

Sampling Edge Covers in 3-Regular Graphs

Ivona Bezáková

(Rochester Institute of Technology)

William A. Rummel

(Rochester Institute of Technology -> University of Rochester)

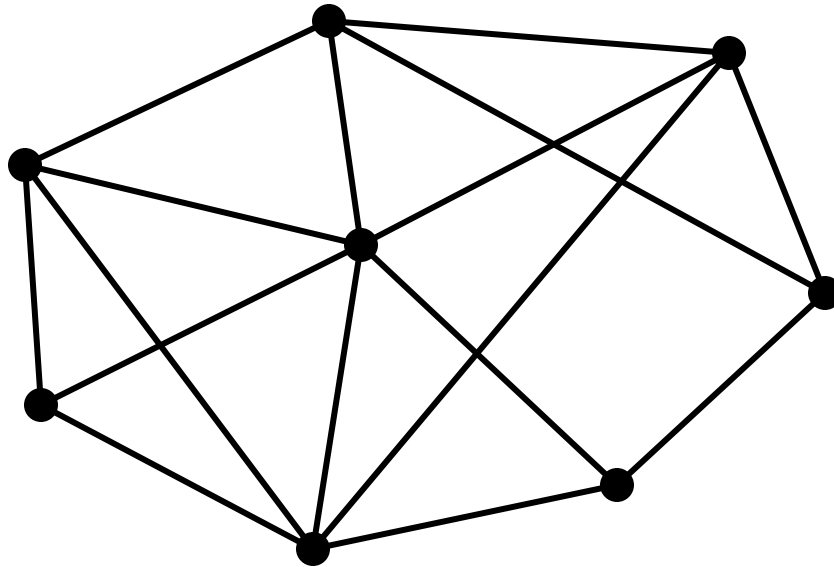
MFCS 2009, August 24, 2009

Parsing the title: Edge Covers

Given: an undirected graph $G=(V,E)$

An **edge cover** of G : a set of edges covering all vertices

Example:

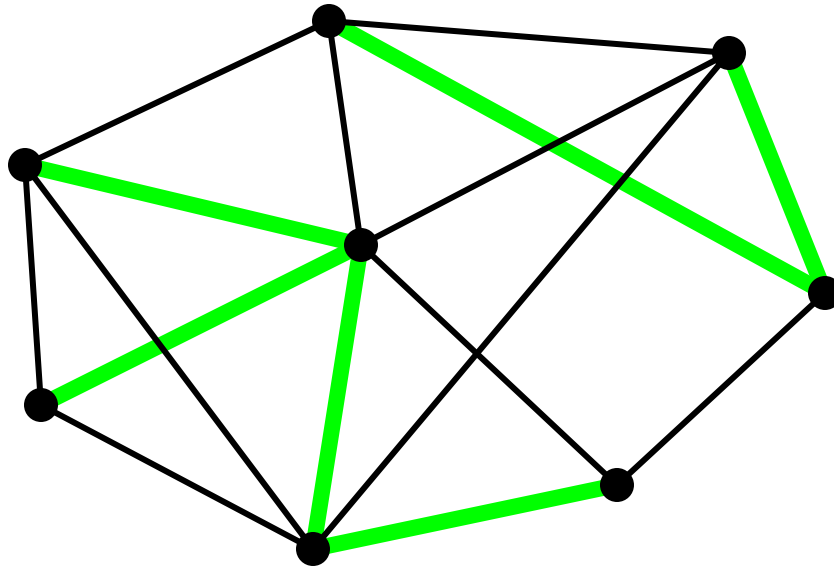


Parsing the title: Edge Covers

Given: an undirected graph $G=(V,E)$

An **edge cover** of G : a set of edges covering all vertices

Example:



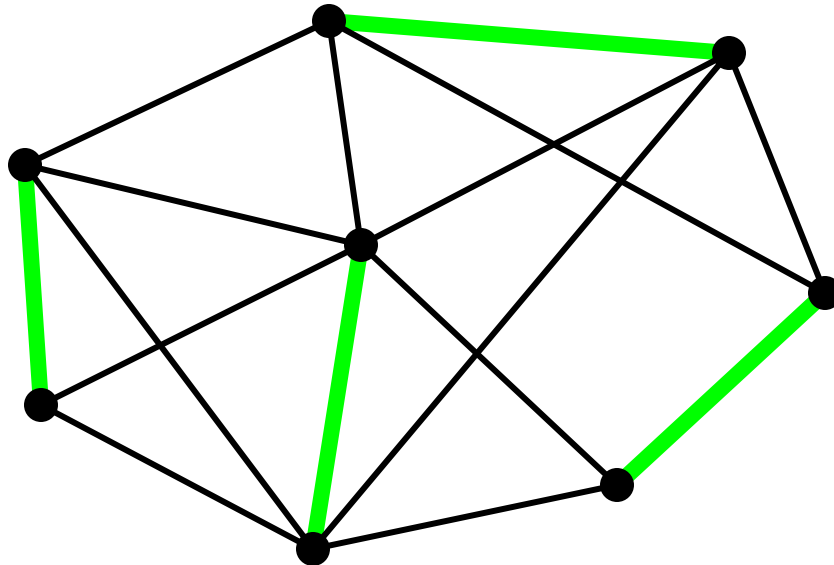
a possible edge cover

Parsing the title: Edge Covers

Given: an undirected graph $G=(V,E)$

An **edge cover** of G : a set of edges covering all vertices

Example:

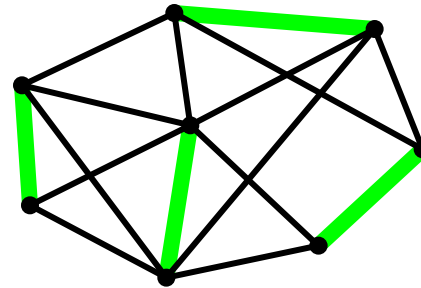
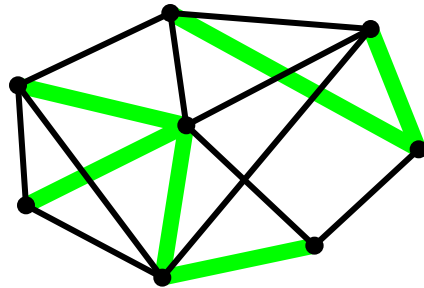


another possible edge cover

Parsing the title: Sampling Edge Covers

The problem: given an undirected graph G , return a uniformly random sample from all edge covers of G .

- let Ω be the set of all edge covers of G
- uniformly random: every edge cover is sampled with probability $1/|\Omega|$



Motivation

Edge covers are related to other problems:

- (perfect) matchings of a graph G
- graphs with a given degree sequence (the bipartite case also known as binary or 0/1 contingency tables)
- vertex covers, independent sets

These problems are well-studied sampling problems, with applications, e.g., in statistical physics, biology, and various areas of CS.

Motivation & Related Work

Sampling version of the match./deg.seq. problem is **open** for:

- perfect matchings for arbitrary graphs
- graphs with a given (arbitrary) degree sequence

Most relevant related works - known poly-time samplers:

- matchings:
 - all in arbitrary graphs, perfect in dense graphs: [Jerrum-Sinclair '89]
 - perfect in bipartite graphs: [Jerrum-Sinclair-Vigoda '04]
- graphs with a given degree sequence:
 - regular seq., bipartite graphs: [Kannan-Tetali-Vempala '89]
 - arbitrary seq., dense or bipartite graphs: [JS '89, JSV '04]
 - regular sequence, non-bipartite graphs: [Cooper-Dyer-Greenhill '07]

Motivation & Related Work

Sampling version of the match./deg.seq. problem is **open** for:

- perfect matchings for arbitrary graphs
- graphs with a given (arbitrary) degree sequence

Our motivation: issues arising when attempting to work with these open problems are similar to issues arising with analyzing edge covers

Note:

$$|\text{min vertex cover}| + |\text{max indep. set}| = n$$

$$|\text{min edge cover}| + |\text{max matching}| = n$$

where n = number of vertices.

However, the above relationship cannot be directly used for deriving sampling algorithms for edge covers from matchings, or vice versa.

Motivation & Related Work

Graphs with bounded degree are of interest in statistical physics - most relevant related works:

- perfect matchings: [Luby-Randall-Sinclair '01]
- independent sets: [Luby-Vigoda '99],
[Dyer-Greenhill '00],
[Vigoda '01]

Note: These works (except DG) use a Glauber-dynamics-type Markov chain approach.

Our contributions

Thm: There is a fully polynomial approximate sampler for edge covers of any graph with maximum degree 3.

Where:

- **fully polynomial approximate sampler (fpas)** = a randomized algo returning a sample from a distribution at most ϵ -distant from the uniform distribution, running time is polynomial in the size of the input and $1/\epsilon$
- **fully polynomial randomized approximation scheme (fpras)** = a randomized $(1+\epsilon)$ -approximation algo with success probability $1-\eta$, running time is polynomial in the size of the input, $1/\epsilon$, and $1/\log \eta$

Our contributions

Thm: There is a fully polynomial approximate sampler for edge covers of any graph with maximum degree 3.

In the extended version:

Cor: There is an fpras for counting all edge covers of graphs with maximum degree 3.

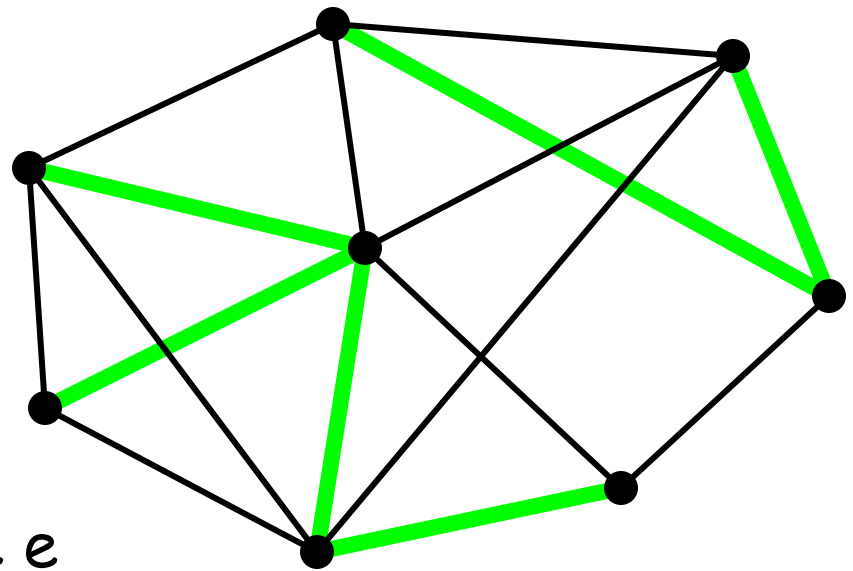
Thm: There is a fully polynomial approximate sampler (and fpras for counting) all 1-2-edge covers (i.e., path-cycle covers) for $n/3$ -dense graphs.

A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

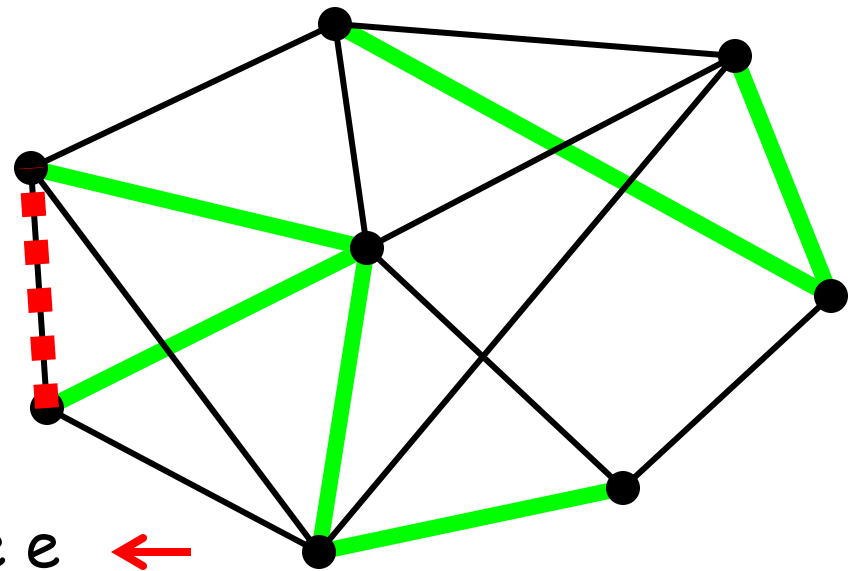


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e ←
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

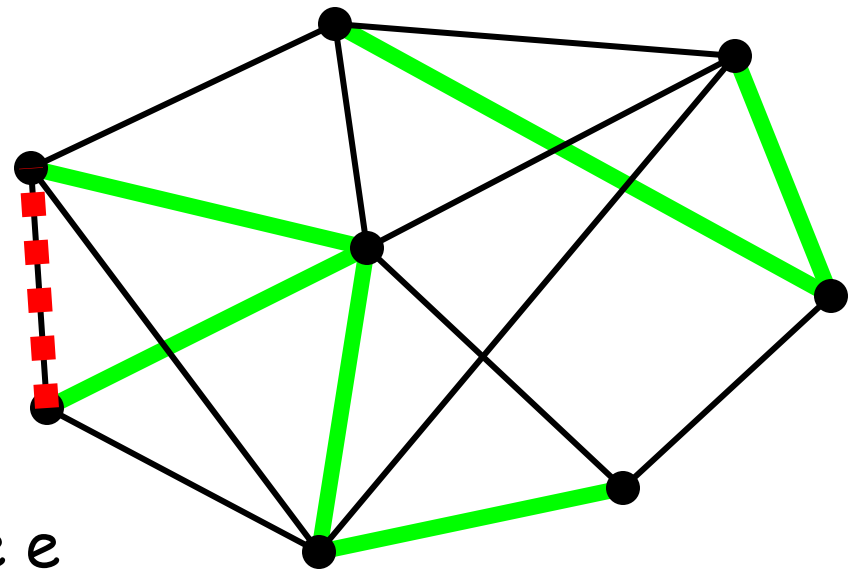


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$ ←
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

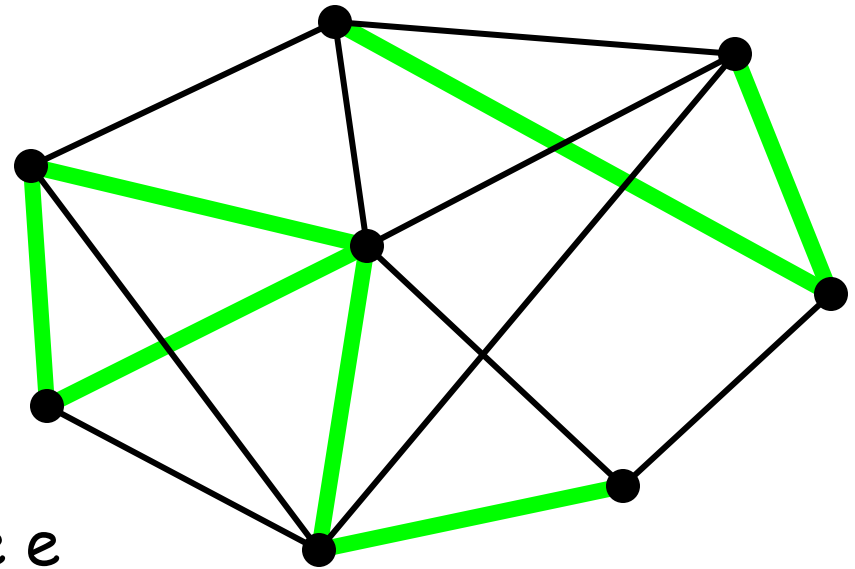


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:


- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

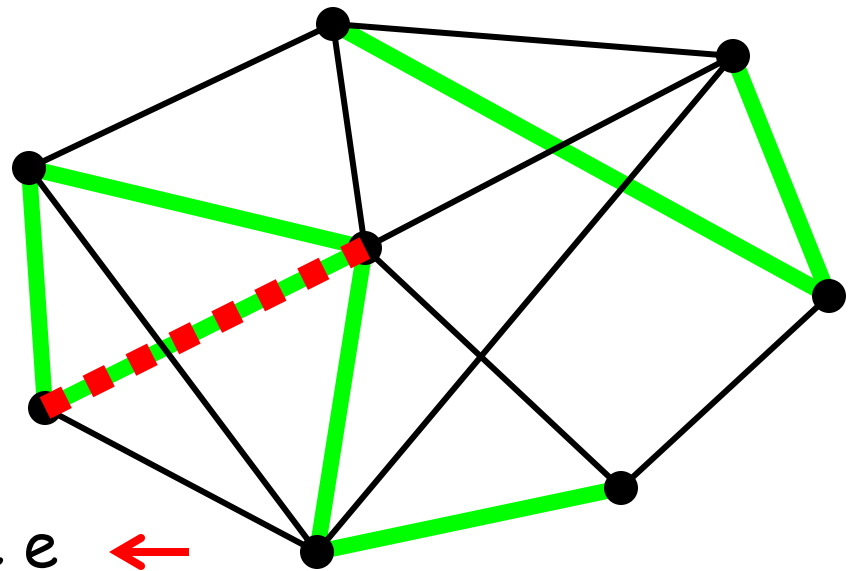


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e 
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

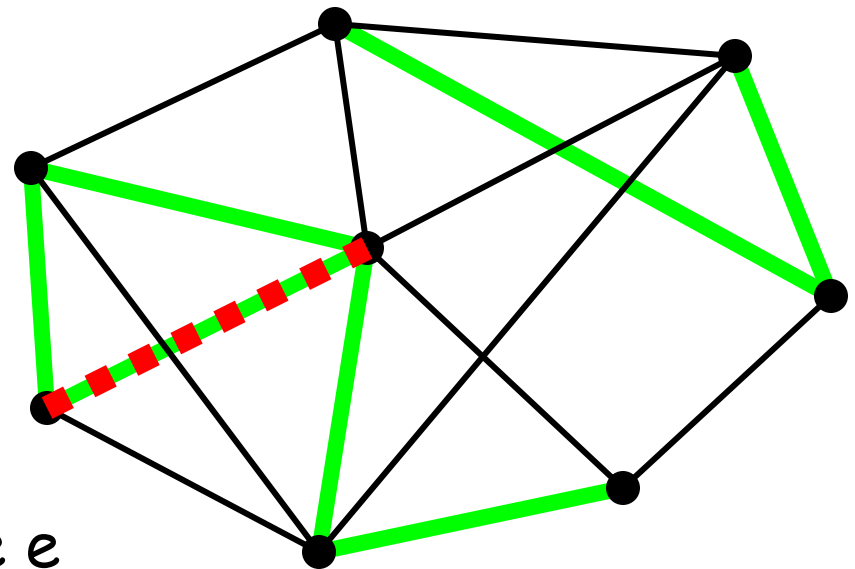


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$ ←
 - else let $X_{i+1}=X_i$

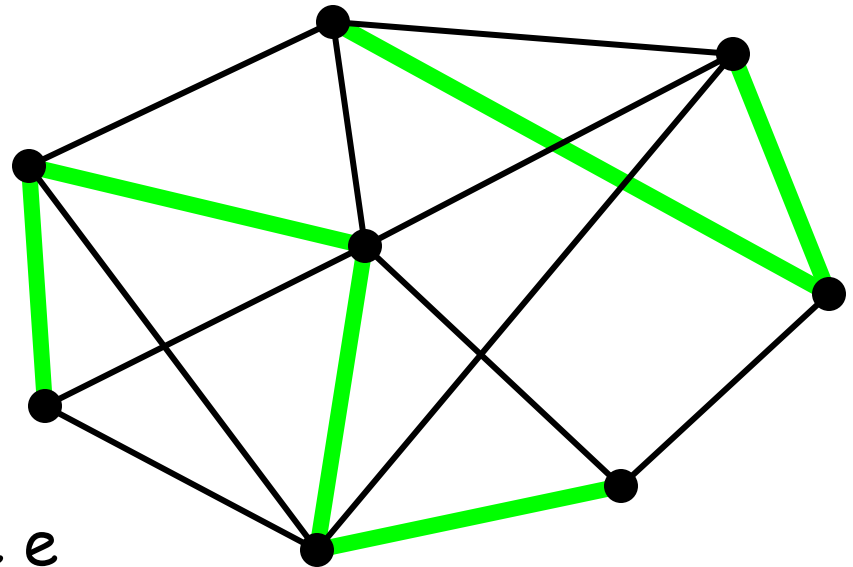


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

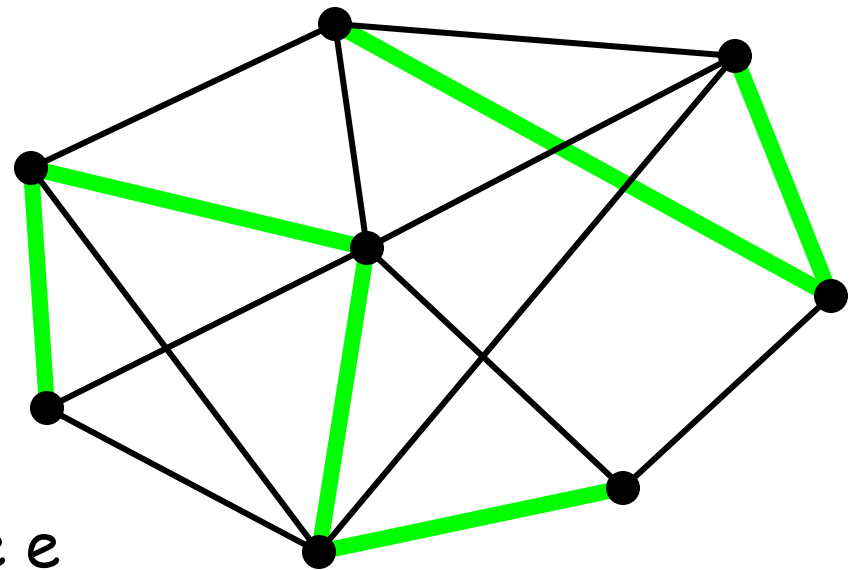


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$ ←
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

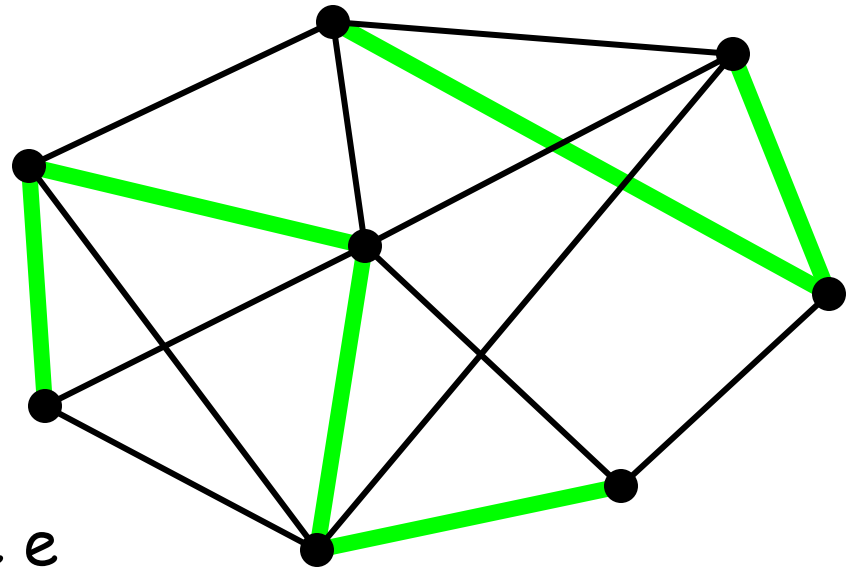


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:


- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

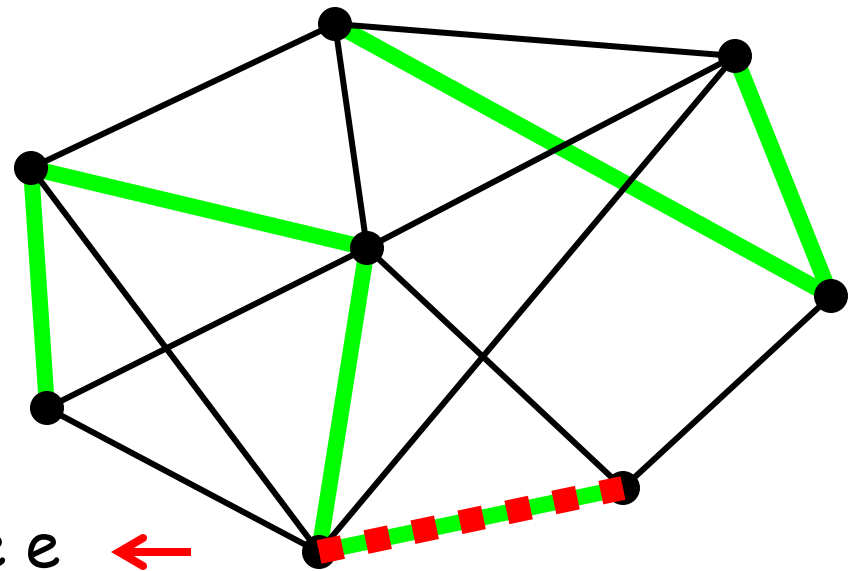


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e 
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

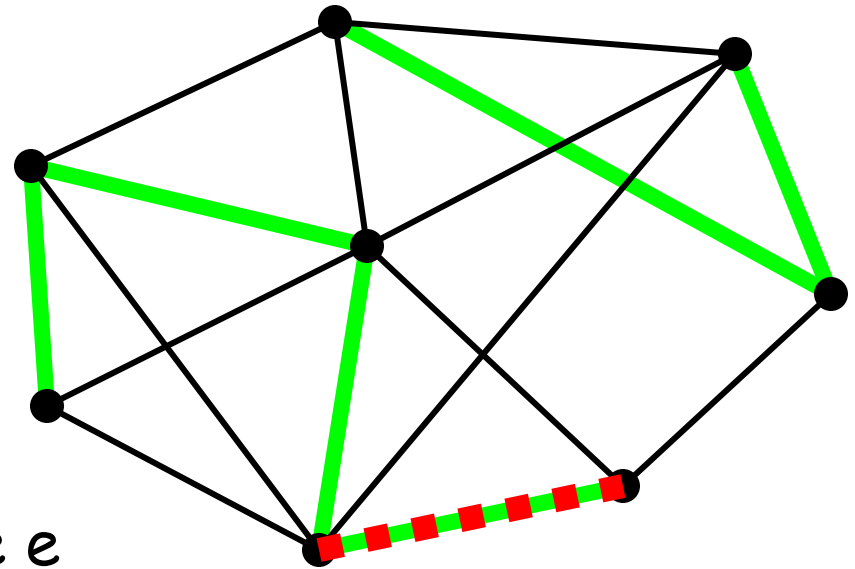


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$ ←

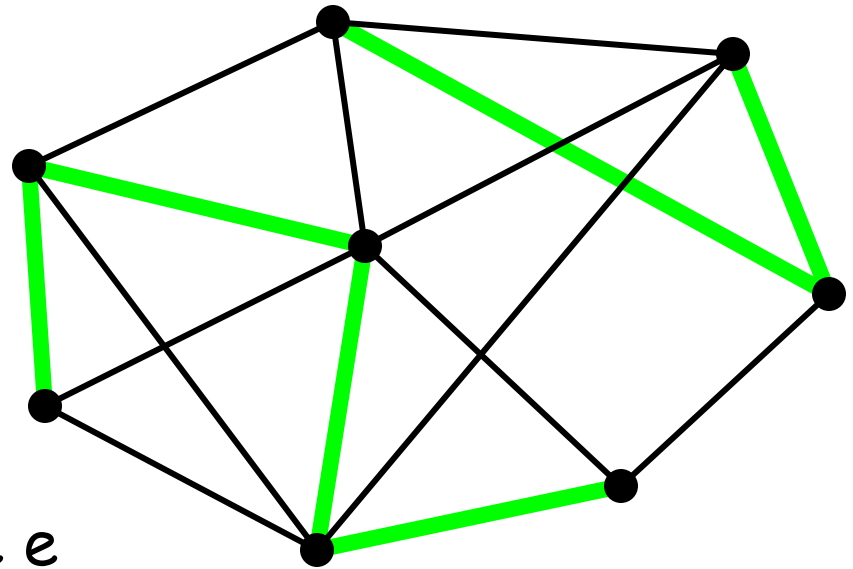


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:


- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

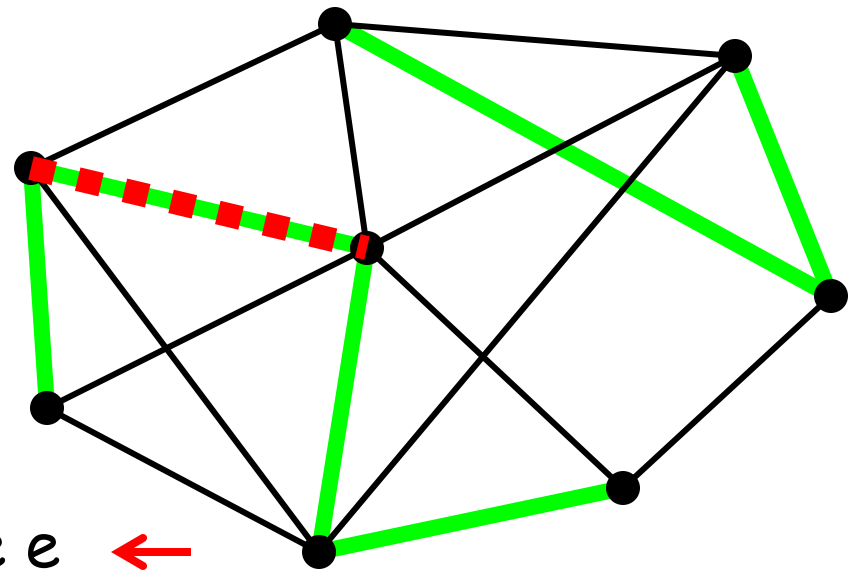


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e 
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

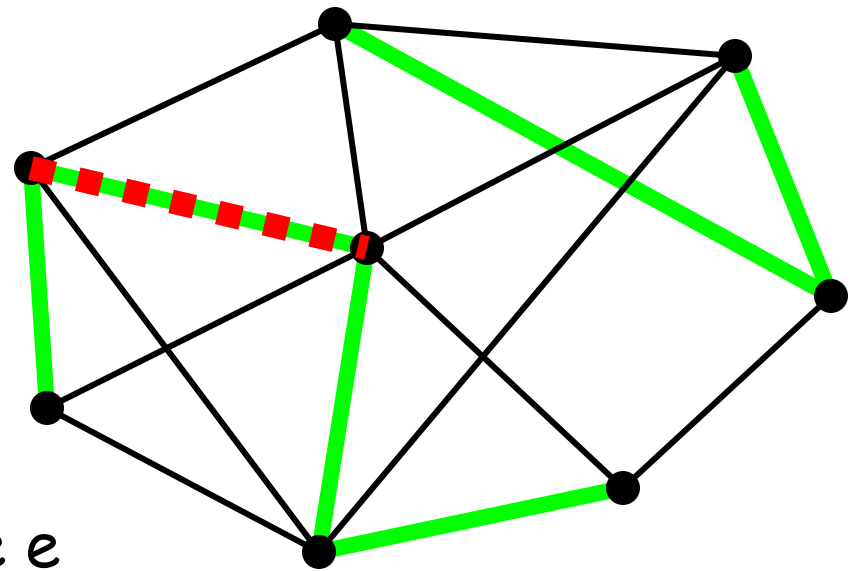


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$ ←
 - else let $X_{i+1}=X_i$

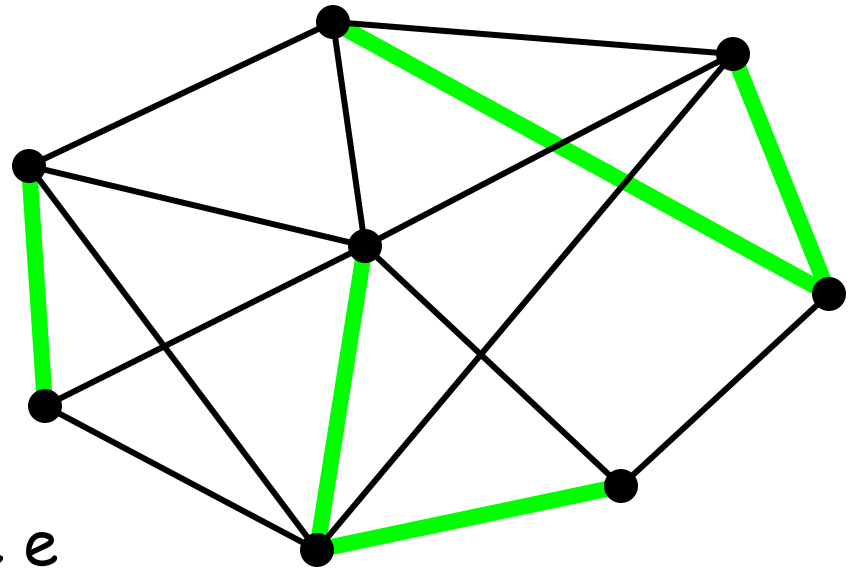


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:


- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

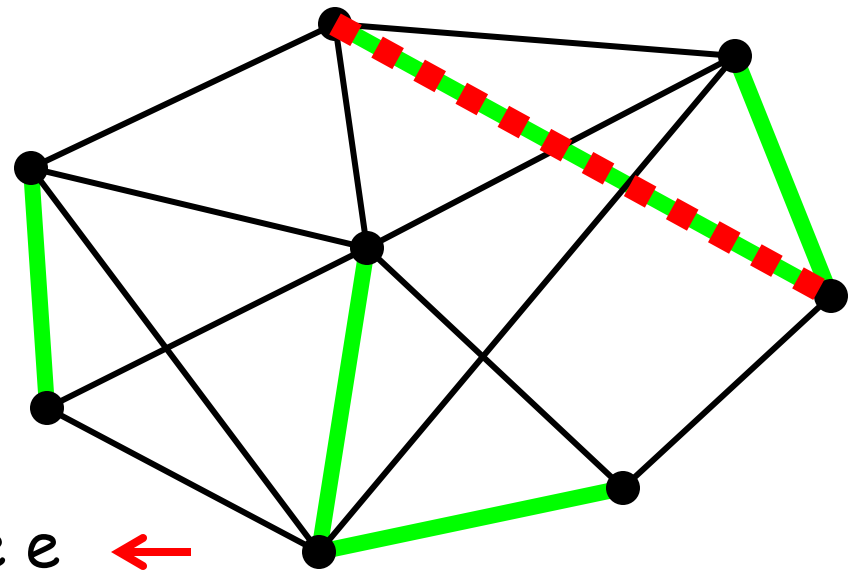


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e 
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

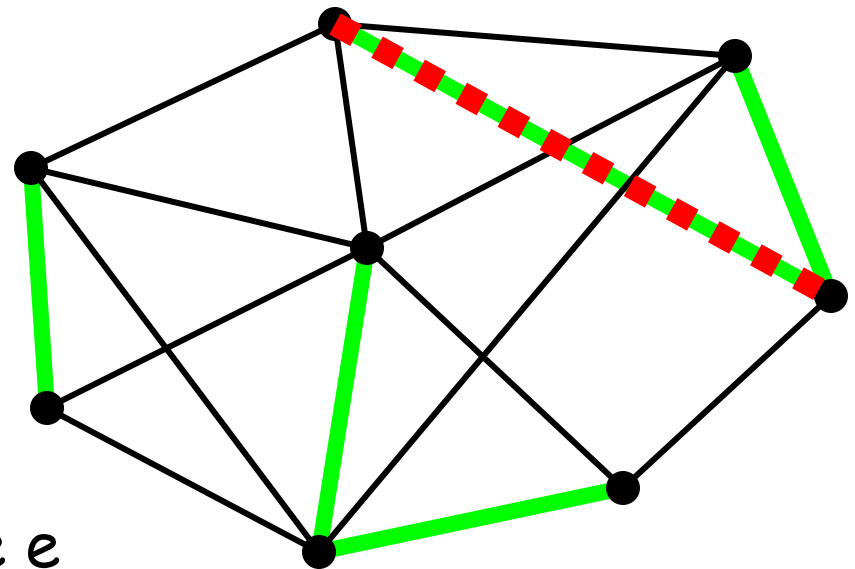


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$ ←

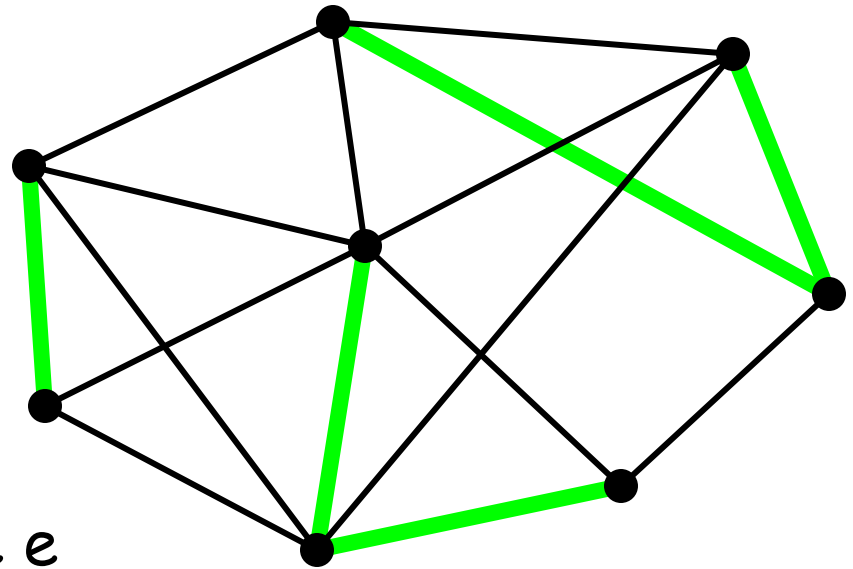


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:


- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

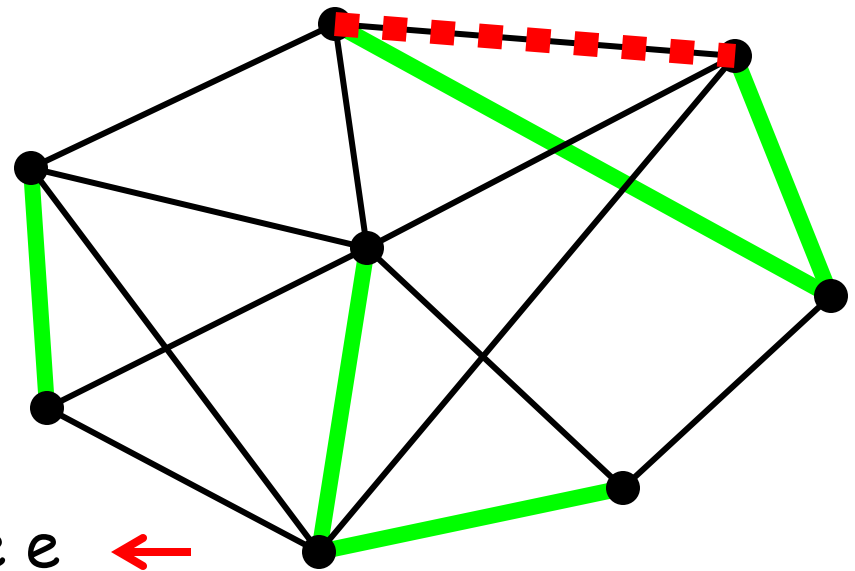


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e 
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

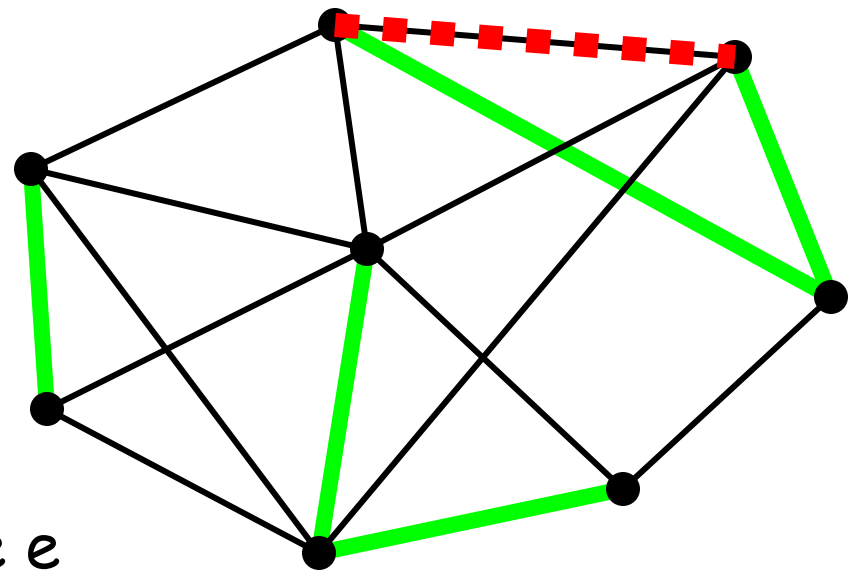


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$ ←
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

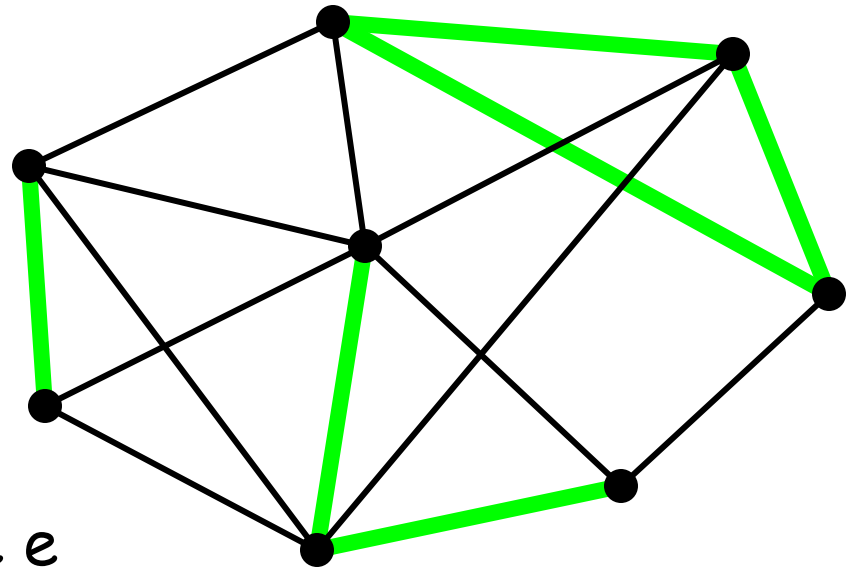


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:


- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

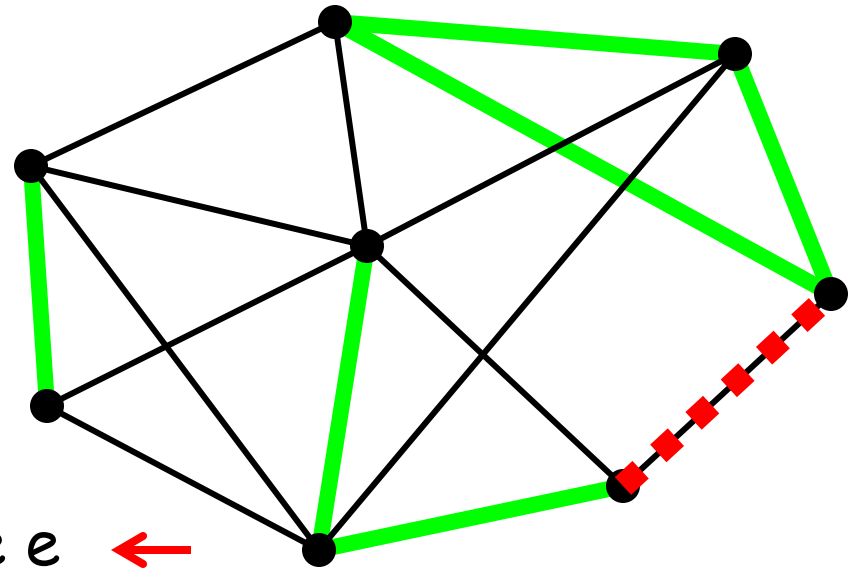


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e 
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

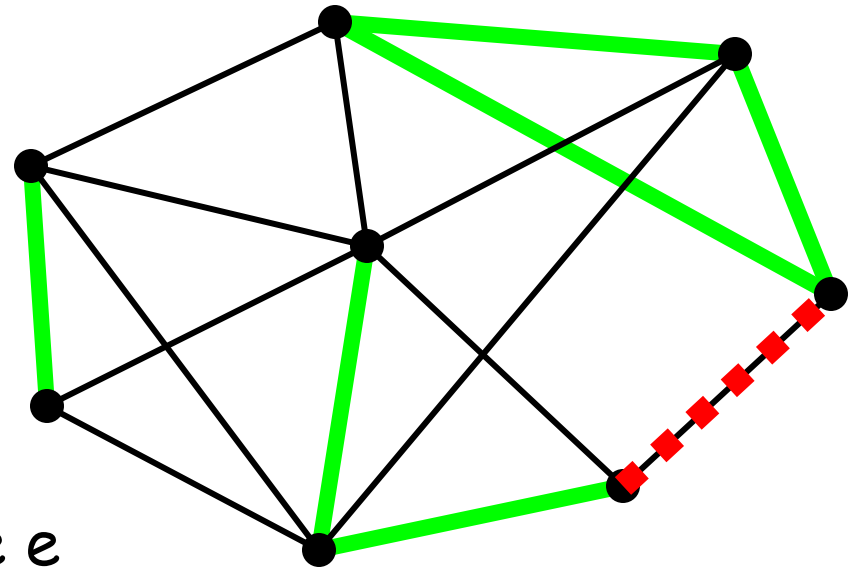


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$ ←
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

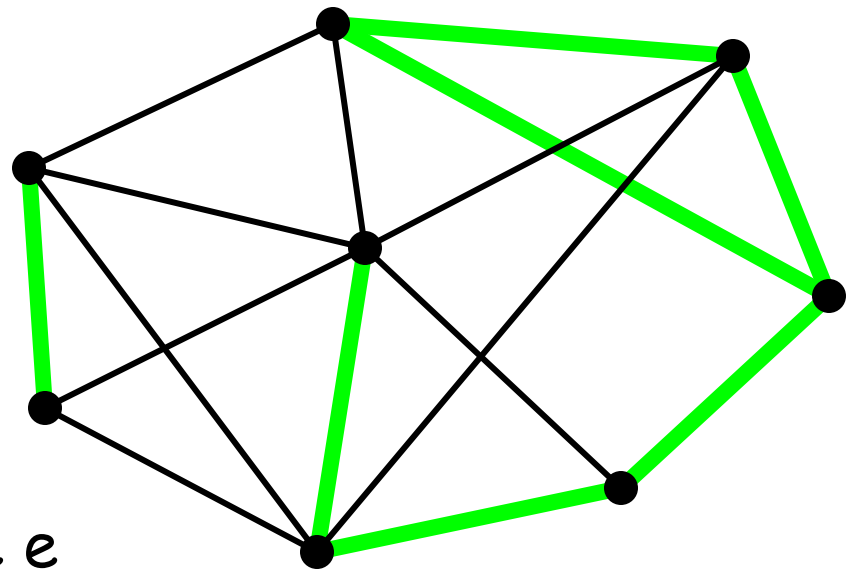


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:


- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

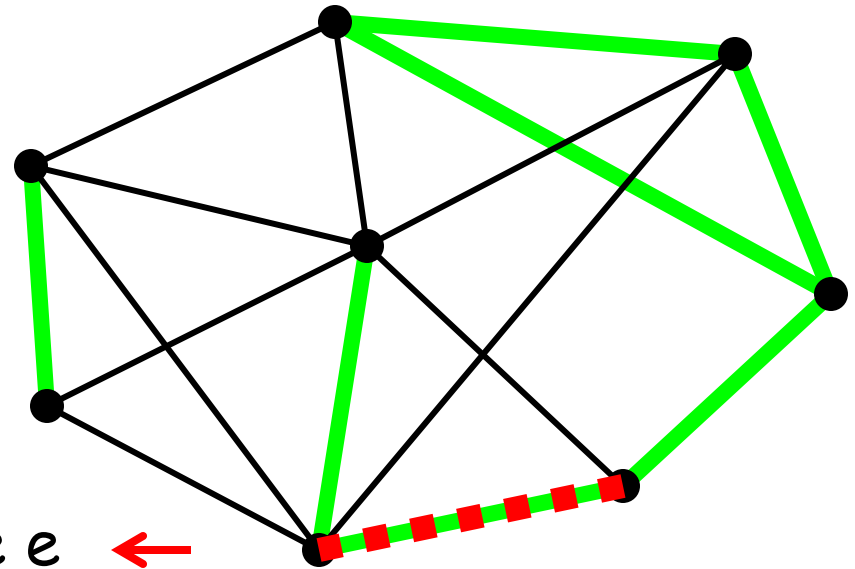


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e 
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

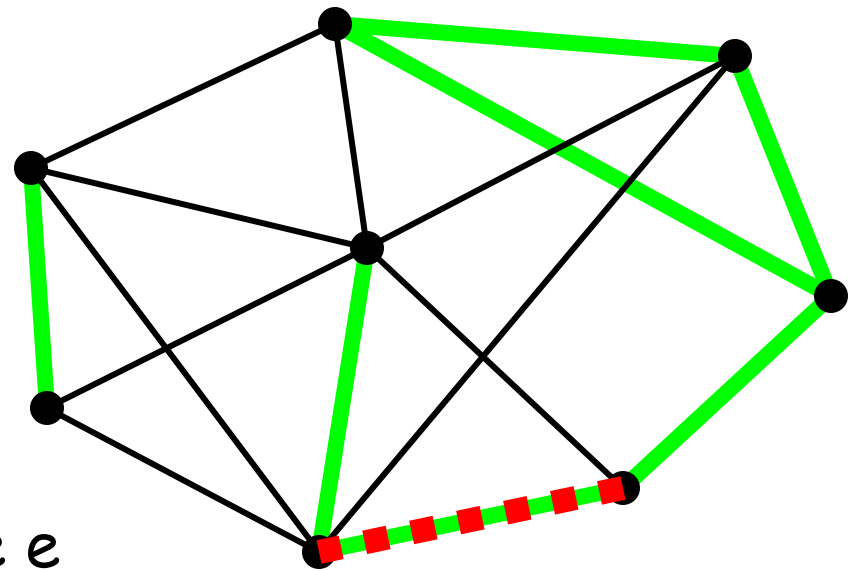


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$ ←
 - else let $X_{i+1}=X_i$

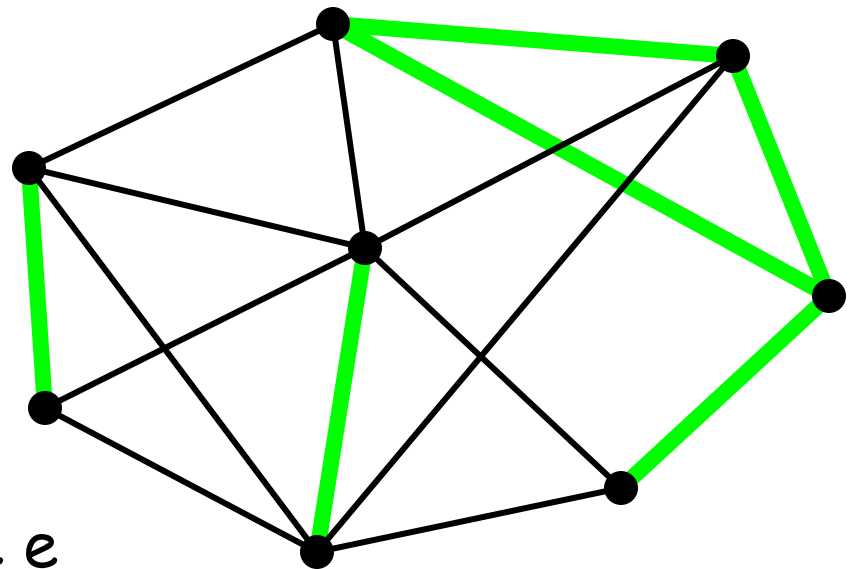


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:


- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

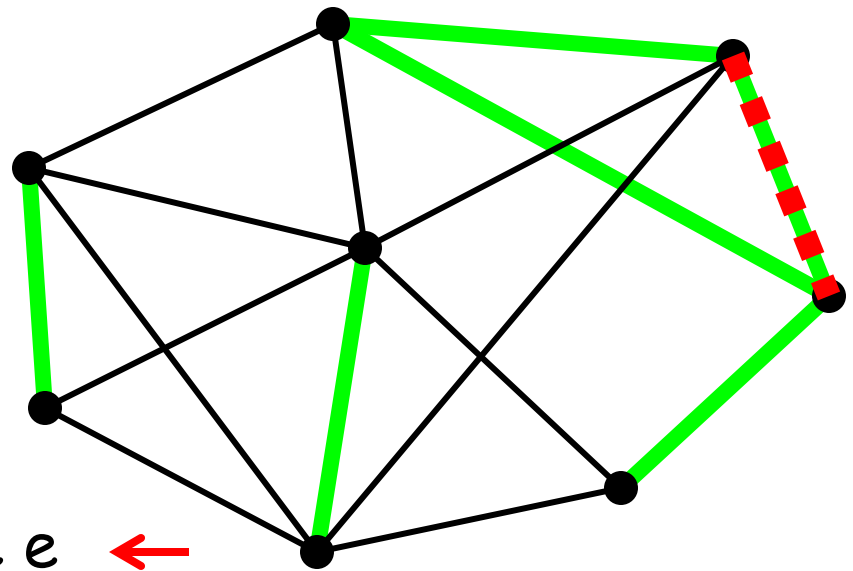


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e 
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

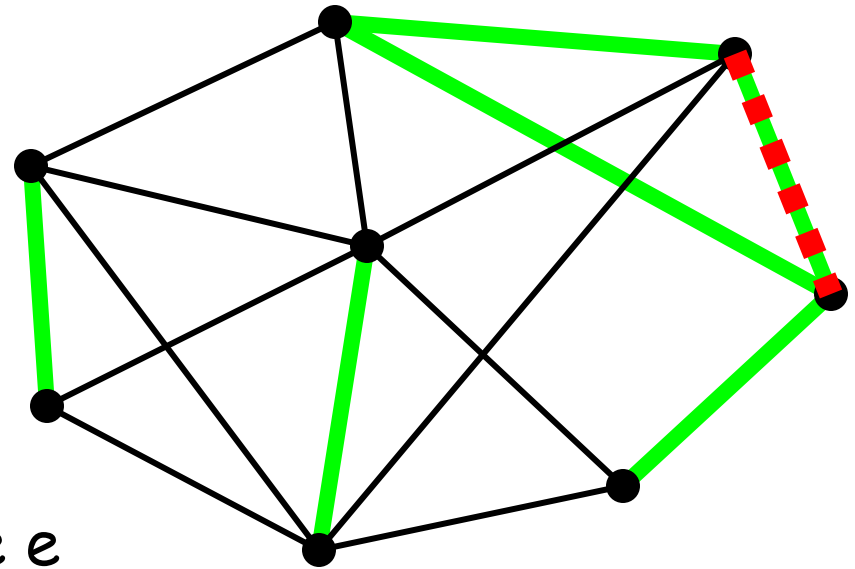


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$ ←
 - else let $X_{i+1}=X_i$

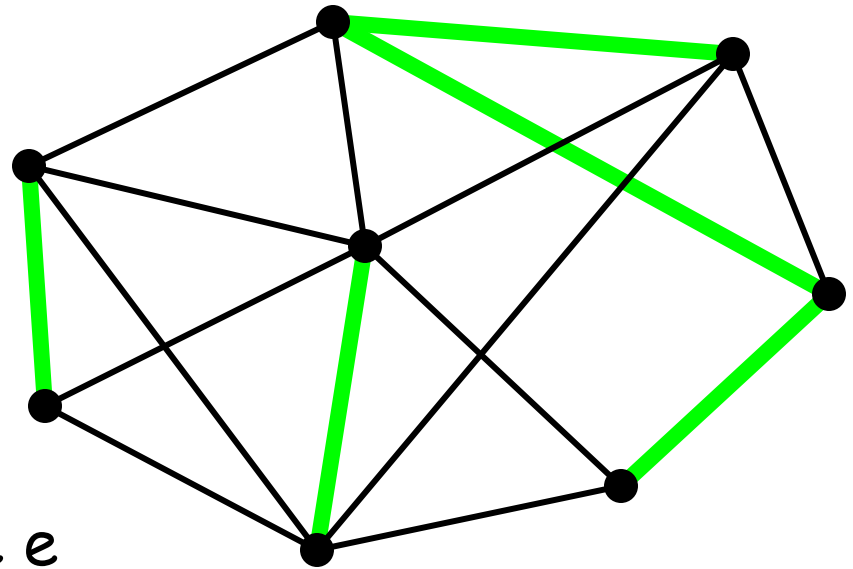


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:


- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

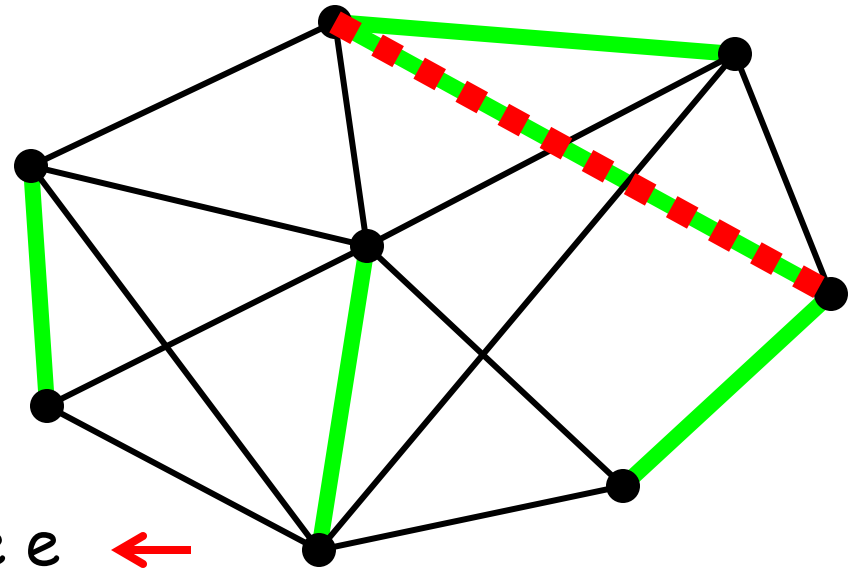


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e 
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

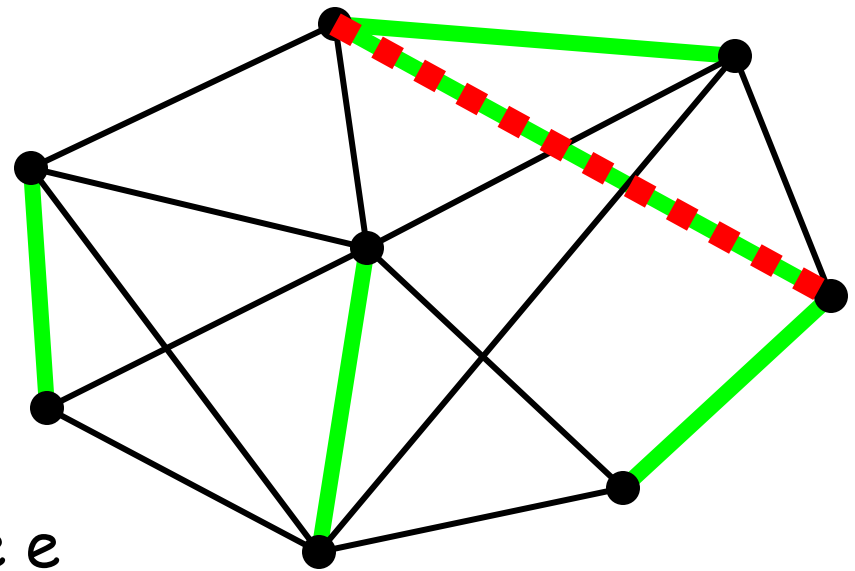


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$ ←
 - else let $X_{i+1}=X_i$

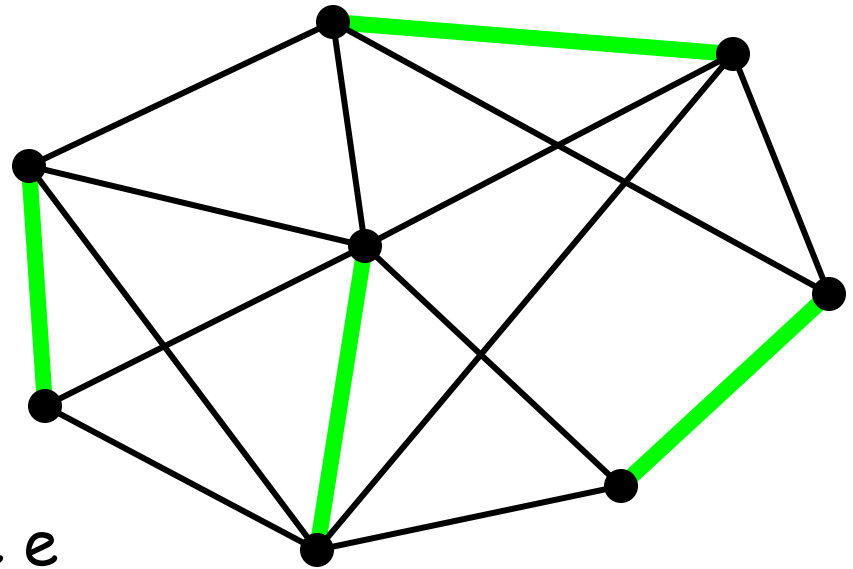


A Glauber-dynamics MC for edge covers

Thm: There exists a rapidly mixing Glauber-dynamics Markov chain for edge covers of any graph with maximum degree 3.

The Markov chain:

- from edge cover X_i :
 - with prob $\frac{1}{2}$ let $X_{i+1}=X_i$
 - else choose a random edge e
 - if e not in X_i , let $X_{i+1}=X_i \cup \{e\}$
 - else if $X_i - \{e\}$ is an edge cover, let $X_{i+1}=X_i - \{e\}$
 - else let $X_{i+1}=X_i$

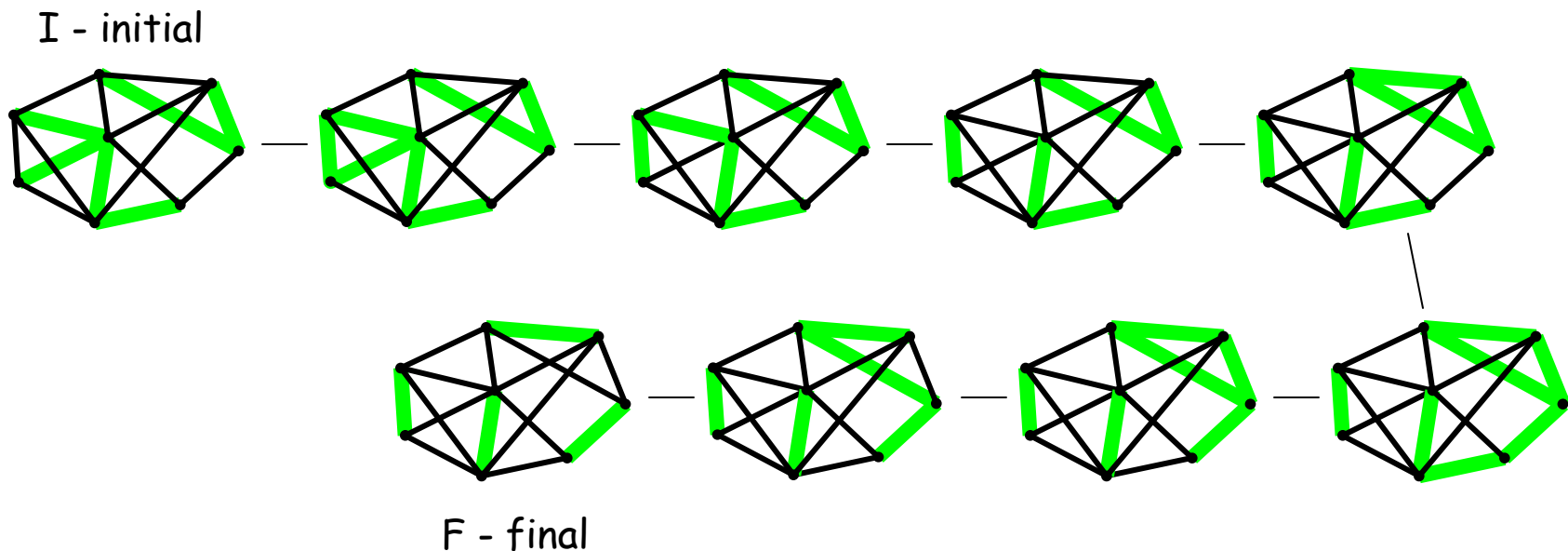


Proving rapid mixing: Canonical Flow

Consider the graph of all edge covers (all configurations), edges connect endpoints distant at one step of the MC.

Canonical path:

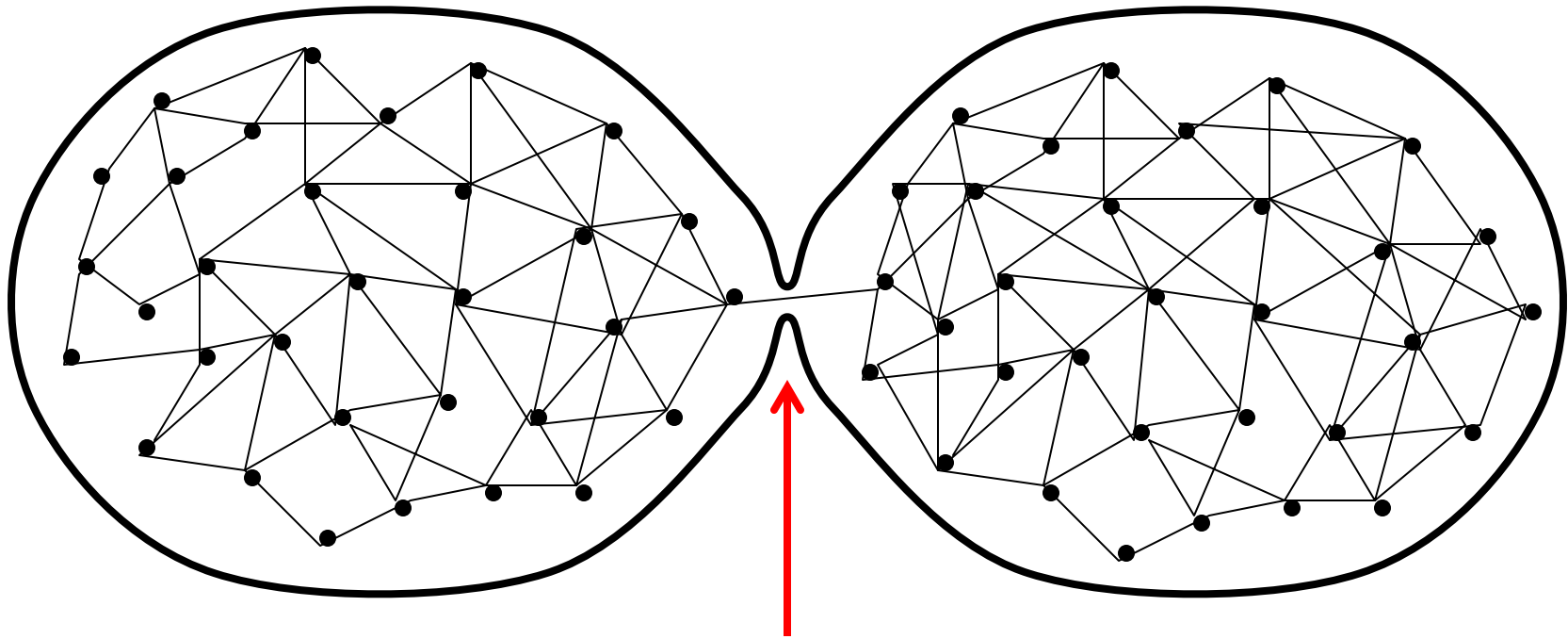
for two configs I, F , a sequence of configs from I to F , uniquely given by a canonical order (of edges, etc).



Proving rapid mixing: Canonical Flow

Why canonical paths ?

Consider the graph of all configurations ($|\Omega|$ vertices) and all I-F canonical paths:

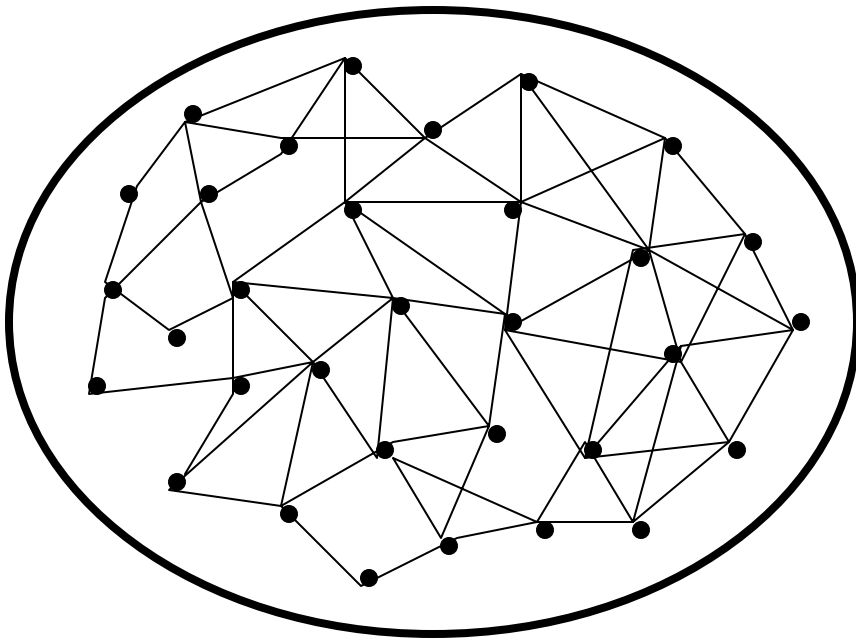


Transition used by $\sim |\Omega|^2/4$ paths \rightarrow MC not mixing rapidly.

Proving rapid mixing: Canonical Flow

Why canonical paths ?

Consider the graph of all configurations ($|\Omega|$ vertices) and all I-F canonical paths:



If all transitions used by poly number of paths times $|\Omega|$, then MC rapidly mixing.

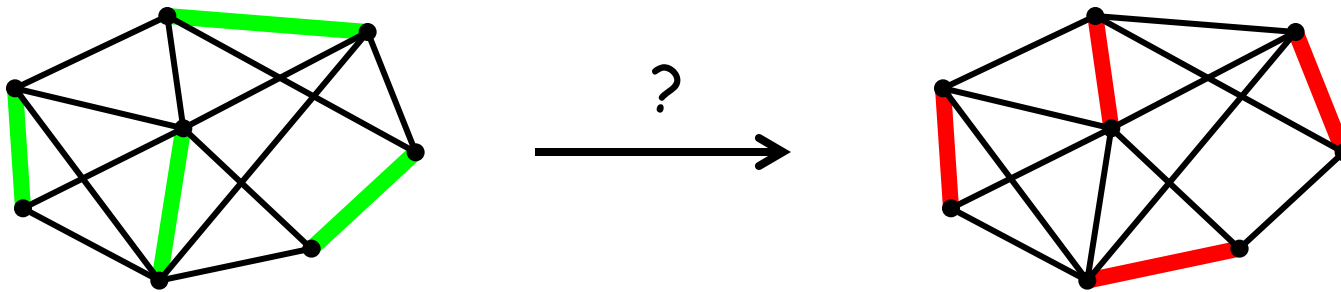
[Diaconis-Stroock '91, Sinclair '92]

Note: the number of vertices is often exponential (in the input size)

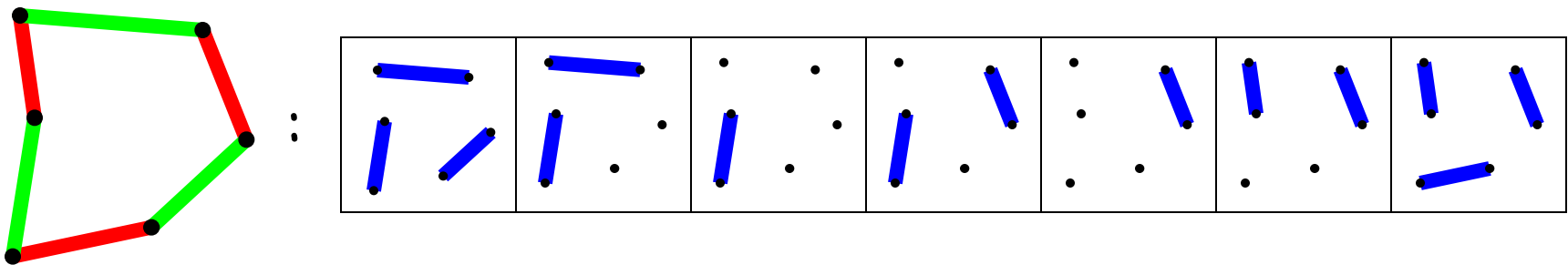
Note: canonical flow = unit flow from I to F through several I-F paths.

Canonical paths for matchings

Let I , F be two different matchings of a graph G :



- take symmetric difference
- order cycles/paths (using a predetermined canonical order)
- deal with each component

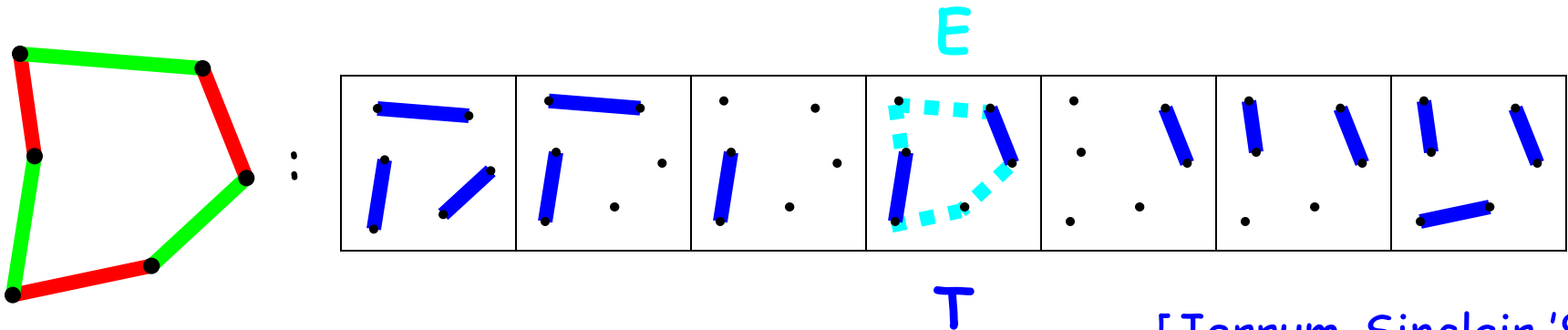


[Jerrum-Sinclair '89]

Canonical paths for matchings

Why these specific canonical paths ?

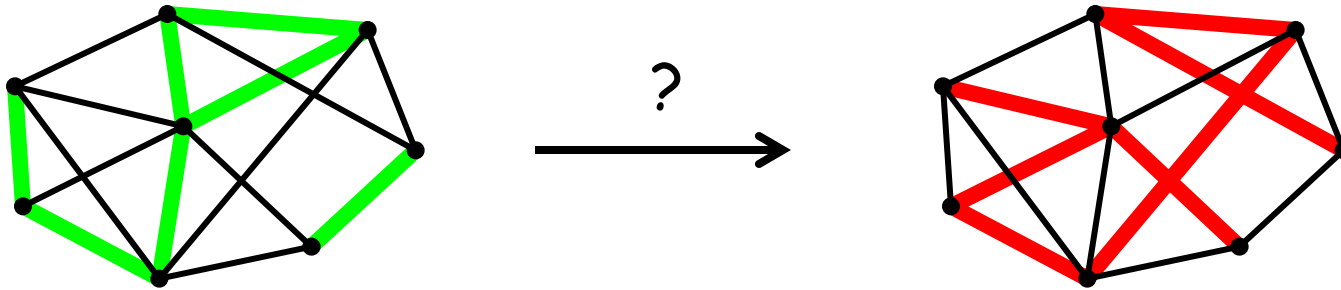
- let T be any transition on the path from I to F
- we need $E = I \oplus F \oplus T$ to be close to a valid matching
- given T , E "encodes" (uniquely determines) I and F
- thus the number of paths through T is $|\Omega|$ times a polynomial in the input size
- implies small congestion and rapid mixing



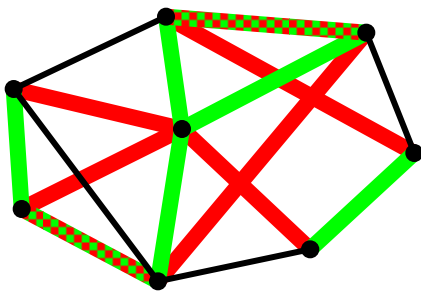
[Jerrum-Sinclair '89]

Canonical flow for graphs w. given deg.seq.

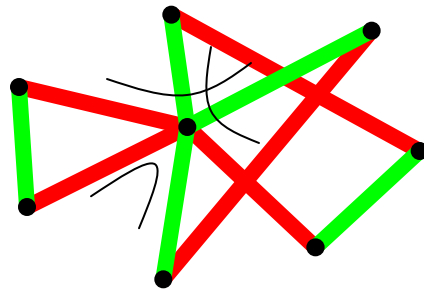
Consider I, F :



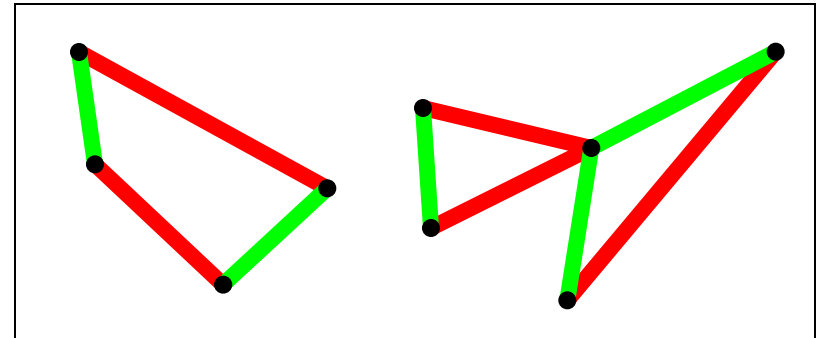
- take sym. diff. and an **alternating** cycle decomposition
- decomposition: **pair** green with red edges at every vertex



$I \cup F$



$I \oplus F$ and
a pairing at the middle vertex

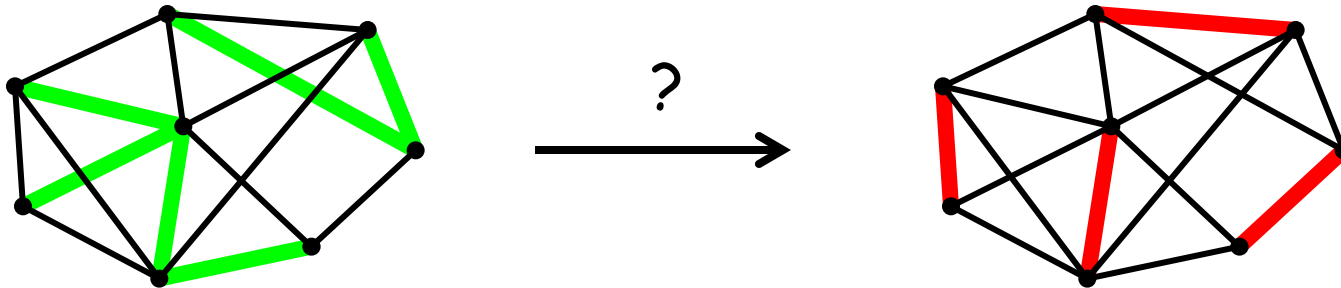


decomposition

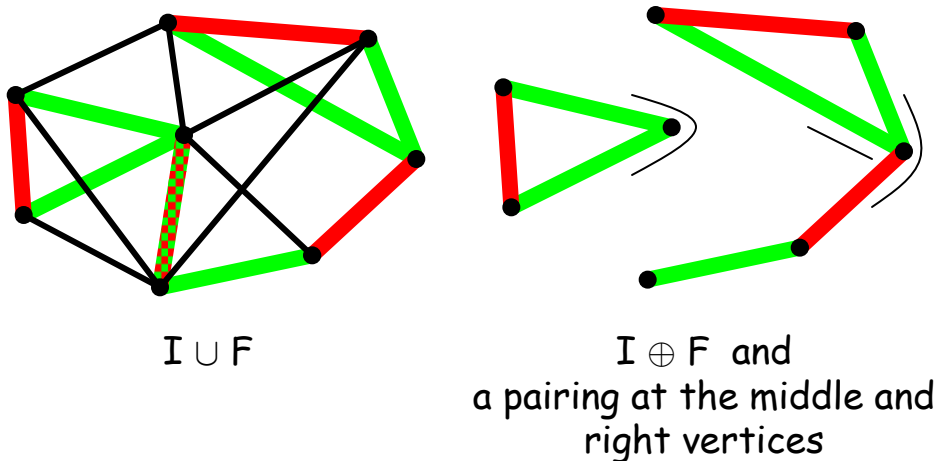
[Cooper-Dyer-Greenhill '07]

Canonical flow for edge covers

Consider I, F :



- take sym. diff. and a cycle/path-decomposition, not necessarily alternating, as follows:

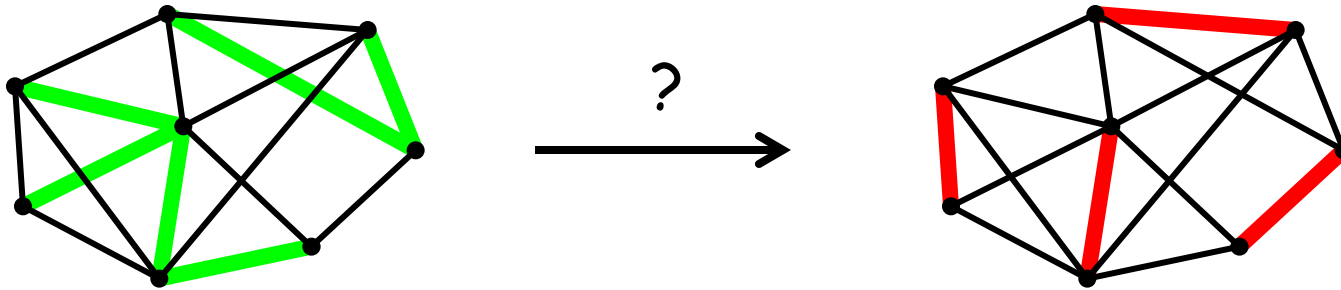


pair green with red edges at every vertex, if left-over edges, pair the same color together

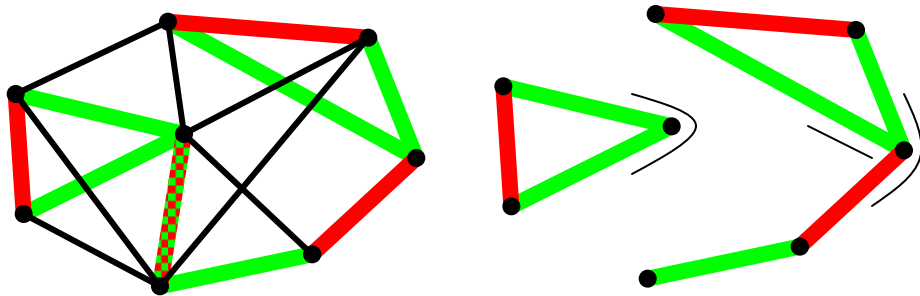
[this paper]

Canonical flow for edge covers

Consider I, F :



- take sym. diff. and a cycle/path-decomposition, not necessarily alternating, as follows:



$I \cup F$

$I \oplus F$ and
a pairing at the middle and
right vertices

The catch: prove it works 😊

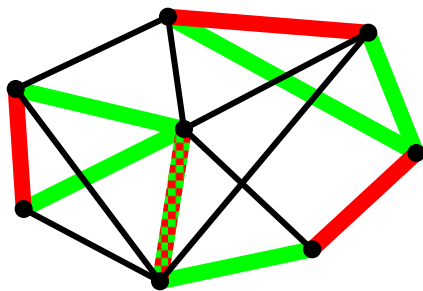
pair green with red
edges at every vertex,
if left-over edges, pair
the same color together

[this paper]

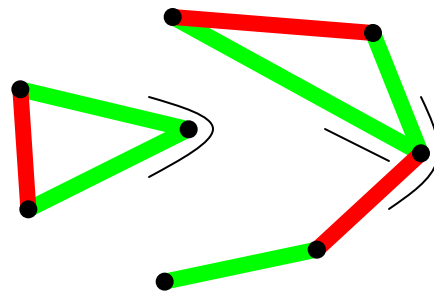
Canonical flow for edge covers

A little bit about the proof:

- alternation is critical to show that $I \rightarrow F$ moves only through valid configurations
- with degrees ≤ 3 we succeeded to show valid canonical flow with non-alternating decomposition
- encoding is "close" to an edge cover



$I \cup F$



$I \oplus F$ and
a pairing at the middle and
right vertices

The catch: prove it works ☺

pair green with red
edges at every vertex,
if left-over edges, pair
the same color together

[this paper]

Open problems

- edge covers for
 - constant degree graphs
 - bipartite graphs
 - general graphs (implications for matchings ?)
- minimum edge covers ? weighted edge covers ?
- non-alternating decomposition for other problems ?

