PATTERNS IN ART

Mathematics Informs the History of Art

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• Millville, New Jersey, USA

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The Golden Ratio

Pentagon and Pentagram Calculation

\[ LL = L = M = 1.618 \ldots \]

\[ L \quad L \quad M \quad S \]

\[ b = 2 \]

\[ a \]

\[ \sqrt{5} \]

Easiest Golden Numbers Construction

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Divine Proportion
Fibonacci Numbers

Phi = 1.618…
Architecture in Antiquity

Golden Rectangles
Architecture in Antiquity

Golden Rectangles

The Golden Ratio
The Taj Mahal
Churches in Romania

St. Michael’s Cathedral

St. John the Baptist
Architecture in Paris

Paris, France

Notre Dame Cathedral

Eiffel Tower
North America

United Nations in New York

CN Tower in Toronto
Leonardo Da Vinci

Self Portrait

The Vitruvian Man
Leonardo Da Vinci

Self Portrait  The Vitruvian Man
Leonardo Da Vinci

The Annunciation
Renaissance Painters

Leonardo Da Vinci

Lady with an Ermine
Portrait of a Lady
Leonardo Da Vinci

The Last Supper
Renaissance Painters

The Mona Lisa
The Mona Lisa
Michelangelo

Creation of Man
Renaissance Painters

Michelangelo

Holy Family

Raphael

Crucifixion
Renaissance Painters

Michelangelo

Holy Family

Crucifixion

Raphael

Crucifixion
Botticelli

Self Portrait

Birth of Venus
Botticelli

Birth of Venus
Baroque Painters

Caravaggio

Portrait of Caravaggio

The Incredulity of Saint Thomas
Baroque Painters

Caravaggio

Portrait of Caravaggio  The Incredulity of Saint Thomas
Post–Impressionism

George Seurat

Self Portrait

Bridge of Courbevoie
Post-Impressionism

George Seurat

Bathers at Asnières
Post–Impressionism

George Seurat

La Parade

Young Woman Powdering Herself
Post–Impressionism

Cezanne

Self Portrait

Still Life with Apples
and a Pot of Primroses
Post–Impressionism

Cézanne

Basket of Apples

Still Life, Drapery, Pitcher, and Fruit Bowl
The Algonquin School

Catalogue

Autumn In Orillia
by Franklin Carmichael
Tom Thomson

Woodland Waterfall

Northern River
Salvador Dali

Photo of Dali

The Persistence of Memory
Salvador Dali

The Sacrament of the Last Supper
Abstract Expressionism

De Kooning

Photo of De Kooning

Seated Woman
Abstract Art

Mondrian

Photo of Mondrian

Composition with Gray and Light Brown
Abstract Art

Mondrian

Photo of Mondrian

Composition with Gray and Light Brown
Abstract Art

Mondrian

Composition in Red, Yellow, and Blue

Composition with Yellow, Blue and Red
Abstract Expressionism

Painters Eleven

Snap by Harold Town

Licorice by William Ronald
Abstract Expressionism

Jack Bush

Tail Spread

Big A
Abstract – Color Fields

Gene Davis

Passion Flower

Red Devil
Abstract Expressionism

Frank Stella

West Broadway

Grape Island
Haunch of Venison
Abstract Expressionism

Frank Stella

Mitered Squares

Hyena Stomp
Ellsworth Kelly

Red Yellow Blue White

Colors for a Large Wall
Abstract Expressionism

Kandinsky

Color of Squares

Color Study of Squares
Pop Art

Warhol

Repetition

Marylin Monroe
Op Art

Anni Albers

Movement in Squares

Second Movement

Bridget Riley

Movement in Squares
Op Art

Vasarely

Titan

Escher

Balcony
Tesselations

Escher

Horseman

Fish
Tesselations

Bird/Fish (manipulated)
Folk Art

Quilts
Gerda de Vries

Sierpinski Meets Mondrian  Bubbillusion
Voudon Banner – Haiti
First Nation Art – Salmon Spirits
Melencolia

Albrecht Dürer's Magic Square

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Circles on Grid
## Number Arrays

### Wythoff array.

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### $k$-Zeckendorf array at $k = 3$.

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Array $\mathcal{X}_k$ modulo $m$

Row periods in Arrays $x_{r,c} \pmod m$

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<td>48</td>
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</table>

Triple cycle $\pi(\mathcal{X}_2) = 8$ for $x_{r,c} \pmod 3$.

Color key: green = 0, blue = 1, red = 2, white = 3.

Double cycle $\pi(\mathcal{X}_3) = 14$ for $x_{r,c} \pmod 4$. 
Modular Arrays

Quilt Pattern
Group of Seven
The Ash Can School

The Eight
Artist Groups

Glasstown Nine
Larry Ericksen

Still Life
Thank you