1. For the graph given below, darken the edges of the minimum spanning tree found using Kruskal’s algorithm. Give the total cost of the MST you have found.

2. Apply Prim’s algorithm to the following graph. Include in the queue only the fringe vertices (the vertices not in the current tree which are adjacent to at least one tree vertex). Begin at vertex $a$. For every vertex in the queue, give the parent of that vertex and the cost to bring that vertex into the minimum spanning tree. For example, once the start vertex, $a$, is in the minimum spanning tree, the queue of fringe vertices consists of $b(a, 3), c(a, 5)$ and $d(a, 4)$. List the edges that are in the minimum spanning tree.

Questions 3 and 4 on reverse side
3. Provide a logical argument that for Kruskal’s algorithm, it is always, never, or sometimes the case that if a connected graph has one edge with a weight smaller than all of the other edges, that edge will always be part of the minimum spanning tree for that graph.

4. In a city there are $N$ houses, each of which is in need of a water supply. It costs $w[i]$ dollars to build a well at house $i$, and it costs $c[i][j]$ to build a pipe in between houses $i$ and $j$. A house can receive water if either there is a well built there or if there is some path of pipes to a house with a well. Design an efficient algorithm to find the minimum amount of money needed to supply every house with water. What is the efficiency of your algorithm?