

1 Syntax

<i>Constant Qualifiers</i>	$q \in \text{Quals} = \{U, R, A, L\}$	
<i>Qualifiers</i>	$q ::= \xi \mid q$	
<i>Locations</i>	$l \in \text{Locs}$	
<i>Constant Region Identifiers</i>	$\tau \in \text{RIds}$	
<i>Region Identifiers</i>	$r ::= \varrho \mid \tau$	
<i>Expressions</i>	$e ::= l \mid$ $x \mid {}^{qa} \lambda x:\tau. e \mid e_1 e_2 \mid$ ${}^{qa} \langle \rangle \mid \text{let } \langle \rangle = e_1 \text{ in } e_2 \mid {}^{qa} \langle e_1, e_2 \rangle \mid \text{let } \langle x_1, x_2 \rangle = e_1 \text{ in } e_2 \mid$ ${}^{qa} \langle \rangle \mid {}^{qa} \langle e_1, e_2 \rangle \mid \text{fst } e \mid \text{snd } e \mid$ $\text{abort } e \mid {}^{qa} \text{inl } e \mid {}^{qa} \text{inr } e \mid \text{case } e_1 \text{ of inl } x \Rightarrow e_l \parallel \text{inr } y \Rightarrow e_r \mid$ ${}^{qa} \Lambda \xi. e \mid e[q] \mid {}^{qa} \text{pack}(q, e) \mid \text{let pack}(\xi, x) = e_1 \text{ in } e_2 \mid$ ${}^{qa} \Lambda \bar{\alpha}. e \mid e[\bar{\tau}] \mid {}^{qa} \text{pack}(\bar{\tau}, e) \mid \text{let pack}(\bar{\alpha}, x) = e_1 \text{ in } e_2 \mid$ ${}^{qa} \Lambda \alpha. e \mid e[\tau] \mid {}^{qa} \text{pack}(\tau, e) \mid \text{let pack}(\alpha, x) = e_1 \text{ in } e_2 \mid$ ${}^{qa} \text{fold } e \mid \text{unfold } e \mid$ ${}^{qc, qh} \text{newrgn} \mid \text{freergn } e_1 e_2 \mid$ ${}^{qa} \text{new } e_1 e_2 e_3 \mid \text{free } e_1 e_2 \mid \text{read } e_1 e_2 \mid \text{swap } e_1 e_2 e_3 \mid$ ${}^{qa} \Lambda \varrho. e \mid e[r] \mid {}^{qa} \text{pack}(r, e) \mid \text{let pack}(\varrho, x) = e_1 \text{ in } e_2$	
<i>Pointers</i>	$p \in \text{Ptrs}$	
<i>Values</i>	$v ::= \lambda x:\tau. e \mid$ $\langle \rangle \mid \langle l_1, l_2 \rangle \mid$ $\langle \rangle \mid \langle e_1, e_2 \rangle \mid$ $\text{inl } l \mid \text{inr } l \mid$ $\Lambda \xi. e \mid \text{pack}(q, e) \mid \Lambda \bar{\alpha}. e \mid \text{pack}(\bar{\tau}, e) \mid \Lambda \alpha. e \mid \text{pack}(\tau, e) \mid$ $\text{fold } l \mid$ $\text{cap} \mid \text{hnd } r \mid \text{ref } r p \mid$ $\Lambda \varrho. e \mid \text{pack}(r, e)$	
<i>PreTypes</i>	$\bar{\tau} ::= \bar{\alpha} \mid$ $\tau_1 \multimap \tau_2 \mid$ $\mathbf{1}_{\otimes} \mid \tau_1 \otimes \tau_2 \mid$ $\mathbf{1}_{\oplus} \mid \tau_1 \oplus \tau_2 \mid$ $\mathbf{0} \mid \tau_1 \oplus \tau_2 \mid$ $\forall \xi. \tau \mid \exists \xi. \tau \mid \forall \bar{\alpha}. \tau \mid \exists \bar{\alpha}. \tau \mid \forall \alpha. \tau \mid \exists \alpha. \tau \mid$ $\mu \bar{\alpha}. \tau \mid$ $\text{cap } r \mid \text{hnd } r \mid \text{ref } r \tau \mid$ $\forall \varrho. \tau \mid \exists \varrho. \tau$	
<i>Types</i>	$\tau ::= \alpha \mid {}^{q\bar{\tau}}$	
<i>Type-level Contexts</i>	$\Delta ::= \bullet \mid \Delta, \xi \mid \Delta, \bar{\alpha} \mid \Delta, \alpha \mid \Delta, \varrho$	
<i>Expression-level Contexts</i>	$\Gamma ::= \bullet \mid \Gamma, x:\tau$	
<i>Flags</i>	$f \in \{\text{unused}, \text{used}\}$	
<i>Stores</i>	$\sigma ::= \bullet \mid \sigma, l \mapsto (q, v, f)$	
<i>Store Typings</i>	$\Sigma ::= \bullet \mid \Sigma, l \mapsto \tau$	
<i>Regions</i>	$\theta ::= \bullet \mid \theta, p \mapsto (q, l)$	
<i>Region Typings</i>	$\Theta ::= \bullet \mid \Theta, p \mapsto (q, \tau)$	
<i>Region Marks</i>	$v ::= {}^q \text{live} \mid \text{dead}$	
<i>Region Capability Tokens</i>	$\Upsilon ::= {}^q \text{pre} \mid \text{abs}$	
<i>Heaps</i>	$\psi ::= \bullet \mid \psi, \tau \mapsto (v, \theta)$	
<i>Heap Typings</i>	$\Psi ::= \bullet \mid \Psi, \tau \mapsto (\Upsilon, \Theta)$	

2 Dynamic Semantics

2.1 $(\sigma; \mathbf{q}; v) \xrightarrow{\text{alloc}} (\sigma'; l')$

$$\frac{l \notin \text{dom}(\sigma)}{(\sigma; \mathbf{q}; v) \xrightarrow{\text{alloc}} (\sigma, l \mapsto (\mathbf{q}, v, \text{unused}); l)}$$

2.2 $(\sigma; l) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}'; v')$

$$\frac{\mathbf{q} \sqsubseteq R}{(\sigma, l \mapsto (\mathbf{q}, v, f); l) \xrightarrow{\text{fetch}} (\sigma, l \mapsto (\mathbf{q}, v, \text{used}); \mathbf{q}; v)} \qquad \frac{A \sqsubseteq \mathbf{q}}{(\sigma, l \mapsto (\mathbf{q}, v, f); l) \xrightarrow{\text{fetch}} (\sigma; \mathbf{q}; v)}$$

$$\frac{l \neq l' \quad (\sigma_1; l) \xrightarrow{\text{fetch}} (\sigma_2; \mathbf{q}; v)}{(\sigma_1, l' \mapsto (\mathbf{q}', v', f'); l) \xrightarrow{\text{fetch}} (\sigma_2, l' \mapsto (\mathbf{q}', v', f'); \mathbf{q}; v)}$$

$$2.3 \quad (\psi; \mathbf{q}_r) \xrightarrow{\text{newrgn}} (\psi'; \mathbf{r}')$$

$$\frac{\mathbf{r} \notin \text{dom}(\psi)}{\psi \xrightarrow{\text{newrgn}} (\psi, \mathbf{r} \mapsto (\text{qr live}, \bullet); \mathbf{r})}$$

$$2.4 \quad (\psi; \mathbf{r}) \xrightarrow{\text{freergn}} \psi'$$

$$\frac{}{(\psi, \mathbf{r} \mapsto (\text{qr live}, \theta); \mathbf{r}) \xrightarrow{\text{freergn}} \psi, \mathbf{r} \mapsto (\text{dead}, \theta)} \quad \frac{\mathbf{r} \neq \mathbf{r}' \quad (\psi_1; \mathbf{r}) \xrightarrow{\text{freergn}} \psi_2}{(\psi_1, \mathbf{r}' \mapsto (\mathbf{v}', \theta'); \mathbf{r}) \xrightarrow{\text{freergn}} \psi_2, \mathbf{r}' \mapsto (\mathbf{v}', \theta')}$$

$$2.5 \quad (\theta; \mathbf{q}; \mathbf{l}_r) \xrightarrow{\text{new}} (\theta'; \mathbf{p}') \quad \text{and} \quad (\psi; \mathbf{r}; \mathbf{q}; \mathbf{l}_r) \xrightarrow{\text{new}} (\psi'; \mathbf{p}')$$

$$\frac{\mathbf{p} \notin \text{dom}(\theta)}{(\theta; \mathbf{q}; \mathbf{l}_r) \xrightarrow{\text{new}} (\theta, \mathbf{p} \mapsto (\mathbf{q}, \mathbf{l}_r); \mathbf{p})}$$

$$\frac{(\theta; \mathbf{q}; \mathbf{l}_r) \xrightarrow{\text{new}} (\theta'; \mathbf{p}')}{(\psi, \mathbf{r} \mapsto (\text{qr live}, \theta); \mathbf{r}; \mathbf{q}; \mathbf{l}_r) \xrightarrow{\text{new}} (\psi, \mathbf{r} \mapsto (\text{qr live}, \theta'); \mathbf{p}')} \quad \frac{\mathbf{r} \neq \mathbf{r}' \quad (\psi_1; \mathbf{r}; \mathbf{q}; \mathbf{l}_r) \xrightarrow{\text{new}} (\psi_2; \mathbf{p})}{(\psi_1, \mathbf{r}' \mapsto (\mathbf{v}', \theta'); \mathbf{r}; \mathbf{q}; \mathbf{l}_r) \xrightarrow{\text{new}} (\psi_2, \mathbf{r}' \mapsto (\mathbf{v}', \theta'); \mathbf{p})}$$

$$2.6 \quad (\theta; \mathbf{p}) \xrightarrow{\text{free}} (\theta'; \mathbf{l}'_r) \quad \text{and} \quad (\psi; \mathbf{r}; \mathbf{p}) \xrightarrow{\text{free}} (\psi'; \mathbf{l}'_r)$$

$$\frac{}{(\theta, \mathbf{p} \mapsto (\mathbf{q}, \mathbf{l}_r); \mathbf{p}) \xrightarrow{\text{free}} (\theta; \mathbf{l}_r)} \quad \frac{\mathbf{p} \neq \mathbf{p}' \quad (\theta_1; \mathbf{p}) \xrightarrow{\text{free}} (\theta_2; \mathbf{l}_r)}{(\theta_1, \mathbf{p}' \mapsto (\mathbf{q}', \mathbf{l}'_r); \mathbf{p}) \xrightarrow{\text{free}} (\theta_2, \mathbf{p}' \mapsto (\mathbf{q}', \mathbf{l}'_r); \mathbf{l}_r)}$$

$$\frac{(\theta; \mathbf{p}) \xrightarrow{\text{free}} (\theta'; \mathbf{l}'_r)}{(\psi, \mathbf{r} \mapsto (\text{qr live}, \theta); \mathbf{r}; \mathbf{p}) \xrightarrow{\text{free}} (\psi, \mathbf{r} \mapsto (\text{qr live}, \theta'); \mathbf{l}'_r)} \quad \frac{\mathbf{r} \neq \mathbf{r}' \quad (\psi_1; \mathbf{r}; \mathbf{p}) \xrightarrow{\text{free}} (\psi_2; \mathbf{l}_r)}{(\psi_1, \mathbf{r}' \mapsto (\mathbf{v}', \theta'); \mathbf{r}; \mathbf{p}) \xrightarrow{\text{free}} (\psi_2, \mathbf{r}' \mapsto (\mathbf{v}', \theta'); \mathbf{l}_r)}$$

$$2.7 \quad (\theta; \mathbf{p}) \xrightarrow{\text{read}} \mathbf{l}'_r \quad \text{and} \quad (\psi; \mathbf{r}; \mathbf{p}) \xrightarrow{\text{read}} \mathbf{l}'_r$$

$$\frac{}{(\theta, \mathbf{p} \mapsto (\mathbf{q}, \mathbf{l}_r); \mathbf{p}) \xrightarrow{\text{read}} \mathbf{l}_r} \quad \frac{\mathbf{p} \neq \mathbf{p}' \quad (\theta_1; \mathbf{p}) \xrightarrow{\text{read}} \mathbf{l}_r}{(\theta_1, \mathbf{p}' \mapsto (\mathbf{q}', \mathbf{l}'_r); \mathbf{p}) \xrightarrow{\text{read}} \mathbf{l}_r}$$

$$\frac{(\theta; \mathbf{p}) \xrightarrow{\text{read}} \mathbf{l}'_r}{(\psi, \mathbf{r} \mapsto (\text{qr live}, \theta); \mathbf{r}; \mathbf{p}) \xrightarrow{\text{read}} \mathbf{l}'_r} \quad \frac{\mathbf{r} \neq \mathbf{r}' \quad (\psi_1; \mathbf{r}; \mathbf{p}) \xrightarrow{\text{read}} \mathbf{l}_r}{(\psi_1, \mathbf{r}' \mapsto (\mathbf{v}', \theta'); \mathbf{r}; \mathbf{p}) \xrightarrow{\text{read}} \mathbf{l}_r}$$

$$2.8 \quad (\theta; \mathbf{p}; \mathbf{l}_{r*}) \xrightarrow{\text{swap}} (\theta'; \mathbf{l}'_r) \quad \text{and} \quad (\psi; \mathbf{r}; \mathbf{p}; \mathbf{l}_{r*}) \xrightarrow{\text{swap}} (\psi'; \mathbf{l}'_r)$$

$$\frac{}{(\theta, \mathbf{p} \mapsto (\mathbf{q}, \mathbf{l}_r); \mathbf{p}; \mathbf{l}_{r*}) \xrightarrow{\text{swap}} (\theta, \mathbf{p} \mapsto (\mathbf{q}, \mathbf{l}_{r*}); \mathbf{l}_r)} \quad \frac{\mathbf{p} \neq \mathbf{p}' \quad (\theta_1; \mathbf{p}; \mathbf{l}_{r*}) \xrightarrow{\text{swap}} (\theta_2; \mathbf{l}_r)}{(\theta_1, \mathbf{p}' \mapsto (\mathbf{q}', \mathbf{l}'_r); \mathbf{p}; \mathbf{l}_{r*}) \xrightarrow{\text{swap}} (\theta_2, \mathbf{p}' \mapsto (\mathbf{q}', \mathbf{l}'_r); \mathbf{l}_r)}$$

$$\frac{(\theta; \mathbf{p}; \mathbf{l}_{r*}) \xrightarrow{\text{swap}} (\theta'; \mathbf{l}'_r)}{(\psi, \mathbf{r} \mapsto (\text{qr live}, \theta); \mathbf{r}; \mathbf{p}; \mathbf{l}_{r*}) \xrightarrow{\text{swap}} (\psi, \mathbf{r} \mapsto (\text{qr live}, \theta'); \mathbf{l}'_r)} \quad \frac{\mathbf{r} \neq \mathbf{r}' \quad (\psi_1; \mathbf{r}; \mathbf{p}; \mathbf{l}_{r*}) \xrightarrow{\text{swap}} (\psi_2; \mathbf{l}_r)}{(\psi_1, \mathbf{r}' \mapsto (\mathbf{v}', \theta'); \mathbf{r}; \mathbf{p}; \mathbf{l}_{r*}) \xrightarrow{\text{swap}} (\psi_2, \mathbf{r}' \mapsto (\mathbf{v}', \theta'); \mathbf{l}_r)}$$

2.9 $(\sigma, \psi, e) \mapsto (\sigma', \psi', e')$

$$\begin{array}{c}
\frac{(\sigma; \mathbf{q}_a; \lambda x:\tau. e) \xrightarrow{\text{alloc}} (\sigma'; \mathbf{l})}{(\sigma, \psi, {}^{\mathbf{q}_a} \lambda x:\tau. e) \mapsto (\sigma', \psi, \mathbf{l}')} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi', e_1 e_2) \mapsto (\sigma', \psi', e'_1 e_2)} \quad \frac{(\sigma, \psi, e_2) \mapsto (\sigma', \psi', e'_2)}{(\sigma, \psi, \mathbf{l}_1 e_2) \mapsto (\sigma', \psi', \mathbf{l}_1 e'_2)} \quad \frac{(\sigma; \mathbf{l}_1) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \lambda x:\tau. e)}{(\sigma, \psi, \mathbf{l}_1 \mathbf{l}_2) \mapsto (\sigma', \psi, e[\mathbf{l}_2/x])} \\
\\
\frac{(\sigma; \mathbf{q}_a; \langle \rangle) \xrightarrow{\text{alloc}} (\sigma'; \mathbf{l}')}{(\sigma, \psi, {}^{\mathbf{q}_a} \langle \rangle) \mapsto (\sigma', \psi', \mathbf{l}')} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \mathbf{let} \langle \rangle = e_1 \mathbf{in} e_2) \mapsto (\sigma', \psi', \mathbf{let} \langle \rangle = e'_1 \mathbf{in} e_2)} \quad \frac{(\sigma; \mathbf{l}_1) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \langle \rangle)}{(\sigma, \psi, \mathbf{let} \langle \rangle = \mathbf{l}_1 \mathbf{in} e_2) \mapsto (\sigma', \psi, e_2)} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, {}^{\mathbf{q}_a} (e_1, e_2)) \mapsto (\sigma', \psi', {}^{\mathbf{q}_a} (e'_1, e_2))} \quad \frac{(\sigma, \psi, e_2) \mapsto (\sigma', \psi', e'_2)}{(\sigma, \psi, {}^{\mathbf{q}_a} \langle \mathbf{l}_1, e_2 \rangle) \mapsto (\sigma', \psi', {}^{\mathbf{q}_a} \langle \mathbf{l}_1, e'_2 \rangle)} \quad \frac{(\sigma; \mathbf{q}_a; \langle \mathbf{l}_1, \mathbf{l}_2 \rangle) \xrightarrow{\text{alloc}} (\sigma'; \mathbf{l}')}{(\sigma, \psi, {}^{\mathbf{q}_a} \langle \mathbf{l}_1, \mathbf{l}_2 \rangle) \mapsto (\sigma', \psi, \mathbf{l}')} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \mathbf{let} \langle x, y \rangle = e_1 \mathbf{in} e_2) \mapsto (\sigma', \psi', \mathbf{let} \langle x, y \rangle = e'_1 \mathbf{in} e_2)} \\
\\
\frac{(\sigma; \mathbf{l}_1) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \langle \mathbf{l}_x, \mathbf{l}_y \rangle)}{(\sigma, \psi, \mathbf{let} \langle x, y \rangle = \mathbf{l}_1 \mathbf{in} e_2) \mapsto (\sigma', \psi, e_2[\mathbf{l}_x/x][\mathbf{l}_y/y])} \\
\\
\frac{(\sigma; \mathbf{q}_a; \langle \rangle) \xrightarrow{\text{alloc}} (\sigma'; \mathbf{l}')}{(\sigma, \psi, {}^{\mathbf{q}_a} \langle \rangle) \mapsto (\sigma', \psi, \mathbf{l}')} \\
\\
\frac{(\sigma; \mathbf{q}_a; \langle e_1, e_2 \rangle) \xrightarrow{\text{alloc}} (\sigma'; \mathbf{l}')}{(\sigma, \psi, {}^{\mathbf{q}_a} \langle e_1, e_2 \rangle) \mapsto (\sigma', \psi, \mathbf{l}')} \\
\\
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, \mathbf{fst} e) \mapsto (\sigma', \psi', \mathbf{fst} e')} \quad \frac{(\sigma; \mathbf{l}) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \langle e_1, e_2 \rangle)}{(\sigma, \psi, \mathbf{fst} \mathbf{l}) \mapsto (\sigma', \psi, e_1)} \\
\\
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, \mathbf{snd} e) \mapsto (\sigma', \psi', \mathbf{snd} e')} \quad \frac{(\sigma; \mathbf{l}) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \langle e_1, e_2 \rangle)}{(\sigma, \psi, \mathbf{snd} \mathbf{l}) \mapsto (\sigma', \psi, e_2)}
\end{array}$$

$$\begin{array}{c}
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, \mathbf{abort} \ e) \mapsto (\sigma', \psi', \mathbf{abort} \ e')} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, {}^{qa} \mathbf{inl} \ e_1) \mapsto (\sigma', \psi', {}^{qa} \mathbf{inl} \ e'_1)} \qquad \frac{(\sigma; \mathbf{q}_a; \mathbf{inl} \ l_1) \xrightarrow{\mathbf{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{qa} \mathbf{inl} \ l_1) \mapsto (\sigma', \psi, l')} \\
\\
\frac{(\sigma, \psi, e_2) \mapsto (\sigma', \psi', e'_2)}{(\sigma, \psi, {}^{qa} \mathbf{inr} \ e_2) \mapsto (\sigma', \psi', {}^{qa} \mathbf{inr} \ e'_2)} \qquad \frac{(\sigma; \mathbf{q}_a; \mathbf{inr} \ l_2) \xrightarrow{\mathbf{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{qa} \mathbf{inr} \ l_2) \mapsto (\sigma', \psi, l')} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \mathbf{case} \ e_1 \ \mathbf{of} \ \mathbf{inl} \ x \Rightarrow e_l \ \parallel \ \mathbf{inr} \ y \Rightarrow e_r) \mapsto (\sigma', \psi', \mathbf{case} \ e'_1 \ \mathbf{of} \ \mathbf{inl} \ x \Rightarrow e_l \ \parallel \ \mathbf{inr} \ y \Rightarrow e_r)} \\
\\
\frac{(\sigma; l_1) \xrightarrow{\mathbf{fetch}} (\sigma'; \mathbf{q}_a; \mathbf{inl} \ l_x)}{(\sigma, \psi, \mathbf{case} \ l_1 \ \mathbf{of} \ \mathbf{inl} \ x \Rightarrow e_l \ \parallel \ \mathbf{inr} \ y \Rightarrow e_r) \mapsto (\sigma', \psi, e_l[l_x/x])} \\
\\
\frac{(\sigma; l_1) \xrightarrow{\mathbf{fetch}} (\sigma'; \mathbf{q}_a; \mathbf{inr} \ l_y)}{(\sigma, \psi, \mathbf{case} \ l_1 \ \mathbf{of} \ \mathbf{inl} \ x \Rightarrow e_l \ \parallel \ \mathbf{inr} \ y \Rightarrow e_r) \mapsto (\sigma', \psi, e_r[l_y/y])}
\end{array}$$

$$\begin{array}{c}
\frac{(\sigma; \mathbf{q}_a; \Lambda \xi. e) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \Lambda \xi. e) \mapsto (\sigma', \psi, l')} \quad \frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, e [q]) \mapsto (\sigma', \psi', e' [q])} \quad \frac{(\sigma; l) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \Lambda \xi. e)}{(\sigma, \psi, l [q]) \mapsto (\sigma', \psi, e [q/\xi])} \\
\\
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{pack}(q, e)) \mapsto (\sigma', \psi', {}^{\mathbf{q}_a} \text{pack}(q, e'))} \quad \frac{(\sigma; \mathbf{q}_a; \text{pack}(q, l)) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{pack}(q, l)) \mapsto (\sigma', \psi, l')} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \text{let pack}(\xi, x) = e_1 \text{ in } e_2) \mapsto (\sigma', \psi', \text{let pack}(\xi, x) = e'_1 \text{ in } e_2)} \\
\\
\frac{(\sigma; l_1) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \text{pack}(q, l_x))}{(\sigma, \psi, \text{let pack}(\xi, x) = l_1 \text{ in } e_2) \mapsto (\sigma', \psi, e_2[q/\xi][l_x/x])} \\
\\
\frac{(\sigma; \mathbf{q}_a; \Lambda \bar{\alpha}. e) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \Lambda \bar{\alpha}. e) \mapsto (\sigma', \psi', l')} \quad \frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, e [\bar{\tau}]) \mapsto (\sigma', \psi', e' [\bar{\tau}])} \quad \frac{(\sigma; l) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \Lambda \bar{\alpha}. e)}{(\sigma, \psi, l [\bar{\tau}]) \mapsto (\sigma', \psi, e[\bar{\tau}/\bar{\alpha}])} \\
\\
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{pack}(\bar{\tau}, e)) \mapsto (\sigma', \psi', {}^{\mathbf{q}_a} \text{pack}(\bar{\tau}, e'))} \quad \frac{(\sigma; \mathbf{q}_a; \text{pack}(\bar{\tau}, l)) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{pack}(\bar{\tau}, l)) \mapsto (\sigma', \psi, l')} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \text{let pack}(\bar{\alpha}, x) = e_1 \text{ in } e_2) \mapsto (\sigma', \psi', \text{let pack}(\bar{\alpha}, x) = e'_1 \text{ in } e_2)} \\
\\
\frac{(\sigma; l_1) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \text{pack}(\bar{\tau}, l_x))}{(\sigma, \psi, \text{let pack}(\bar{\alpha}, x) = l_1 \text{ in } e_2) \mapsto (\sigma', \psi, e_2[\bar{\tau}/\bar{\alpha}][l_x/x])} \\
\\
\frac{(\sigma; \mathbf{q}_a; \Lambda \alpha. e) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \Lambda \alpha. e) \mapsto (\sigma', \psi, l')} \quad \frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, e [\tau]) \mapsto (\sigma', \psi', e' [\tau])} \quad \frac{(\sigma; l) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \Lambda \alpha. e)}{(\sigma, \psi, l [\tau]) \mapsto (\sigma', \psi, e[\tau/\alpha])} \\
\\
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{pack}(\tau, e)) \mapsto (\sigma', \psi', {}^{\mathbf{q}_a} \text{pack}(\tau, e'))} \quad \frac{(\sigma; \mathbf{q}_a; \text{pack}(\tau, l)) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{pack}(\tau, l)) \mapsto (\sigma', \psi, l')} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \text{let pack}(\alpha, x) = e_1 \text{ in } e_2) \mapsto (\sigma', \psi', \text{let pack}(\alpha, x) = e'_1 \text{ in } e_2)} \\
\\
\frac{(\sigma; l_1) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \text{pack}(\tau, l_x))}{(\sigma, \psi, \text{let pack}(\alpha, x) = l_1 \text{ in } e_2) \mapsto (\sigma', \psi, e_2[\tau/\alpha][l_x/x])} \\
\\
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{fold } e) \mapsto (\sigma', \psi', {}^{\mathbf{q}_a} \text{fold } e')} \quad \frac{(\sigma; \mathbf{q}_a; \text{fold } l) \xrightarrow{\text{alloc}} (\sigma'; l')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{fold } l) \mapsto (\sigma', \psi, l')} \\
\\
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, \text{unfold } e) \mapsto (\sigma', \psi', \text{unfold } e')} \quad \frac{(\sigma; l) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \text{fold } l')}{(\sigma, \psi, \text{unfold } l) \mapsto (\sigma', \psi, l')}
\end{array}$$

$$\begin{array}{c}
\frac{(\psi; \mathbf{q}_c) \xrightarrow{\text{newrgn}} (\psi'; \mathbf{r}') \quad (\sigma; \mathbf{q}_c; \mathbf{cap}) \xrightarrow{\text{alloc}} (\sigma'_c; \mathbf{l}'_c)}{(\sigma'_c; \mathbf{q}_h; \mathbf{hnd} \mathbf{r}') \xrightarrow{\text{alloc}} (\sigma'_h; \mathbf{l}'_h) \quad (\sigma'_h; \mathbf{L}; \langle \mathbf{l}_c, \mathbf{l}_h \rangle) \xrightarrow{\text{alloc}} (\sigma'_z; \mathbf{l}'_z) \quad (\sigma'_z; \mathbf{L}; \mathbf{pack}(\mathbf{r}', \mathbf{l}'_z)) \xrightarrow{\text{alloc}} (\sigma'; \mathbf{l}')} \\
(\sigma, \psi, \mathbf{q}_c, \mathbf{q}_h \mathbf{newrgn}) \mapsto (\sigma', \psi', \mathbf{l}')
\end{array}$$

$$\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \mathbf{freergn} e_1 e_2) \mapsto (\sigma', \psi', \mathbf{freergn} e'_1 e_2)} \quad \frac{(\sigma, \psi, e_2) \mapsto (\sigma', \psi', e'_2)}{(\sigma, \psi, \mathbf{freergn} l_1 e_2) \mapsto (\sigma', \psi', \mathbf{freergn} l_1 e'_2)}$$

$$\frac{(\sigma; \mathbf{l}_c) \xrightarrow{\text{fetch}} (\sigma_c; \mathbf{q}_c; \mathbf{cap}) \quad (\sigma_c; \mathbf{l}_h) \xrightarrow{\text{fetch}} (\sigma_h; \mathbf{q}_h; \mathbf{hnd} \mathbf{r}') \quad (\psi; \mathbf{r}') \xrightarrow{\text{freergn}} \psi' \quad (\sigma_h; \mathbf{L}; \langle \rangle) \xrightarrow{\text{alloc}} (\sigma'; \mathbf{l}')}{(\sigma, \psi, \mathbf{freergn} l_c l_h) \mapsto (\sigma', \psi', \mathbf{l}')}$$

$$\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \mathbf{q}_a \mathbf{new} e_1 e_2 e_3) \mapsto (\sigma', \psi', \mathbf{q}_a \mathbf{new} e'_1 e_2 e_3)} \quad \frac{(\sigma, \psi, e_2) \mapsto (\sigma', \psi', e'_2)}{(\sigma, \psi, \mathbf{q}_a \mathbf{new} l_1 e_2 e_3) \mapsto (\sigma', \psi', \mathbf{q}_a \mathbf{new} l_1 e'_2 e_3)}$$

$$\frac{(\sigma, \psi, e_3) \mapsto (\sigma', \psi', e'_3)}{(\sigma, \psi, \mathbf{q}_a \mathbf{new} l_1 l_2 e_3) \mapsto (\sigma', \psi', \mathbf{q}_a \mathbf{new} l_1 l_2 e'_3)}$$

$$\frac{(\sigma; \mathbf{l}_c) \xrightarrow{\text{fetch}} (\sigma_c; \mathbf{q}_c; \mathbf{cap}) \quad (\sigma_c; \mathbf{l}_h) \xrightarrow{\text{fetch}} (\sigma_h; \mathbf{q}_h; \mathbf{hnd} \mathbf{r}') \quad (\psi; \mathbf{r}'; \mathbf{q}_a; \mathbf{l}_r) \xrightarrow{\text{new}} (\psi'; \mathbf{p}')}{(\sigma_h; \mathbf{q}_c; \mathbf{cap}) \xrightarrow{\text{alloc}} (\sigma'_c; \mathbf{l}'_c) \quad (\sigma'_c; \mathbf{q}_a; \mathbf{ref} \mathbf{r}' \mathbf{p}') \xrightarrow{\text{alloc}} (\sigma'_p; \mathbf{l}'_p) \quad (\sigma'_p; \mathbf{L}; \langle \mathbf{l}'_c, \mathbf{l}'_p \rangle) \xrightarrow{\text{alloc}} (\sigma'; \mathbf{l}')} \\
(\sigma, \psi, \mathbf{q}_a \mathbf{new} l_c l_h l_r) \mapsto (\sigma', \psi', \mathbf{l}')$$

$$\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \mathbf{free} e_1 e_2) \mapsto (\sigma', \psi', \mathbf{free} e'_1 e_2)} \quad \frac{(\sigma, \psi, e_2) \mapsto (\sigma', \psi', e'_2)}{(\sigma, \psi, \mathbf{free} l_1 e_2) \mapsto (\sigma', \psi', \mathbf{free} l_1 e'_2)}$$

$$\frac{(\sigma; \mathbf{l}_c) \xrightarrow{\text{fetch}} (\sigma_c; \mathbf{q}_c; \mathbf{cap}) \quad (\sigma_c; \mathbf{l}_p) \xrightarrow{\text{fetch}} (\sigma_p; \mathbf{q}_p; \mathbf{ref} \mathbf{r}' \mathbf{p}') \quad (\psi; \mathbf{r}'; \mathbf{p}') \xrightarrow{\text{free}} (\psi'; \mathbf{l}'_r) \quad (\sigma_p; \mathbf{q}_c; \mathbf{cap}) \xrightarrow{\text{alloc}} (\sigma'_c; \mathbf{l}'_c) \quad (\sigma'_c; \mathbf{L}; \langle \mathbf{l}'_c, \mathbf{l}'_r \rangle) \xrightarrow{\text{alloc}} (\sigma'; \mathbf{l}')}{(\sigma, \psi, \mathbf{free} l_c l_p) \mapsto (\sigma', \psi', \mathbf{l}')}$$

$$\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \mathbf{read} e_1 e_2) \mapsto (\sigma', \psi', \mathbf{read} e'_1 e_2)} \quad \frac{(\sigma, \psi, e_2) \mapsto (\sigma', \psi', e'_2)}{(\sigma, \psi, \mathbf{read} l_1 e_2) \mapsto (\sigma', \psi', \mathbf{read} l_1 e'_2)}$$

$$\frac{(\sigma; \mathbf{l}_c) \xrightarrow{\text{fetch}} (\sigma_c; \mathbf{q}_c; \mathbf{cap}) \quad (\sigma_c; \mathbf{l}_p) \xrightarrow{\text{fetch}} (\sigma_p; \mathbf{q}_p; \mathbf{ref} \mathbf{r}' \mathbf{p}') \quad (\psi; \mathbf{r}'; \mathbf{p}') \xrightarrow{\text{read}} \mathbf{l}'_r \quad (\sigma_p; \mathbf{q}_c; \mathbf{cap}) \xrightarrow{\text{alloc}} (\sigma'_c; \mathbf{l}'_c)}{(\sigma'_c; \mathbf{q}_p; \mathbf{ref} \mathbf{r}' \mathbf{p}') \xrightarrow{\text{alloc}} (\sigma'_p; \mathbf{l}'_p) \quad (\sigma'_p; \mathbf{L}; \langle \mathbf{l}'_c, \mathbf{l}'_p \rangle) \xrightarrow{\text{alloc}} (\sigma'_z; \mathbf{l}'_z) \quad (\sigma'_z; \mathbf{L}; \langle \mathbf{l}'_z, \mathbf{l}'_r \rangle) \xrightarrow{\text{alloc}} (\sigma'; \mathbf{l}')} \\
(\sigma, \psi, \mathbf{read} l_c l_p) \mapsto (\sigma', \psi', \mathbf{l}')$$

$$\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \mathbf{swap} e_1 e_2 e_3) \mapsto (\sigma', \psi', \mathbf{swap} e'_1 e_2 e_3)} \quad \frac{(\sigma, \psi, e_2) \mapsto (\sigma', \psi', e'_2)}{(\sigma, \psi, \mathbf{swap} l_1 e_2 e_3) \mapsto (\sigma', \psi', \mathbf{swap} l_1 e'_2 e_3)}$$

$$\frac{(\sigma, \psi, e_3) \mapsto (\sigma', \psi', e'_3)}{(\sigma, \psi, \mathbf{swap} l_1 l_2 e_3) \mapsto (\sigma', \psi', \mathbf{swap} l_1 l_2 e'_3)}$$

$$\frac{(\sigma; \mathbf{l}_c) \xrightarrow{\text{fetch}} (\sigma_c; \mathbf{q}_c; \mathbf{cap}) \quad (\sigma_c; \mathbf{l}_p) \xrightarrow{\text{fetch}} (\sigma_p; \mathbf{q}_p; \mathbf{ref} \mathbf{r}' \mathbf{p}') \quad (\psi; \mathbf{r}'; \mathbf{p}'; \mathbf{l}_{r*}) \xrightarrow{\text{swap}} (\psi'; \mathbf{l}'_r) \quad (\sigma_p; \mathbf{q}_c; \mathbf{cap}) \xrightarrow{\text{alloc}} (\sigma'_c; \mathbf{l}'_c)}{(\sigma'_c; \mathbf{q}_p; \mathbf{ref} \mathbf{r}' \mathbf{p}') \xrightarrow{\text{alloc}} (\sigma'_p; \mathbf{l}'_p) \quad (\sigma'_p; \mathbf{L}; \langle \mathbf{l}'_c, \mathbf{l}'_p \rangle) \xrightarrow{\text{alloc}} (\sigma'_z; \mathbf{l}'_z) \quad (\sigma'_z; \mathbf{L}; \langle \mathbf{l}'_z, \mathbf{l}'_r \rangle) \xrightarrow{\text{alloc}} (\sigma'; \mathbf{l}')} \\
(\sigma, \psi, \mathbf{swap} l_c l_p l_{r*}) \mapsto (\sigma', \psi', \mathbf{l}')$$

$$\begin{array}{c}
\frac{(\sigma; \mathbf{q}_a; \Lambda \varrho. e) \xrightarrow{\text{alloc}} (\sigma'; \mathbf{l}')}{(\sigma, \psi, {}^{\mathbf{q}_a} \Lambda \varrho. e) \mapsto (\sigma', \psi, \mathbf{l}')} \quad \frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, e[r]) \mapsto (\sigma', \psi', e'[r])} \quad \frac{(\sigma; \mathbf{l}) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \Lambda \varrho. e)}{(\sigma, \psi', \mathbf{l}[r]) \mapsto (\sigma', \psi, e[r/\varrho])} \\
\\
\frac{(\sigma, \psi, e) \mapsto (\sigma', \psi', e')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{pack}(r, e)) \mapsto (\sigma', \psi', {}^{\mathbf{q}_a} \text{pack}(r, e'))} \quad \frac{(\sigma; \mathbf{q}_a; \text{pack}(r, \mathbf{l})) \xrightarrow{\text{alloc}} (\sigma'; \mathbf{l}')}{(\sigma, \psi, {}^{\mathbf{q}_a} \text{pack}(r, \mathbf{l})) \mapsto (\sigma', \psi, \mathbf{l}')} \\
\\
\frac{(\sigma, \psi, e_1) \mapsto (\sigma', \psi', e'_1)}{(\sigma, \psi, \text{let pack}(\varrho, x) = e_1 \text{ in } e_2) \mapsto (\sigma', \psi', \text{let pack}(\varrho, x) = e'_1 \text{ in } e_2)} \\
\\
\frac{(\sigma; \mathbf{l}_1) \xrightarrow{\text{fetch}} (\sigma'; \mathbf{q}_a; \text{pack}(r, \mathbf{l}_x))}{(\sigma, \psi, \text{let pack}(\varrho, x) = \mathbf{l}_1 \text{ in } e_2) \mapsto (\sigma', \psi, e_2[r/\varrho][\mathbf{l}_x/x])}
\end{array}$$

2.10 $(\sigma, \psi, e) \mapsto^* (\sigma', \psi', e')$

$$\overline{(\sigma, \psi, e) \mapsto^* (\sigma, \psi, e)}$$

$$\frac{(\sigma_1, \psi_1, e_1) \mapsto^* (\sigma_2, \psi_2, e_2) \quad (\sigma_2, \psi_2, e_2) \mapsto^* (\sigma_3, \psi_3, e_3)}{(\sigma_1, \psi_1, e_1) \mapsto^* (\sigma_3, \psi_3, e_3)}$$

$$\frac{(\sigma_1, \psi_1, e_1) \mapsto (\sigma_2, \psi_2, e_2)}{(\sigma_1, \psi_1, e_1) \mapsto^* (\sigma_2, \psi_2, e_2)}$$

3 Static Semantics

3.1 $\Delta \vdash q \preceq q'$

$$\frac{FV(q) \subseteq \Delta}{\Delta \vdash U \preceq q}$$

$$\frac{q_1 \sqsubseteq q_2}{\Delta \vdash q_1 \preceq q_2}$$

$$\frac{FV(q) \subseteq \Delta}{\Delta \vdash q \preceq L}$$

$$\frac{FV(q) \subseteq \Delta}{\Delta \vdash q \preceq q}$$

$$\frac{\Delta \vdash q_1 \preceq q_2 \quad \Delta \vdash q_2 \preceq q_3}{\Delta \vdash q_1 \preceq q_3}$$

3.2 $\Delta \vdash \tau \preceq q'$

$$\frac{FV(\tau) \subseteq \Delta}{\Delta \vdash \tau \preceq L}$$

$$\frac{\Delta \vdash q \preceq q' \quad FV(\bar{\tau}) \subseteq \Delta}{\Delta \vdash {}^q \bar{\tau} \preceq q'}$$

3.3 $\Delta \vdash \Gamma \preceq q'$

$$\frac{FV(q') \subseteq \Delta}{\Delta \vdash \bullet \preceq q'}$$

$$\frac{\Delta \vdash \Gamma \preceq q' \quad \Delta \vdash \tau \preceq q'}{\Delta \vdash \Gamma, x:\tau \preceq q'}$$

3.4 $\Delta \vdash \Sigma \preceq q'$

$$\frac{FV(q') \subseteq \Delta}{\Delta \vdash \bullet \preceq q'}$$

$$\frac{\Delta \vdash \Sigma \preceq q' \quad \Delta \vdash \tau \preceq q'}{\Delta \vdash \Sigma, l \mapsto \tau \preceq q'}$$

3.5 $\vdash \Theta \preceq q'$

$$\overline{\vdash \bullet \preceq q'}$$

$$\frac{\vdash \Theta \preceq q' \quad \vdash q \preceq q'}{\vdash \Theta, p \mapsto (q, \tau) \preceq q'}$$

3.6 $\vdash \Upsilon \preceq q'$

$$\frac{\bullet \vdash q \preceq q'}{\vdash {}^q \text{pre} \preceq q'}$$

$$\overline{\vdash \text{abs} \preceq q'}$$

3.7 $\vdash \Psi \preceq q'$

$$\overline{\vdash \bullet \preceq q'}$$

$$\frac{\vdash \Psi \preceq q' \quad \vdash \Upsilon \preceq q' \quad \vdash \Theta \preceq q'}{\vdash \Psi, \tau \mapsto (\Upsilon, \Theta) \preceq q'}$$

3.8 $\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma$

$$\overline{\Delta \vdash \bullet \boxplus \bullet \rightsquigarrow \bullet}$$

$$\frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma}{\Delta \vdash \Gamma_1, x:\tau \boxplus \Gamma_2 \rightsquigarrow \Gamma, x:\tau} \quad \frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma}{\Delta \vdash \Gamma_1 \boxplus \Gamma_2, x:\tau \rightsquigarrow \Gamma, x:\tau}$$

$$\frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \tau \preceq R}{\Delta \vdash \Gamma_1, x:\tau \boxplus \Gamma_2, x:\tau \rightsquigarrow \Gamma, x:\tau}$$

3.9 $\Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma$

$$\overline{\Delta \vdash \bullet \odot \bullet \rightsquigarrow \bullet}$$

$$\frac{\Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma}{\Delta \vdash \Sigma_1, l \mapsto \tau \odot \Sigma_2 \rightsquigarrow \Sigma, l \mapsto \tau} \quad \frac{\Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma}{\Delta \vdash \Sigma_1 \odot \Sigma_2, l \mapsto \tau \rightsquigarrow \Sigma, l \mapsto \tau}$$

$$\frac{\Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta \vdash \tau \preceq R}{\Delta \vdash \Sigma_1, l \mapsto \tau \odot \Sigma_2, l \mapsto \tau \rightsquigarrow \Sigma, l \mapsto \tau}$$

3.10 $\vdash \Upsilon_1 \odot \Upsilon_2 \rightsquigarrow \Upsilon$

$$\frac{q \sqsubseteq R}{\vdash {}^q\text{pre} \odot {}^q\text{pre} \rightsquigarrow {}^q\text{pre}} \quad \frac{}{\vdash {}^q\text{pre} \odot \text{abs} \rightsquigarrow {}^q\text{pre}} \quad \frac{}{\vdash \text{abs} \odot {}^q\text{pre} \rightsquigarrow {}^q\text{pre}} \quad \frac{}{\vdash \text{abs} \odot \text{abs} \rightsquigarrow \text{abs}}$$

3.11 $\vdash \Theta_1 \odot \Theta_2 \rightsquigarrow \Theta$

$$\overline{\vdash \bullet \odot \bullet \rightsquigarrow \bullet}$$

$$\frac{\vdash \Theta_1 \odot \Theta_2 \rightsquigarrow \Theta}{\vdash \Theta_1, p \mapsto (q, \tau) \odot \Theta_2 \rightsquigarrow \Theta, p \mapsto (q, \tau)} \quad \frac{\vdash \Theta_1 \odot \Theta_2 \rightsquigarrow \Theta}{\vdash \Theta_1 \odot \Theta_2, p \mapsto (q, \tau) \rightsquigarrow \Theta, p \mapsto (q, \tau)}$$

$$\frac{\vdash \Theta_1 \odot \Theta_2 \rightsquigarrow \Theta \quad q \sqsubseteq R}{\vdash \Theta_1, p \mapsto (q, \tau) \odot \Theta_2, p \mapsto (q, \tau) \rightsquigarrow \Theta, p \mapsto (q, \tau)}$$

3.12 $\vdash \Psi_1 \odot \Psi_2 \rightsquigarrow \Psi$

$$\overline{\vdash \bullet \odot \bullet \rightsquigarrow \bullet}$$

$$\frac{\vdash \Psi_1 \odot \Psi_2 \rightsquigarrow \Psi}{\vdash \Psi_1, r \mapsto (\Upsilon, \Theta) \odot \Psi_2 \rightsquigarrow \Psi, r \mapsto (\Upsilon, \Theta)} \quad \frac{\vdash \Psi_1 \odot \Psi_2 \rightsquigarrow \Psi}{\vdash \Psi_1 \odot \Psi_2, r \mapsto (\Upsilon, \Theta) \rightsquigarrow \Psi, r \mapsto (\Upsilon, \Theta)}$$

$$\frac{\vdash \Psi_1 \odot \Psi_2 \rightsquigarrow \Psi \quad \vdash \Upsilon_1 \odot \Upsilon_2 \rightsquigarrow \Upsilon \quad \vdash \Theta_1 \odot \Theta_2 \rightsquigarrow \Theta}{\vdash \Psi_1, r \mapsto (\Upsilon_1, \Theta_1) \odot \Psi_2, r \mapsto (\Upsilon_2, \Theta_2) \rightsquigarrow \Psi, r \mapsto (\Upsilon, \Theta)}$$

3.13 $\Delta; \Sigma \vdash \iota : \tau$

$$\frac{FV(\tau) \subseteq \Delta}{\Delta; \bullet, \iota \mapsto \tau \vdash \iota : \tau}$$

3.14 $\Delta; \Gamma \vdash x : \tau$

$$\frac{FV(\tau) \subseteq \Delta}{\Delta; \bullet, x : \tau \vdash x : \tau}$$

3.15 $\Delta; \Gamma; \Sigma \vdash e : \tau$

$$\frac{\frac{\Delta; \Sigma \vdash \iota : \tau}{\Delta; \bullet; \Sigma \vdash \iota : \tau} \quad \frac{\Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta \vdash \Sigma_1 \preceq A \quad \Delta; \Gamma; \Sigma_2 \vdash e : \tau}{\Delta; \Gamma; \Sigma \vdash e : \tau}}{\frac{\Delta; \Gamma \vdash x : \tau}{\Delta; \Gamma; \bullet \vdash x : \tau} \quad \frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta; \Gamma_1 \preceq A \quad \Delta; \Gamma_2; \Sigma \vdash e : \tau}{\Delta; \Gamma; \Sigma \vdash e : \tau}}{\frac{\Delta \vdash \Gamma \preceq q_a \quad \Delta \vdash \Sigma \preceq q_a \quad \frac{\Delta; \Gamma, x : \tau_x; \Sigma \vdash e : \tau}{\Delta; \Gamma; \Sigma \vdash {}^{q_a} \lambda x : \tau_x. e : {}^{q_a} (\tau_x \multimap \tau)}}{\Delta; \Gamma; \Sigma \vdash {}^{q_a} \langle \lambda x : \tau_x. e : {}^{q_a} (\tau_x \multimap \tau) \rangle} \quad \frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{q_a} (\tau_x \multimap \tau) \quad \Delta; \Gamma_2; \Sigma_2 \vdash e_2 : \tau_x}{\Delta; \Gamma; \Sigma \vdash e_1 e_2 : \tau}}{\frac{\frac{FV(q_a) \subseteq \Delta}{\Delta; \bullet; \bullet \vdash {}^{q_a} \langle \rangle : {}^{q_a} \mathbf{1}_\otimes} \quad \frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{q_a} \mathbf{1}_\otimes \quad \Delta; \Gamma_2; \Sigma_2 \vdash e_2 : \tau}{\Delta; \Gamma; \Sigma \vdash \mathbf{let} \langle \rangle = e_1 \mathbf{in} e_2 : \tau}}{\frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : \tau_1 \quad \Delta \vdash \tau_1 \preceq q_a \quad \Delta; \Gamma_2; \Sigma_2 \vdash e_2 : \tau_2 \quad \Delta \vdash \tau_2 \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{q_a} \langle e_1, e_2 \rangle : {}^{q_a} (\tau_1 \otimes \tau_2)}}{\frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{q_a} (\tau_x \otimes \tau_y) \quad \Delta; \Gamma_2, x : \tau_x, y : \tau_y; \Sigma_2 \vdash e_2 : \tau}{\Delta; \Gamma; \Sigma \vdash \mathbf{let} \langle x, y \rangle = e_1 \mathbf{in} e_2 : \tau}}{\frac{\Delta \vdash \Gamma \preceq q_a \quad \Delta \vdash \Sigma \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{q_a} \langle \rangle : {}^{q_a} \mathbf{1}_\otimes}}{\frac{\Delta \vdash \Gamma \preceq q_a \quad \Delta \vdash \Sigma \preceq q_a \quad \Delta; \Gamma; \Sigma \vdash e_1 : \tau_1 \quad \Delta; \Gamma; \Sigma \vdash e_2 : \tau_2}{\Delta; \Gamma; \Sigma \vdash {}^{q_a} \langle e_1, e_2 \rangle : {}^{q_a} (\tau_1 \otimes \tau_2)}}{\frac{\frac{\Delta; \Gamma; \Sigma \vdash e : {}^{q_a} (\tau_1 \otimes \tau_2)}{\Delta; \Gamma; \Sigma \vdash \mathbf{fst} e : \tau_1} \quad \frac{\Delta; \Gamma; \Sigma \vdash e : {}^{q_a} (\tau_1 \otimes \tau_2)}{\Delta; \Gamma; \Sigma \vdash \mathbf{snd} e : \tau_2}}{\Delta; \Gamma; \Sigma \vdash e : {}^{q_a} \mathbf{0}} \quad \frac{\Delta; \Gamma; \Sigma \vdash e : {}^{q_a} \mathbf{0}}{\Delta; \Gamma; \Sigma \vdash \mathbf{abort} e : \tau}}{\frac{\Delta; \Gamma; \Sigma \vdash e_1 : \tau_1 \quad \Delta \vdash \tau_1 \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{q_a} \mathbf{inl} e_1 : {}^{q_a} (\tau_1 \oplus \tau_2)} \quad \frac{\Delta; \Gamma; \Sigma \vdash e_2 : \tau_2 \quad \Delta \vdash \tau_2 \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{q_a} \mathbf{inr} e_2 : {}^{q_a} (\tau_1 \oplus \tau_2)}}{\frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{q_a} (\tau_1 \oplus \tau_2) \quad \Delta; \Gamma_2, x : \tau_1 \vdash e_l \vdash \tau \quad \Delta; \Gamma_2, y : \tau_2 \vdash e_r \vdash \tau}{\Delta; \Gamma; \Sigma \vdash \mathbf{case} e_1 \mathbf{of} \mathbf{inl} x \Rightarrow e_l \parallel \mathbf{inr} y \Rightarrow e_r : \tau}}$$

$$\begin{array}{c}
\frac{\Delta \vdash \Gamma \preceq q_a \quad \Delta \vdash \Sigma \preceq q_a}{\Delta, \xi; \Gamma; \Sigma \vdash e : \tau} \\
\frac{\Delta; \Gamma; \Sigma \vdash e_1 : \tau[q_1/\xi] \quad \Delta \vdash \tau[q_1/\xi] \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}\text{pack}(q_1, e_2) : {}^{q_a}(\exists \xi. \tau)}
\end{array}$$

$$\begin{array}{c}
\frac{\Delta; \Gamma; \Sigma \vdash e_1 : {}^{q_a}(\forall \xi. \tau)}{\Delta; \Gamma; \Sigma \vdash e_1 [q_2] : \tau[q_2/\xi]} \\
\frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma}{\Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{q_a}(\exists \xi. \tau)} \\
\frac{\Delta, \xi; \Gamma_2, x: \tau; \Sigma_2 \vdash e_2 : \tau' \quad FV(\tau') \subseteq \Delta}{\Delta; \Gamma; \Sigma \vdash \text{let pack}(\xi, x) = e_1 \text{ in } e_2 : \tau'}
\end{array}$$

$$\begin{array}{c}
\frac{\Delta \vdash \Gamma \preceq q_a \quad \Delta \vdash \Sigma \preceq q_a}{\Delta, \bar{\alpha}; \Gamma; \Sigma \vdash e : \tau} \\
\frac{\Delta; \Gamma; \Sigma \vdash e_1 : {}^{q_a}(\forall \bar{\alpha}. \tau)}{\Delta; \Gamma; \Sigma \vdash e_1 [\bar{\tau}_2] : \tau[\bar{\tau}_2/\bar{\alpha}]}
\end{array}$$

$$\begin{array}{c}
\frac{\Delta; \Gamma; \Sigma \vdash e_2 : \tau[\bar{\tau}_1/\bar{\alpha}] \quad \Delta \vdash \tau[\bar{\tau}_1/\bar{\alpha}] \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}\text{pack}(\bar{\tau}_1, e_2) : {}^{q_a}(\exists \bar{\alpha}. \tau)} \\
\frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma}{\Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{q_a}(\exists \bar{\alpha}. \tau)} \\
\frac{\Delta, \bar{\alpha}; \Gamma_2, x: \tau; \Sigma_2 \vdash e_2 : \tau' \quad FV(\tau') \subseteq \Delta}{\Delta; \Gamma; \Sigma \vdash \text{let pack}(\bar{\alpha}, x) = e_1 \text{ in } e_2 : \tau'}
\end{array}$$

$$\begin{array}{c}
\frac{\Delta \vdash \Gamma \preceq q_a \quad \Delta \vdash \Sigma \preceq q_a}{\Delta, \alpha; \Gamma; \Sigma \vdash e : \tau} \\
\frac{\Delta; \Gamma; \Sigma \vdash e_1 : {}^{q_a}(\forall \alpha. \tau)}{\Delta; \Gamma; \Sigma \vdash e_1 [\tau_2] : \tau[\tau_2/\alpha]}
\end{array}$$

$$\begin{array}{c}
\frac{\Delta; \Gamma; \Sigma \vdash e_2 : \tau[\tau_1/\alpha] \quad \Delta \vdash \tau[\tau_1/\alpha] \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}\text{pack}(\tau_1, e_2) : {}^{q_a}(\exists \alpha. \tau)} \\
\frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma}{\Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{q_a}(\exists \alpha. \tau)} \\
\frac{\Delta, \alpha; \Gamma_2, x: \tau; \Sigma_2 \vdash e_2 : \tau' \quad FV(\tau') \subseteq \Delta}{\Delta; \Gamma; \Sigma \vdash \text{let pack}(\alpha, x) = e_1 \text{ in } e_2 : \tau'}
\end{array}$$

$$\begin{array}{c}
\frac{\Delta; \Gamma; \Sigma \vdash e : \tau[\mu \bar{\alpha}. \tau / \bar{\alpha}] \quad \Delta \vdash \tau[\mu \bar{\alpha}. \tau / \bar{\alpha}] \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{q_a}\text{fold } e : {}^{q_a}(\mu \bar{\alpha}. \tau)} \\
\frac{\Delta; \Gamma; \Sigma \vdash e : {}^{q_a}(\mu \bar{\alpha}. \tau)}{\Delta; \Gamma; \Sigma \vdash \text{unfold } e : \tau[\mu \bar{\alpha}. \tau / \bar{\alpha}]}
\end{array}$$

$$\begin{array}{c}
\frac{}{\Delta; \bullet, \bullet \vdash {}^{qc, qh} \text{newrgn} : \mathbb{L}(\exists \varrho. \mathbb{L}({}^{qc}(\text{cap } \varrho) \otimes {}^{qh}(\text{hnd } \varrho)))} \\
\frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{qc}(\text{cap } r) \quad \Delta \vdash A \preceq q_c \quad \Delta; \Gamma_2; \Sigma_2 \vdash e_2 : {}^{qh}(\text{hnd } r)}{\Delta; \Gamma; \Sigma \vdash \text{freergn } e_1 e_2 : \mathbb{L}1_{\otimes}} \\
\frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \boxplus \Gamma_3 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \odot \Sigma_3 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{qc}(\text{cap } r) \quad \Delta; \Gamma_2; \Sigma_2 \vdash e_2 : {}^{qh}(\text{hnd } r) \quad \Delta; \Gamma_3; \Sigma_3 \vdash e_3 : \tau \quad \Delta \vdash \tau \preceq A}{\Delta; \Gamma; \Sigma \vdash {}^{qa} \text{new } e_1 e_2 e_3 : \mathbb{L}({}^{qc}(\text{cap } r) \otimes {}^{qa}(\text{ref } r \tau))} \\
\frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \boxplus \Gamma_3 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \odot \Sigma_3 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{qc}(\text{cap } r) \quad \Delta; \Gamma_2; \Sigma_2 \vdash e_2 : {}^{qh}(\text{hnd } r) \quad \Delta \vdash R \preceq q_a \quad \Delta; \Gamma_3; \Sigma_3 \vdash e_3 : \tau}{\Delta; \Gamma; \Sigma \vdash {}^{qa} \text{new } e_1 e_2 e_3 : \mathbb{L}({}^{qc}(\text{cap } r) \otimes {}^{qa}(\text{ref } r \tau))} \\
\frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{qc}(\text{cap } r) \quad \Delta; \Gamma_2; \Sigma_2 \vdash e_2 : {}^{qa}(\text{ref } r \tau) \quad \Delta \vdash A \preceq q_a}{\Delta; \Gamma; \Sigma \vdash \text{free } e_1 e_2 : \mathbb{L}({}^{qc}(\text{cap } r) \otimes \tau)} \\
\frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{qc}(\text{cap } r) \quad \Delta; \Gamma_2; \Sigma_2 \vdash e_2 : {}^{qa}(\text{ref } r \tau) \quad \Delta \vdash \tau \preceq R}{\Delta; \Gamma; \Sigma \vdash \text{read } e_1 e_2 : \mathbb{L}(\mathbb{L}({}^{qc}(\text{cap } r) \otimes {}^{qa}(\text{ref } r \tau)) \otimes \tau)} \\
\frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \boxplus \Gamma_3 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \odot \Sigma_3 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{qc}(\text{cap } r) \quad \Delta; \Gamma_2; \Sigma_2 \vdash e_2 : {}^{qa}(\text{ref } r \tau_2) \quad \Delta; \Gamma_3; \Sigma_3 \vdash e_3 : \tau_3}{\Delta; \Gamma; \Sigma \vdash \text{swap } e_1 e_2 e_3 : \mathbb{L}(\mathbb{L}({}^{qc}(\text{cap } r) \otimes {}^{qa}(\text{ref } r \tau_3)) \otimes \tau_2)} \\
\frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \boxplus \Gamma_3 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \odot \Sigma_3 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{qc}(\text{cap } r) \quad \Delta; \Gamma_2; \Sigma_2 \vdash e_2 : {}^{qa}(\text{ref } r \tau) \quad \Delta; \Gamma_3; \Sigma_3 \vdash e_3 : \tau}{\Delta; \Gamma; \Sigma \vdash \text{swap } e_1 e_2 e_3 : \mathbb{L}(\mathbb{L}({}^{qc}(\text{cap } r) \otimes {}^{qa}(\text{ref } r \tau)) \otimes \tau)} \\
\frac{\Delta \vdash \Gamma \preceq q_a \quad \Delta \vdash \Sigma \preceq q_a \quad \Delta, \alpha; \Gamma; \Sigma \vdash e : \tau}{\Delta; \Gamma; \Sigma \vdash {}^{qa} \Lambda \varrho. e : {}^{qa}(\forall \varrho. \tau)} \quad \frac{\Delta; \Gamma; \Sigma \vdash e_1 : {}^{qa}(\forall \varrho. \tau)}{\Delta; \Gamma; \Sigma \vdash e_1 [r_2] : \tau [r_2 / \varrho]} \\
\frac{\Delta; \Gamma; \Sigma \vdash e_2 : \tau [r_1 / \varrho] \quad \Delta \vdash \tau [r_1 / \varrho] \preceq q_a}{\Delta; \Gamma; \Sigma \vdash {}^{qa} \text{pack}(r_1, e_2) : {}^{qa}(\exists \varrho. \tau)} \quad \frac{\Delta \vdash \Gamma_1 \boxplus \Gamma_2 \rightsquigarrow \Gamma \quad \Delta \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Delta; \Gamma_1; \Sigma_1 \vdash e_1 : {}^{qa}(\exists \varrho. \tau) \quad \Delta, \varrho; \Gamma_2, x; \tau; \Sigma_2 \vdash e_2 : \tau' \quad FV(\tau') \subseteq \Delta}{\Delta; \Gamma; \Sigma \vdash \text{let pack}(\varrho, x) = e_1 \text{ in } e_2 : \tau'}
\end{array}$$

3.16 $\Psi \vdash_{\text{cap}} \mathfrak{t} : \mathfrak{q}$

$$\overline{\bullet, \mathfrak{t} \mapsto ({}^{\mathfrak{q}}\text{pre}, \bullet) \vdash_{\text{cap}} \mathfrak{t} : \mathfrak{q}}$$

3.17 $\Psi \vdash_{\text{hnd}} \mathfrak{t}$

$$\overline{\bullet, \mathfrak{t} \mapsto (\text{abs}, \bullet) \vdash_{\text{hnd}} \mathfrak{t}}$$

3.18 $\Psi \vdash_{\text{ref}} (\mathfrak{t}, \mathfrak{p}) : (\mathfrak{q}, \tau)$

$$\overline{\bullet, \mathfrak{t} \mapsto (\text{abs}, \bullet, \mathfrak{p} \mapsto (\mathfrak{q}, \tau)) \vdash_{\text{ref}} (\mathfrak{t}, \mathfrak{p}) : (\mathfrak{q}, \tau)}$$

3.19 $\Sigma; \Psi \vdash (\mathfrak{q}, v) : \tau$

$$\frac{\bullet \vdash \Sigma \preceq \mathfrak{q} \quad \bullet, \bullet, x:\tau_x; \Sigma \vdash e : \tau}{\Sigma; \bullet \vdash (\mathfrak{q}, \lambda x:\tau_x. e) : {}^{\mathfrak{q}}(\tau_x \multimap \tau)}$$

$$\frac{\bullet \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \bullet; \Sigma_1 \vdash l_1 : \tau_1 \quad \bullet \vdash \tau_1 \preceq \mathfrak{q}\bullet; \Sigma_2 \vdash l_2 : \tau_2 \quad \bullet \vdash \tau_2 \preceq \mathfrak{q}}{\bullet; \bullet \vdash (\mathfrak{q}, \langle l_1, l_2 \rangle) : {}^{\mathfrak{q}}(\tau_1 \otimes \tau_2)}$$

$$\frac{\bullet \vdash \Sigma \preceq \mathfrak{q}}{\Sigma; \bullet \vdash (\mathfrak{q}, \langle \rangle) : {}^{\mathfrak{q}}\mathbf{1}_{\otimes}}$$

$$\frac{\bullet \vdash \Sigma \preceq \mathfrak{q} \quad \bullet; \bullet; \Sigma \vdash e_1 : \tau_1 \quad \bullet; \bullet; \Sigma \vdash e_2 : \tau_2}{\Sigma; \bullet \vdash (\mathfrak{q}, \langle e_1, e_2 \rangle) : {}^{\mathfrak{q}}(\tau_1 \otimes \tau_2)}$$

$$\frac{\bullet; \Sigma \vdash l : \tau_1 \quad \bullet \vdash \tau_1 \preceq \mathfrak{q}}{\Sigma; \bullet \vdash (\mathfrak{q}, \text{inl } l) : {}^{\mathfrak{q}}(\tau_1 \oplus \tau_2)}$$

$$\frac{\bullet; \Sigma \vdash l : \tau_2 \quad \bullet \vdash \tau_2 \preceq \mathfrak{q}}{\Sigma; \bullet \vdash (\mathfrak{q}, \text{inr } l) : {}^{\mathfrak{q}}(\tau_1 \oplus \tau_2)}$$

$$\frac{\bullet \vdash \Sigma \preceq \mathfrak{q} \quad \bullet, \xi; \bullet; \Sigma \vdash e : \tau}{\Sigma; \bullet \vdash (\mathfrak{q}, \Lambda \xi. e) : {}^{\mathfrak{q}}(\forall \xi. \tau)}$$

$$\frac{\bullet; \Sigma \vdash l_2 : \tau[q_1/\xi] \quad \bullet \vdash \tau[q_1/\xi] \preceq \mathfrak{q}}{\Sigma; \bullet \vdash (\mathfrak{q}, \text{pack}(q_1, l_2)) : {}^{\mathfrak{q}}(\exists \xi. \tau)}$$

$$\frac{\bullet \vdash \Sigma \preceq \mathfrak{q} \quad \bullet, \bar{\alpha}; \bullet; \Sigma \vdash e : \tau}{\Sigma; \bullet \vdash (\mathfrak{q}, \Lambda \bar{\alpha}. e) : {}^{\mathfrak{q}}(\forall \bar{\alpha}. \tau)}$$

$$\frac{\bullet; \Sigma \vdash l_2 : \tau[\bar{\tau}_1/\bar{\alpha}] \quad \bullet \vdash \tau[\bar{\tau}_1/\bar{\alpha}] \preceq \mathfrak{q}}{\Sigma; \bullet \vdash (\mathfrak{q}, \text{pack}(\bar{\tau}_1, l_2)) : {}^{\mathfrak{q}}(\exists \bar{\alpha}. \tau)}$$

$$\frac{\bullet \vdash \Sigma \preceq \mathfrak{q} \quad \bullet, \alpha; \bullet; \Sigma \vdash e : \tau}{\Sigma; \bullet \vdash (\mathfrak{q}, \Lambda \alpha. e) : {}^{\mathfrak{q}}(\forall \alpha. \tau)}$$

$$\frac{\bullet; \Sigma \vdash l_2 : \tau[\tau_1/\alpha] \quad \bullet \vdash \tau[\tau_1/\alpha] \preceq \mathfrak{q}}{\Sigma; \bullet \vdash (\mathfrak{q}, \text{pack}(\tau_1, l_2)) : {}^{\mathfrak{q}}(\exists \alpha. \tau)}$$

$$\frac{\bullet; \Sigma \vdash l : \tau[\mu \bar{\alpha}. \tau / \bar{\alpha}] \quad \bullet \vdash \tau[\mu \bar{\alpha}. \tau / \bar{\alpha}] \preceq \mathfrak{q}}{\Sigma; \bullet \vdash (\mathfrak{q}, \text{fold } l) : {}^{\mathfrak{q}}(\mu \bar{\alpha}. \tau)}$$

$$\frac{\Psi \vdash_{\text{cap}} \mathfrak{t} : \mathfrak{q}}{\bullet; \Psi \vdash (\mathfrak{q}, \text{cap}) : {}^{\mathfrak{q}}(\text{cap } \mathfrak{t})} \quad \frac{\Psi \vdash_{\text{hnd}} \mathfrak{t}}{\bullet; \Psi \vdash (\mathfrak{q}, \text{hnd } \mathfrak{t}) : {}^{\mathfrak{q}}(\text{hnd } \mathfrak{t})} \quad \frac{\Psi \vdash_{\text{ref}} (\mathfrak{t}, \mathfrak{p}) : (\mathfrak{q}, \tau)}{\bullet; \Psi \vdash (\mathfrak{q}, \text{ref } \mathfrak{t} \mathfrak{p}) : {}^{\mathfrak{q}}(\text{ref } \mathfrak{t} \tau)}$$

$$\frac{\bullet \vdash \Sigma \preceq \mathfrak{q} \quad \bullet, \varrho; \bullet; \Sigma \vdash e : \tau}{\Sigma; \bullet \vdash (\mathfrak{q}, \Lambda \varrho. e) : {}^{\mathfrak{q}}(\forall \varrho. \tau)}$$

$$\frac{\bullet; \Sigma \vdash l_2 : \tau[r_1/\varrho] \quad \bullet \vdash \tau[r_1/\varrho] \preceq \mathfrak{q}}{\Sigma; \bullet \vdash (\mathfrak{q}, \text{pack}(r_1, l_2)) : {}^{\mathfrak{q}}(\exists \varrho. \tau)}$$

3.20 $\vdash \Psi$ skel

$$\frac{}{\vdash \bullet \text{ skel}} \qquad \frac{\vdash \Psi \text{ skel}}{\vdash \Psi, \tau \mapsto (\text{abs}, \bullet) \text{ skel}}$$

3.21 $\Psi \vdash \sigma : \Sigma$

$$\frac{\vdash \Psi \text{ skel}}{\Psi \vdash \bullet : \bullet} \qquad \frac{\Sigma_v; \Psi_v \vdash (q, v) : \tau \quad \bullet \vdash \Sigma_v \odot \Sigma \rightsquigarrow \Sigma_* \quad \Psi_* \vdash \sigma : \Sigma_* \quad \vdash \Psi_* \odot \Psi_v \rightsquigarrow \Psi}{\Psi \vdash \sigma, l \mapsto (q, v, f) : \Sigma, l \mapsto \tau}$$

$$\frac{\Psi \vdash \sigma : \Sigma \quad q \sqsubseteq R}{\Psi \vdash \sigma, l \mapsto (q, v, \text{used}) : \Sigma} \qquad \frac{\Psi \vdash \sigma : \Sigma \quad q \sqsubseteq A}{\Psi \vdash \sigma, l \mapsto (q, v, f) : \Sigma}$$

3.22 $\vdash \tau \downarrow q$

$$\frac{\bullet \vdash \tau \preceq A}{\vdash \tau \downarrow U} \qquad \frac{}{\vdash \tau \downarrow R} \qquad \frac{\bullet \vdash \tau \preceq A}{\vdash \tau \downarrow A} \qquad \frac{}{\vdash \tau \downarrow L}$$

3.23 $\Sigma \vdash \theta : \Theta$

$$\frac{}{\bullet \vdash \bullet : \bullet} \qquad \frac{\Sigma_* \vdash \theta : \Theta \quad \vdash \tau \downarrow q \quad \bullet; \Sigma_l \vdash l : \tau \quad \bullet \vdash \Sigma_l \odot \Sigma_* \rightsquigarrow \Sigma}{\Sigma \vdash \theta, p \mapsto (q, l) : \Theta, p \mapsto (q, \tau)}$$

$$\frac{\Sigma \vdash \theta : \Theta \quad q \sqsubseteq A}{\Sigma \vdash \theta, p \mapsto (q, l) : \Theta}$$

3.24 $\vdash v : \Upsilon$

$$\frac{}{\vdash \text{live} : \text{pre}} \qquad \frac{q \sqsubseteq A}{\vdash \text{live} : \text{abs}} \qquad \frac{}{\vdash \text{dead} : \text{abs}}$$

3.25 $\Sigma \vdash \psi : \Psi$

$$\frac{}{\bullet \vdash \bullet : \bullet} \qquad \frac{\vdash v : \Upsilon \quad \Sigma_* \vdash \psi : \Psi \quad \Sigma_r \vdash \theta : \Theta \quad \bullet \vdash \Sigma_r \odot \Sigma_* \rightsquigarrow \Sigma}{\Sigma \vdash \psi, \tau \mapsto (v, \theta) : \Psi, \tau \mapsto (\Upsilon, \Theta)}$$

3.26 $\vdash (\sigma, \psi) : \Sigma$

$$\frac{\Sigma_1 \vdash \psi : \Psi \quad \bullet \vdash \Sigma_1 \odot \Sigma_2 \rightsquigarrow \Sigma \quad \Psi \vdash \sigma : \Sigma}{\vdash (\sigma, \psi) : \Sigma_2}$$

4 Safety

Theorem 1 (Preservation)

If $(\sigma_1, \psi_1, e_1) \mapsto^ (\sigma_2, \psi_2, e_2)$ and $\vdash (\sigma_1, \psi_1) : \Sigma_1$ and $\bullet; \bullet; \Sigma_1 \vdash e_1 : \tau$, then there exists Σ_2 such that $\vdash (\sigma_2, \psi_2) : \Sigma_2$ and $\bullet; \bullet; \Sigma_2 \vdash e_2 : \tau$.*

Theorem 2 (Progress)

If $\vdash (\sigma_1, \psi_1) : \Sigma_1$ and $\bullet; \bullet; \Sigma_1 \vdash e_1 : \tau$, then either there exists l such that $e_1 \equiv l$ or there exists σ_2 and ψ_2 and e_2 such that $(\sigma_1, \psi_1, e_1) \mapsto (\sigma_2, \psi_2, e_2)$.

Theorem 3 (Safety)

If $\vdash (\sigma_1, \psi_1) : \Sigma_1$ and $\bullet; \bullet; \Sigma_1 \vdash e_1 : \tau$ and $(\sigma_1, \psi_1, e_1) \mapsto^ (\sigma_2, \psi_2, e_2)$, then either there exists l such that $e_2 \equiv l$ or there exists σ_3 and ψ_3 and e_3 such that $(\sigma_2, \psi_2, e_2) \mapsto (\sigma_3, \psi_3, e_3)$.*