Problem 1
Consider the following divide-and-conquer algorithm that assumes a global array $A$ of integers:

```
WHATDOIDO(integer left, integer right):
    if left==right:
        if $A[left]<0$ return (0, 0, 0, $A[left]$)
    if left<right:
        m = (left+right)/2 (rounded down)
        (lmaxsum, llmaxsum, lrmaxsum, lsum) = WHATDOIDO(left, m)
        (rmaxsum, rlmaxsum, rrmaxsum, rsum) = WHATDOIDO(m+1, right)
        maxsum = max{lmaxsum, rmaxsum, lrmaxsum+rmaxsum}
        leftalignedmaxsum = max{llmaxsum, lsum+rlmaxsum}
        rightalignedmaxsum = max{rrmaxsum, lrmaxsum+rsum}
        sum = lsum+rsum
        return (maxsum, leftalignedmaxsum, rightalignedmaxsum, sum)
```

Before running the algorithm, we ask the user to enter $n$ integers that we store in the array $A$. Then we run WHATDOIDO(1,n).

a) State the recurrence for $T(n)$ that captures the running time of the algorithm as closely as possible.

b) Use the “unrolling the recurrence” or the mathematical induction to find a tight bound on $T(n)$.

c) What does the algorithm do? Specify the input and the output of the algorithm (see the slides for examples).

Problem 2
You are given $A[1...n]$, a sorted array of distinct integers. Design an algorithm that runs in $O(\log n)$ time that determines if there exists an integer $k$ such that $A[k] = k$. 
Problem 3

For each of the following recurrences, use the Master theorem to express $T(n)$ as a Theta of a simple function. State what the corresponding values of $a$, $b$, and $f(n)$ are and how you determined which case of the theorem applies. Do not worry about the base case or rounding.

1. $T(n) = T(n/3) + 1$
2. $T(n) = 2T(n/4) + n$
3. $T(n) = 4T(n/2) + n\log n$

Problem 4

Given is a set of $n$ rectangles, all with their bottom edge aligned along the $x$-axis. Moreover, the union of these rectangles forms a single polygon. Design an $O(n \log n)$ algorithm that computes the perimeter of this polygon.

For example, on the figure above, we are given four rectangles: first with bottom left corner at (2,0) and top right corner at (4,4), second with corners at (3,0) and (10,3), third with corners at (8,0) and (11,10), and the fourth with corners at (6,0) and (12,8). The perimeter of the union is $4 + 2 + 1 + 2 + 5 + 2 + 2 + 3 + 2 + 1 + 8 + 10 = 42$ (the sum follows the perimeter clockwise from coordinates (2,0)).