



## Performance Evaluation and Experimental Comparisons for Classifiers

Prof. Richard Zanibbi

## Performance Evaluation

#### Goal

We wish to determine the number and type of errors our classifier makes

#### Problem

Often the feature space (i.e. the input space) is vast; impractical to obtain a data set with labels for all possible inputs

Compromise (solution?...)

Estimate errors using a (ideally representative) labeled test set



# The Counting Estimator of the Error Rate

#### Definition

For a labelled test data set Z, this the percentage of inputs from Z that are misclassified (#errors / |Z|)

#### Question

Does the counting estimator provide a complete picture of the errors made by a classifier?



# A More General Error Rate Formulation

$$Error(D) = \frac{1}{|Z|} \sum_{j=1}^{|Z|} \{1 - I(l(z_j), s_j)\}, \quad z_j \in Z$$

where  $I(a, b) = \begin{cases} 1, & \text{if } a=b \\ 0, & \text{otherwise} \end{cases}$ 

is an indicator function, and  $l(z_j)$  returns the label (true class) for test sample  $z_j \in Z$ 

\*The indicator fn can be replaced by one returning values in [0,1], to smooth (reduce variation in) the error estimates (e.g. using proximity of input to closest instance in correct class)



## Confusion Matrix for a Binary Classifier (Kuncheva, 2004)

a gnugadaiacida	$D(\mathbf{x})$			
True Class	$\omega_1$	ω2		
$\omega_1$	7	0		
$\omega_2$	1	7		

Our test set Z has 15 instances

One error (confusion) is made: a class I instance is confused for a class 2 instance





## Larger Example: Letter Recognition (Kuncheva, 2004)

TABLE 1.1 The "H"-Row in the Confusion Matrix for the Letter Data Set Obtained from a Linear Classifier Trained on 10,000 Points.

"H" mistaken for:	А	В	С	D	Е	F	G	Н		J	К	L	Μ
No of times:	2	12	0	27	0	2	1	165	0	0	26	0	1
"H" mistaken for:	Ν	0	Ρ	Q	R	S	Т	U	V	W	Х	Y	Z
No of times:	31	37	4	8	17	1	1	13	3	1	27	0	0

Full Table: 26 x 26 entries



# The "Reject" Option

#### Purpose

Avoid error on difficult inputs by allowing the classifier to reject inputs, making no decision. Can be achieved by thresholding the discriminant function scores (e.g. estimated probabilities), rejecting inputs whose scores fall below the threshold

Confusion matrix: add a row for rejection: size (c+1) x c

#### Trade-off



In general, the more we reject, the fewer errors are made, but rejection often has its own associated cost (e.g. human inspection of OCR results, medical diagnosis)

## **Reject** Rate

#### **Reject Rate**

Percentage of inputs rejected

#### Reporting

Recognition results should be reported with *no* rejection as a base/control case, and then if used, rejection parameters and the rejection rate should be reported along with error estimates. A binary classification example:

- No rejection: error rate of 10%
- Discriminant scores both <= 0.5 : 30% reject rate, 2% error rate
- One discriminant score < 0.9 : 70% reject rate, 0% error rate









## Using Available Labeled Data: Training, Validation, and Test Set Creation

# Using Available Data

#### Labeled Data

Expensive to produce, as it often involves people (e.g. image labeling)

#### Available Data

Is finite; we want a large sample to learn model parameters accurately, but also want a large sample to estimate errors accurately





## Common Division of Available Data into (Disjoint) Sets

**Training Set** 

To learn model parameters

**Testing Set** 

To estimate error rates

Validation Set

"Pseudo" test set used during training; stop training when improvements on training set do not lead to improvements on validation set (avoids overtraining)





## Methods for Data Use

#### Resubstitution (avoid!)

Use all data for training and testing: optimistic error estimate

#### Hold-Out Method

Randomly split data into two sets. Use one half as training, the other as testing (pessimistic estimate)

- Can split into 3 sets, to produce validation set
- Data shuffle: split data randomly L times, and average the results





## Methods for Data Use (Cont'd)

#### **Cross-Validation**

Randomly partition the data into K sets. Treat each partition as a test set, using the remaining data for training, then average the K error estimates.

 Leave-one-out: K=N (the number of samples), we "test" on each sample individually

#### **Error Distribution**

For the hold-out and cross-validation techniques, we obtain an error rate *distribution* that characterizes the stability of the estimates (e.g. variance in errors across samples)







# Experimental Comparison of Classifiers

## Factors to Consider for Classifier Comparisons

#### Choice of test set

Different sets can rank classifiers differently, even though they have the same accuracy over the population (over all possible inputs)

• Dangerous to draw conclusions from a single experiment, esp. if data size is small

#### Choice of training set

Some classifiers are *instable*: small changes in training set can cause significant changes in accuracy

 must account for variation with respect to training data





# Factors, Cont'd

#### Randomization in Learning Algorithms

Some learning algorithms involve randomization (e.g. initial parameters in a neural network, use of genetic algorithm to modify parameters)

• For a fixed training set, the classifier may perform differently! Need multiple training runs to obtain a complete picture (distribution)

#### Ambiguity and Mislabeling Data

In complex data, there are often ambiguous patterns that have more than one acceptable interpretation, or errors in labeling (human error)



### Guidelines for Comparing Classifiers (Kuncheva pp. 24-25)

- I. Fix the training and testing procedure before starting an experiment. Give enough detail in papers so that other researchers can replicate your experiment
- 2. Include controls ("baseline" versions of classifiers) along with more sophisticated versions (e.g. see earlier binary classifier with "reject" example)
- 3. Use available information to largest extent possible, e.g. best possible (fair) initializations
- 4. Make sure the test set has not been seen during the training phase
- 5. Report the run-time and space complexity of algorithms (e.g. big 'O'), actual running times and space usage

 $R \cdot I \cdot T$ 



## Experimental Comparisons: Hypothesis Testing

#### The Best Performance on a Test Set

....does not imply the best performance over the population (entire input space)

#### Example

Two classifiers run on a test set have accuracies 96% and 98%. Can we claim that the error distributions for these are significantly different?





# Testing the Null Hypothesis

#### Null Hypothesis

That the distributions in question (accuracies) do *not* differ in a statistically significant fashion (i.e. insufficient evidence)

#### Hypothesis Tests

Depending on the distribution types, there are a tests intended to determine whether we can *reject* the null hypothesis at a given *significance level (p*, the probability that we incorrectly reject the null hypothesis, e.g. p < 0.05 or p < 0.01)

#### **Example Tests**

chi-square, t-test, f-test, ANOVA, McNemar test, etc.



