Abstract

We discuss a branch of Ramsey theory concerning edge Folkman numbers and how computer algorithms could help to solve some problems therein. We write $G \rightarrow (a_1, \ldots, a_k; p)^e$ if for every edge k-coloring of an undirected simple graph G not containing K_p, a monochromatic K_{a_i} is forced in color i for some $i \in \{1, \ldots, k\}$. The edge Folkman number is defined as $F_e(a_1, \ldots, a_k; p) = \min\{|V(G)| : G \rightarrow (a_1, \ldots, a_k; p)^e\}$. Folkman showed in 1970 that this number exists for $p > \max(a_1, \ldots, a_k)$.

In general, much less is known about edge Folkman numbers than the related and more studied vertex Folkman numbers, where we color vertices instead of edges. $F_e(3, 3; 4)$ involves the smallest parameters for which the problem is open, namely the question, "What is the smallest order N of a K_4-free graph, for which any edge 2-coloring must contain at least one monochromatic triangle?" This is equivalent to finding the order N of the smallest K_4-free graph which is not a union of two triangle-free graphs. It is known that $16 \leq N$ (an easy bound), and it is known through a probabilistic proof by Spencer (later updated by Hovey) that $N \leq 3 \times 10^9$. We suspect that $N \leq 127$.

This talk will present the background, overview some related problems, discuss the difficulties in obtaining better bounds on N, and give some computational evidence why it is very likely that even $N < 100$.

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