Edge Ramsey arrowing and χ

E

Theorem. (Nenov, Lin, others)

$M = R(a_1, \dots, a_r)$

If $G \longrightarrow (a_1, \dots, a_r)^e$ then $\chi(G) \geq M$

Proof

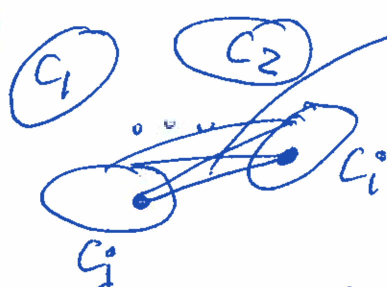
For contradiction, suppose $\chi(G) \leq M-1$.
We will r-partition $E(G)$ so that color i (part i) doesn't contain K_{a_i}

$c: V \longrightarrow \{1, \dots, \chi(G)\}$, an optimal coloring

$C_i = \{v \in V : c(v) = i\}$ is ind-set for color i

graph H: extremal witness for $R(a_1, \dots, a_r)$ on $\chi(G)$ vertices

edge coloring of G:



use color of edge (i,j) in H

Claim: No K_{a_i} in color i (edge-colored) $\square$

Example: $G \to (3,3)^e \Rightarrow \chi(G) \geq 6$, or 5-chromatic G can be Δ-free, 2-edge-colored