Homework #6

Due 24 Oct. 2007

1. Describe each of the languages described by the following grammars in set notation.

   1. \( S \rightarrow aaSB \mid \Lambda; \quad B \rightarrow bB \mid b \quad \{a^ib^j \mid 0 < i <= j \text{ or } i = j = 0 \} \)
   2. \( S \rightarrow aSbb \mid A; \quad A \rightarrow cA \mid c \quad \{a^ic^jb^i \mid i >= 0; j > 0 \} \)
   3. \( S \rightarrow aSB \mid aB; \quad B \rightarrow bb \mid b \quad \{a^ib^j \mid i <= j <= 2i; i,j > 0 \} \)

2. For the grammar

   \( S \rightarrow aSB \mid \Lambda \)
   \( B \rightarrow bB \mid \Lambda \)

   1. Describe the grammar using set notation. \( \{a^ib^j \mid i,j >= 0 \text{ and } (i > 0 \text{ or } j = 0) \} \)
   2. Show that the grammar is ambiguous by example.
      Two different left-most derivations of the string aabb:
      \( S \Rightarrow aSB \Rightarrow aaSB \Rightarrow aaBB \Rightarrow aabBB \Rightarrow aabB \Rightarrow aabbB \Rightarrow aabb \)
      \( S \Rightarrow aSB \Rightarrow aaSB \Rightarrow aaBB \Rightarrow aabBB \Rightarrow aabbBB \Rightarrow aabbB \Rightarrow aabb \)
   3. Design an unambiguous grammar for the language.
      For example:
      \( S \rightarrow aAB \mid \Lambda \)
      \( A \rightarrow aA \mid \Lambda \)
      \( B \rightarrow bB \mid \Lambda \)

For each of the following problems you must draw a diagram for the machine you're designing and give a brief description of how it works.

3. Design a PDA for non-palindromes, i.e., \( \{w \in \{0,1\}^* \mid w \neq wR \} \).

4. Design a DPDA for the language \( \{a^ib^j\mid i + k = j \} \).

5. Transform the machine in Table 7.5 (p. 263) to a DPDA where \( \Gamma=\{Z0,X\} \).
   (Such a machine is called a counter automaton.)