

# Assignment 3

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## Part 1

See Tables 1 and 2 for the automorphism groups of Graphs 2 and 3, respectively, listed in cycle notation and as mappings from vertices to vertices.

Cycle	Mapping
<i>id</i>	0 1 2 3 4 5 6 7 0 1 2 3 4 5 6 7
(2 3)(5 6)	0 1 2 3 4 5 6 7 0 1 3 2 4 6 5 7
(0 1)(4 7)	0 1 2 3 4 5 6 7 1 0 2 3 7 5 6 4
(0 2)(1 3)(4 5)(6 7)	0 1 2 3 4 5 6 7 2 3 0 1 5 4 7 6
(2 3)(5 6)(0 1)(4 7)	0 1 2 3 4 5 6 7 1 0 3 2 7 6 5 4
(0 2 1 3)(4 5 7 6)	0 1 2 3 4 5 6 7 2 3 1 0 5 7 4 6
(0 3 1 2)(4 6 7 5)	0 1 2 3 4 5 6 7 3 2 0 1 6 4 7 5
(0 3)(1 2)(4 6)(5 7)	0 1 2 3 4 5 6 7 3 2 1 0 6 7 4 5

Table 1: Automorphism group of Graph 2.

Cycle	Mapping
<i>id</i>	0 1 2 3 4 5 6 7 0 1 2 3 4 5 6 7
(1 2)(4 5)(6 7)	0 1 2 3 4 5 6 7 0 2 1 3 5 4 7 6
(0 1)(2 4)(3 6)(5 7)	0 1 2 3 4 5 6 7 1 0 4 6 2 7 3 5
(1 6)(0 5)(4 3)	0 1 2 3 4 5 6 7 5 6 2 4 3 0 1 7
(0 3)(2 6)(1 7)	0 1 2 3 4 5 6 7 3 7 6 0 4 5 2 1
(0 4)(2 7)(5 3)	0 1 2 3 4 5 6 7 4 1 7 5 0 3 6 2
(1 4)(0 7)(2 3)(5 6)	0 1 2 3 4 5 6 7 7 4 3 2 1 6 5 0
(4 7)(1 3)(0 6)(2 5)	0 1 2 3 4 5 6 7 6 3 5 1 7 2 0 4
(7 3)(4 6)(1 5)(0 2)	0 1 2 3 4 5 6 7 2 5 0 7 6 1 4 3
(0 1 4 7 3 6 5 2)	0 1 2 3 4 5 6 7 1 4 0 6 7 2 5 3
(0 2 5 6 3 7 4 1)	0 1 2 3 4 5 6 7 2 0 5 7 1 6 3 4
(0 4 3 5)(1 7 6 2)	0 1 2 3 4 5 6 7 4 7 1 5 3 0 2 6
(0 5 3 4)(2 6 7 1)	0 1 2 3 4 5 6 7 5 2 6 4 0 3 7 1
(0 7 5 1 3 2 4 6)	0 1 2 3 4 5 6 7 7 3 4 2 6 1 0 5
(0 6 4 2 3 1 5 7)	0 1 2 3 4 5 6 7 6 5 3 1 2 7 4 0
(0 3)(1 6)(4 5)(7 2)	0 1 2 3 4 5 6 7 4 6 7 0 5 4 1 2

Table 2: Automorphism group of Graph 3.

## Part 2

See Table 3 for the best upper bounds of  $R(s, t)$  that can be derived from the fact that  $R(s, t) \leq R(s, t-1) + R(s-1, t)$ , and from the known bounds  $R(s, 2) = s$ ,  $R(3, 3) = 6$ ,  $R(3, 4) = 9$ ,  $R(3, 5) = 14$ , and  $R(3, 6) = 18$ , which have been discussed and proved in class and/or in Assignment 2. These exact bounds are indicated in bold. Bounds deriving from the strict equality  $R(s, t) < R(s, t-1) + R(s-1, t)$ , which holds when both  $R(s, t-1)$  and  $R(s-1, t)$  are even, are indicated in italics. Only two such bounds hold among those found here, namely  $R(4, 5) < 18 + 14 = 32$  and  $R(3, 10) < 42 + 10 = 52$ .

$s/t$	3	4	5	6	7	8	9	10
3	<b>6</b>	<b>9</b>	<b>14</b>	<b>18</b>	25	33	42	<i>51</i>
4		18	<i>31</i>	49	74	107	149	200
5			62	111	185	292	441	641
6				222	407	699	1140	1781
7					814	1513	2653	4434
8						3026	5679	10113
9							11358	21471
10								42942

Table 3: Inclusive upper bounds of  $R(s, t)$  for  $3 \leq s \leq t \leq 10$ .