(3,3) >= 4
- S = 4 -> n = 6 -> R(4,4) >= 7
- S = 5 -> n = 11 -> R(5,5) >= 12
- S = 6 -> n = 17 -> R(6,6) >= 18
- S = 7 -> n = 27 -> R(7,7) >= 28
- S = 8 -> n = 42 -> R(8,8) >= 43
- S = 9 -> n = 65 -> R(9,9) >= 66
- S = 10 -> n = 100 -> R(10,10) >= 101