1 Introduction

In this programming assignment, you will implement a number of functions in Standard ML in order to gain familiarity with the language and to practice functional programming.

Download `prog04.sml` and `prog04_tests.sml` (or copy from `/usr/local/pub/mtf/plc/programming/prog04-sml` on the CS Department Linux systems). The former is a template for your submission and also includes a number of supporting functions. The latter is a test suite for the assignment.

2 Description

Complete the following problem. Each one is to define one or more Standard ML functions. See Requirements and Submissions for important restrictions.

A. (10pts) Write functions `unzip` of type `(a * b) list -> 'a list * 'b list` and `zip` of type `'a list * 'b list -> ('a * 'b) list`. The function `unzip` takes a list of pairs and produces a pair of lists, while the function `zip` takes a pair of lists and produces a list of pairs; in each case, the order of elements is preserved. When `zip` is applied to lists of unequal length, the excess elements from the tail of the longer one are ignored. For example, `unzip [(1,2),(3,4),(5,6)]` should return `[(1,3,5),(2,4,6)]` and `zip [(1,3,5),(2,4)]` should return `[(1,2),(3,4)]`. The `unzip` and `zip` functions satisfy the following algebraic laws:

* unzip-zip law: `zip (unzip xys) = xys`
* zip-unzip law: `unzip (zip (xs, ys)) = (xs, ys)` if `length xs = length ys`

B. (5pts) Write a function `compound` of type `int -> ('a -> 'a) -> 'a -> 'a`. The application `compound n f`, for non-negative `n`, should return a function that, when applied to an argument (call it `x`), returns `f` applied `n` times to `x`. The idea of `compound n f` is that it returns `fn x => f (f (f ... (f x)...))`, with `n` copies of `f`. For example, `compound 10 (fn x => x + 1)` should return 20, `compound 5 (fn x => x + 1)` should return 15, `compound 0 (fn x => x + 1)` should return 10, `compound 4 (fn x => x + x)` should return 32, and `compound 2 (fn x => x + x)` should return 2. Note: A function `f` applied 0 times to `x` is equivalent to `x`. Hint: You saw this function in µScheme in Chapter 2 of Programming Languages: Build, Prove, and Compare.

C. (5pts) Write a function `exp` of type `int -> int -> int`. The application `exp b e`, for non-negative `e`, should return `b^e`. For example, `exp 3 2` should return 9 and `exp 2 4` should return 16. Note: `exp` must be implemented with `compound` and must not be recursive. Note: It will be convenient to define `0^0 = 1`.

D. (5pts) Write a function `existsUnique` of type `('a -> bool) -> 'a list -> bool`. The function `existsUnique` should return `true` if the input function returns `true` on exactly one element of the input list and should return `false` if the input function returns `true` on zero elements or more than one element of the input list. For example, `existsUnique (fn x => x mod 2 = 1)` should return `false`, `existsUnique (fn x => x mod 2 = 1)` should return `true`, `existsUnique (fn x => x mod 2 = 1)` should return `false`, and `existsUnique (fn x => x mod 2 = 1)` should return `true`.
E. (5pts) Write a function `allAlt` of type `(a -> bool) -> 'a list -> bool`. The function `allAlt` should return `true` if the input function returns `true` on the elements in odd positions of the input list (first element, third element, fifth element, ...) and returns `false` on the elements in even positions of the input list (second element, fourth element, sixth element, ...). For example, `allAlt (fn x => x mod 2 = 1) []` should return `true`, `allAlt (fn x => x mod 2 = 1) [1]` should return `true`, `allAlt (fn x => x mod 2 = 1) [1,2,3,4,5,6,7,8,9]` should return `true`, and `allAlt (fn x => x mod 2 = 1) [1,2,3,4,6,7,8,9]` should return `false`.

F. (5 pts; bonus 5 pts) Write a function `separate` of type `int * 'a * 'a list -> 'a list` such that `separate (k, x, l)` returns the list that inserts element `x` after each `k` elements of list `l` (counting from the end of the list). For example, `separate (1, 0, [1,2,3,4])` should return `[1,0,2,0,3,4]` and `separate (3, 0, [1,2,3,4])` should return `[1,0,2,3,4]`. Furthermore, `separate` can be used to implement functions that insert ones, tens, and thousands separators in the string representation of integers (`explode` converts a string to a list of characters and `implode` converts a list of characters to a string):

```haskell
fun onesSeparate i = 
  implode (separate (1, " ", explode (Int.toString i)))
fun tensSeparate i = 
  implode (separate (2, " ", explode (Int.toString i)))
fun thousandsSeparate i = 
  implode (separate (3, " ", explode (Int.toString i)))
```

(bonus 5 pts) Define `separate` in terms of `foldl` or `foldr` without using recursion or other list functions.
G. (15pts) The $\mu$Scheme interpreter in Standard ML implements environments using lists (see chunk 361 from Chapter 5 of Programming Languages: Build, Prove, and Compare):

```ml
type name = string
fun nameCompare (n1: name, n2: name) : order = String.compare (n1, n2)
fun nameEquals (n1: name, n2: name) : bool =
    case nameCompare (n1, n2) of EQUAL => true | _ => false
exception NotFound of name

type 'a lenv = (name * 'a) list
val lenvEmpty = []
fun lenvFind (name, rho) =
    case rho of
    [] => raise NotFound name
    | (n, d)::tail =>
        if nameEquals (name, n) then d else lenvFind (name, tail)
fun lenvBind (name, data, rho) = (name, data) :: rho
```

We might call this a list representation for environments.

Now consider a binary search tree representation for environments:

```ml
datatype 'a btree = Leaf | Node of 'a btree * 'a * 'a btree

type 'a tenv = (name * 'a) btree
```

We maintain the invariant that a 'a tenv is a binary search tree, ordered by the name component of the name * 'a elements of the tree. Furthermore, we maintain the invariant that all name * 'a elements of the tree have distinct name components.

(a) (5pts) Write a value tenvEmpty of type 'a tenv that represents the empty environment (i.e., the environment that does not bind any names).

(b) (5pts) Write a function tenvFind of type name * 'a tenv -> 'a such that tenvFind (name, rho) returns the data associated with the name name in the environment rho. If the name name is not bound in the environment rho, then tenvFind (name, rho) should raise the exception NotFound. Note: tenvFind must be implemented so as to efficiently traverse the appropriate portion of the binary search tree.

(c) (5pts) Write a function tenvBind of type name * 'a * 'a tenv -> 'a tenv such that tenvBind (name, data, rho) returns the environment rho extended by a binding of the name name to the data data. Note: tenvBind must be implemented so as to efficiently traverse the appropriate portion of the binary search tree. Note: tenvBind must maintain the invariant that all name * 'a elements of the tree have distinct name components.

Hint: Use the provided nameCompare of type name * name -> order to compare names.

Hint: You may use the provided btreeInsert and btreeLookup functions, either directly in your implementation of tenvFind and tenvBind or indirectly as inspiration for your implementation of tenvFind and tenvBind.
H. (15pts) Now consider a function representation for environments:

\[
\text{type } 'a \text{ fenv} = \text{name} \rightarrow 'a
\]

(a) (5pts) Write a value \text{fenvEmpty} of type \text{'a fenv} that represents the empty environment (i.e., the environment that does not bind any names).

(b) (5pts) Write a function \text{fenvFind} of type \text{name * 'a fenv -> 'a} such that \text{fenvFind (name, rho)} returns the data associated with the name \text{name} in the environment \text{rho}. If the name \text{name} is not bound in the environment \text{rho}, then \text{fenvFind (name, rho)} should raise the exception \text{NotFound}.

(c) (5pts) Write a function \text{fenvBind} of type \text{name * 'a * 'a fenv -> 'a fenv} such that \text{fenvBind (name, data, rho)} returns the environment \text{rho} extended by a binding of the name \text{name} to the data \text{data}.

Hint: This problem is similar in spirit to Problem H from Programming 03: Scheme Programming.

I. (40pts) In class, we noted that list append (\text{append} in Scheme and \text{@} in Standard ML) must copy the first list and, therefore, is a linear time operation. Algorithms that make extensive use of list append may suffer (in running time).

An \text{append} list is a (simple) implementation of the list abstract data type that makes construction cheap (\text{O(1)}), but makes destruction expensive (\text{O(n)}). In particular, appending two append lists is a constant time operation, but splitting a list into a head and a tail is a linear time operation. It is useful for algorithms that construct large lists in an irregular fashion and need only convert to a (Scheme or Standard ML) list at the end.

The \text{'a alistNN} and \text{'a alist} types are defined as follows:

\[
\text{datatype 'a alistNN = Sing of 'a | Append of 'a alistNN * 'a alistNN}
\]

\[
\text{datatype 'a alist = Nil | NonNil of 'a alist}
\]

The \text{'a alistNN} type represents the “non-nil” append lists, while the \text{'a alist} type represents arbitrary (nil or non-nil) append lists.

(a) (5pts) Write a function \text{alistAppend} of type \text{'a alist * 'a alist -> 'a alist} that appends to append lists. The application \text{alistAppend xs ys} should run in \text{O(1)} time.

(b) (5pts) Write a function \text{alistCons} of type \text{'a * 'a alist -> 'a alist} that adds a new element to the front of an append list. The application \text{alistCons (x,xs)} should run in \text{O(1)} time.

(c) (5pts) Write a function \text{alistSnoc} of type \text{'a list * 'a -> 'a alist} that adds a new element to the rear of an append list. (\text{snoc} is “backwards” \text{cons}.) The application \text{alistSnoc (xs, x)} should run in \text{O(1)} time.

(d) (5pts) Write a function \text{alistUnsnoc} of type \text{'a alist -> ('a list * 'a) option} that splits an append list into its last element and the remaining elements (as an append list). The application \text{alistUnsnoc xs} should run in \text{O(n)} time.

(e) (5pts) Write a function \text{alistMap} of type \text{('a -> 'b) -> 'a alist -> 'b alist} that performs a \text{map} on an append list. The function \text{alistMap f xs} should run in \text{O(n * t)} time (where \text{t} is the running time of the function \text{f}).

Note: Repeatedly calling \text{alistUncons} or \text{alistUnsnoc} will not achieve the required running time.

(f) (5pts) Write a function \text{alistFilter} of type \text{('a -> bool) -> 'a alist -> 'a alist} that performs a \text{filter} on an append list. The function \text{alistfilter f xs} should run in \text{O(n * t)} time (where \text{t} is the running time of the function \text{f}).

Note: Repeatedly calling \text{alistUncons} or \text{alistUnsnoc} will not achieve the required running time.

(g) (5pts) Write a function \text{alistFoldl} of type \text{('a * 'b -> 'b) -> 'b -> 'a alist -> 'b} that performs a \text{left-fold} on an append list. The function \text{alistFoldl f b xs} should run in \text{O(n * t)} time (where \text{t} is the running time of the function \text{f}).

Note: Repeatedly calling \text{alistUncons} or \text{alistUnsnoc} will not achieve the required running time.

(h) (5pts) Write a function \text{alistToList} of type \text{'a alist -> 'a list} that converts an append list to a (Standard ML) list. The function \text{alistToList xs} should run in \text{O(n)} time.

Note: \text{alistToList} must be implemented with either \text{alistFoldl} or \text{alistFoldr} and must not be recursive.
J. (15 pts; bonus 5 pts) A propositional-logic formula may be represented by the following Standard ML datatype:

```
datatype fmla = F_Var of name 
| F_Not of fmla 
| F_And of fmla * fmla 
| F_Or of fmla * fmla
```

The `F_Var` constructor represents a propositional variable (identified by a string); the `F_Not` constructor represents the logical negation of a sub-formula; the `F_And` and `F_Or` constructors represent the logical conjunction and disjunction of two sub-formulas.

(a) (5pts) Write a function `fmlaSize` of type `fmla -> int` that returns the size of a propositional-logic formula. A propositional variable has size 1; logical negation has size 1 plus the size of its sub-formula; logical conjunction and disjunction have size 1 plus the sizes of their sub-formulas.

(b) (5pts) Write a function `fmlaVarsOf` of type `fmla -> string list` that returns a list of all variables in a propositional-logic formula. The result list should not contain duplicates.

(c) (5pts) Write a function `fmlaEval` of type `fmla * bool env -> bool` that evaluates a propositional-logic formula. The second component of the argument (of type `bool env`) assigns a truth value to variables; assume that all variables that appear in the propositional-logic formula are bound in the environment.

(d) (bonus 5pts) Write a function `fmlaTautology` of type `fmla -> bool` that determines whether or not a propositional-logic formula is a tautology. (A propositional-logic formula is a tautology if all assignments of truth values to variables causes the propositional-logic formula to evaluate to true.)

Hint: This problem is most easily approached using a direct application of the definition of a tautology.

Hint: `fmlaTautology` cannot be implemented by recursion on the formula; note that `F_Or (F_Var "a", F_Not (F_Var "a"))` is a tautology, but neither `F_Var "a"` nor `F_Not (F_Var "a")` is a tautology.

3 Requirements and Submission

The purpose of this assignment is to practice programming the Standard ML language, not to practice searching the Standard ML libraries. Therefore, if there is a function from the Standard Basis Library that “solves” a particular problem, then you are asked to nonetheless define the function “from scratch” and without looking at the definition of the function from the Standard Basis Library.

Helper functions may be defined at the top level. You may also define local functions using `local` or `let`.

Always use pattern matching to inspect and deconstruct values. The use of `null`, `hd`, `tl`, `#1`, `#2`, etc., in any exercise will result in zero credit for that exercise.

Your submission must be a valid Standard ML program. In particular, it must pass the following test:

```
$ cat prog04.sml | /usr/local/pub/mtf/plc/bin/mosml -P full -quietdec
```

without any error messages. If your submission produces error messages (e.g., syntax errors or type errors), then your submission will not be tested and will result in zero credit for the assignment.

Submit `prog04.sml` to the Programming 04 Dropbox on MyCourses by the due date.
A Interpreter

A reference Standard ML interpreter (MoscowML) is available on the CS Department Linux systems (e.g., glados.cs.rit.edu and queeg.cs.rit.edu and ICLs 1 and 2) at:

/usr/local/pub/mtf/plc/bin/mosml

Use the reference interpreter to check your code.

A.1 Interactive mode

Simply executing

$ /usr/local/pub/mtf/plc/bin/mosml -P full

will run the interpreter interactively, but without line editing.

Executing

$ rlwrap /usr/local/pub/mtf/plc/bin/mosml -P full

or

$ ledit /usr/local/pub/mtf/plc/bin/mosml -P full

will run the interpreter interactively with line editing. (See the manual pages for rlwrap and ledit for more details.)

Within the interactive interpreter, the use function of type string -> unit will read, type check, and evaluate a source file.

Executing

- use "prog04.sml";

will read, type check, and evaluate the file prog04.sml. Executing

- app use ["prog04.sml", "prog04_tests.sml"];

will read, type check, and evaluate the files prog04.sml and prog04_tests.sml. Executing

- app use ["prog04.sml", "util.sml", "test-Aa-unzip.sml"];

will read, type check, and evaluate the files prog04.sml, util.sml, and test-Aa-unzip.sml.

A.2 Batch mode

Executing

$ cat prog04.sml | /usr/local/pub/mtf/plc/bin/mosml -P full

will run the interpreter on the contents of the file prog04.sml, but with prompts printed.

Executing

$ cat prog04.sml | /usr/local/pub/mtf/plc/bin/mosml -P full -quietdec

will run the interpreter on the contents of the file prog04.sml without prompts printed.

Executing

$ cat prog04.sml prog04_tests.sml | /usr/local/pub/mtf/plc/bin/mosml -P full -quietdec

will run the interpreter on the contents of the files prog04.sml and prog04_tests.sml without prompts printed.

Executing

$ cat prog04.sml util.sml test-Aa-unzip.sml | /usr/local/pub/mtf/plc/bin/mosml -P full -quietdec

will run the interpreter on the contents of the files prog04.sml, util.sml, and test-Aa-unzip.sml without prompts printed.