



# Performance Evaluation and Experimental Comparisons for Classifiers

Prof. Richard Zanibbi

## Performance Evaluation

#### Goal

We wish to determine the number and type of errors our classifier makes

#### **Problem**

Often the feature space (i.e. the input space) is vast; impractical to obtain a data set with labels for all possible inputs

Compromise (solution?...)

Estimate errors using a labeled sample (ideally, a representative sample)





## The Counting Estimator of the Error Rate

## **Definition**

For a labelled test data set Z, this the percentage of inputs from Z that are misclassified (#errors / |Z|)

## Question

Does the counting estimator provide a complete picture of the errors made by a classifier?





## A More General Error Rate Formulation

$$Error(D) = \frac{1}{|Z|} \sum_{j=1}^{|Z|} \{1 - I(l(z_j), s_j)\}, \quad z_j \in Z$$

where 
$$I(a, b) = \begin{cases} 1, & \text{if } a = b \\ 0, & \text{otherwise} \end{cases}$$

is an indicator function, and  $l(z_j)$  returns the label (true class) for test sample  $z_i \in Z$ 

\*The indicator function can be replaced by one returning values in [0,1], to smooth (reduce variation in) the error estimates (e.g. using proximity of input to closest instance in the correct class)



## Confusion Matrix for a Binary Classifier (Kuncheva, 2004)

|            | D          | $D(\mathbf{x})$ |  |  |  |
|------------|------------|-----------------|--|--|--|
| True Class | $\omega_1$ | $\omega_2$      |  |  |  |
| $\omega_1$ | 7          | 0               |  |  |  |
| $\omega_2$ | 1          | 7               |  |  |  |

Our test set Z has 15 instances

One error (confusion) is made: a class I instance is confused for a class 2 instance





## Larger Example: Letter Recognition (Kuncheva, 2004)

TABLE 1.1 The "H"-Row in the Confusion Matrix for the Letter Data Set Obtained from a Linear Classifier Trained on 10,000 Points.

| "H" mistaken for: | Α  | В  | С | D  | E  | F | G | Н   | . 1 | J | K  | L | M |
|-------------------|----|----|---|----|----|---|---|-----|-----|---|----|---|---|
| No of times:      | 2  | 12 | 0 | 27 | 0  | 2 | 1 | 165 | 0   | 0 | 26 | 0 | 1 |
| "H" mistaken for: | N  | 0  | Р | Q  | R  | S | Т | U   | ٧   | W | Χ  | Υ | Z |
| No of times:      | 31 | 37 | 4 | 8  | 17 | 1 | 1 | 13  | 3   | 1 | 27 | 0 | 0 |

Full Table: 26 x 26 entries



## The "Reject" Option

## Purpose

Avoid errors on ambiguous inputs through rejection (i.e. 'skip' the input). One approach: threshold discriminant function scores (e.g. estimated probabilities), reject inputs with max discriminant function score below the threhold

Confusion matrix: adding rejection: size  $(c+1) \times c$ 

#### Trade-off

Rejection avoids error, but often has its own cost (e.g. human inspection of OCR results, medical diagnosis)



## Reject Rate

### Reject Rate

Percentage of inputs rejected

## Reporting

Initial recognition results should be reported with no rejection. Rejection may then be used, but with parameters and the rejection rate reported along with error estimates. A binary classification example:

- No rejection: error rate of 10%
- Discriminant scores < 0.5 : 30% reject rate, 2% error rate</li>
- Discriminant scores < 0.9 : 70% reject rate, 0% error rate







# Using Available Labeled Data: Training, Test and Validation Set Creation

## Using Available Data

### Labeled Data

Expensive to produce, as it often involves people (e.g. image labeling)

## Available Data

Is finite; we want a large sample to learn model parameters accurately, but also want a large sample to estimate errors accurately





## Common Division of Available Data into (Disjoint) Sets

## Training Set

To learn model parameters

## Testing Set

To estimate error rates

### Validation Set

"Pseudo" test set used during training; stop training when improvements on training set do not lead to improvements on validation set (avoid overtraining)





## Methods for Data Use

## Resubstitution (avoid!)

Use all data for training and testing: optimistic error estimate

### Hold-Out Method

Randomly split data into two sets. Use one half as training, the other as testing (pessimistic estimate)

- Can split into 3 sets, to produce validation set
- Data shuffle: split data randomly L times, and average the results





## Methods for Data Use (Cont'd)

### **Cross-Validation**

Randomly partition the data into K sets. Treat each partition as a test set, using the remaining data for training, then average the K error estimates.

 Leave-one-out: K=N (the number of samples), we "test" on each sample individually

#### **Error Distribution**

For hold-out and cross-validation, obtain an error rate distribution that characterizes the stability of the estimates (e.g. variance in error across samples)







## Experimental Comparison of Classifiers

## Factors to Consider for Classifier Comparisons

#### Choice of test set

Different sets can rank classifiers differently, even though they have the same accuracy over the population (over all possible inputs)

 Dangerous to draw conclusions from a single experiment, esp. if data size is small

## Choice of training set

Some classifiers are *instable*: small changes in training set can cause significant changes in accuracy

must account for variation with respect to training data





## Factors, Cont'd

## Randomization in Learning Algorithms

Some learning algorithms involve randomization (e.g. initial weights in a neural network, use genetic algorithm to modify parameters)

 For a fixed training set, the classifier may perform differently!
Need multiple training runs to obtain a complete picture (distribution)

## Ambiguity and Mislabeling Data

In complex data, often ambiguous patterns that have more than one acceptable interpretation, or errors in labeling (human error)





## Guidelines for Comparing Classifiers (Kuncheva pp. 24-25)

- I. Fix the training and testing procedure before starting an experiment. Give enough detail in papers so that other researchers can replicate your experiment
- 2. Include controls ("baseline" versions of classifiers) along with more sophisticated versions (e.g. see earlier binary classifier with "reject" example)
- 3. Use available information to largest extent possible, e.g. best possible (fair) initializations
- 4. Make sure the test set has not been seen during training
- 5. Report the run-time and space complexity of algorithms (e.g. big 'O'), actual running times and space usage



## Experimental Comparisons: Hypothesis Testing

### The Best Performance on a Test Set

....does not imply best performance over the entire feature space

## Example

Two classifiers run on a test set have accuracies 96% and 98%. Can we claim that the error distributions for these are significantly different?





# Testing the Null Hypothesis

### **Null Hypothesis**

That the distributions in question (accuracies) do not differ in a statistically significant fashion (i.e. insufficient evidence)

## Hypothesis Tests

Depending on the distribution types, there are a tests intended to determine whether we can *reject* the null hypothesis at a given *significance level* (p, the probability that we incorrectly reject the null hypothesis, e.g. p < 0.05 or p < 0.01)

#### **Example Tests**

chi-square, t-test, f-test, ANOVA, McNemar test, etc.

