CSCI-665 Foundations of Algorithms

Homework 1

due Thursday, February 9, 2017

Reading Chapters 1 through 4

1. Consider algorithms A_1, A_2, A_3, and A_4 that have running times of $f_1(n) = 50000 \log_2 n$, $f_2(n) = n^3/50 + n^2$, $f_3(n) = 30n^2 + n$, and $f_4(n) = 2^n/10000$, respectively (in microseconds). Draw a table showing the size of a problem that can be solved in 1 second, 1 hour and 1 year by each algorithm. It is similar to, but different from the tables on page 15 of CLRS, or a handout.

Hint: You don't have to solve it analytically; you can write a simple program to obtain the approximate answers. Give at least two most significant digits of the answers.

2. Prove or disprove the following statements. Use compact arguments, no more than just a few lines for each case.
a) $n^3 = O(n^3 + n \log n)$
b) $n^4 = \Omega(n \log n + n^4)$
c) $n^2 + n = \Theta(n^3 + n^2)$
d) $n!n^2 = \Omega(2^n)$
e) $\lg n = \Theta(\log_{10} n^{1/3})$
f) $\log_{10} n = O(\lg \sqrt{n} / \lg \lg n)$

3. Solve exercise 2.3-7 page 39. Be brief, but to the point.

4. Solve exercise 3.2-5 page 60. Justify your answer.

5. Solve problems 3.4/aceg page 62. Justify your answers.

6. Find the number of comparisons performed by the MAXMIN algorithm for the number of elements in the set S equal to 7, 19, and 47. Show the details of the computation for $n = 12$. The answer for 0, 1, 2 is 0, 0, 1, respectively, and the details as in the figure in class.

7. Prove that Karatsuba's integer multiplication algorithm takes time proportional to $n^{\log_2 3}$. Do not use Master Theorem for recurrences.