Foundations of CS Theory—Homework 8 answers

1. All but c and d are CFLs.

2. Suppose for a contradiction that DCFLs are closed under difference. Let $A$ and $B$ be DCFLs. $A \cap B = A - B'$. Since DCFLs are closed under complementation and difference, it follows that $A \cap B$ is a DCFL. So, we have shown that DCFLs are closed under intersection. But that is a contradiction. It follows that DCFLs are not closed under difference.

3. Easy.

4. $S \Rightarrow ACaB \Rightarrow AaaCB \Rightarrow AaaDB \Rightarrow AaDaB \Rightarrow ADaaB \Rightarrow ACaaB \Rightarrow AaaCaB \Rightarrow AaaaaCB \Rightarrow AaaaaDB \Rightarrow AaaaDaB \Rightarrow AaDaab \Rightarrow ADaabaB \Rightarrow ACaabaB \Rightarrow AaaCaaB \Rightarrow AaaaaCaB \Rightarrow AaaaaaaCB \Rightarrow AaaaaaaaE \Rightarrow^{8} AEaaaaaaa \Rightarrow aaaaaaa$

5. See the answers to the practice final.

6. Let $G = (V, \Sigma, S, P)$. It is easy to see that we can compute $M$, the set of marked variables, as follows:

\[
M = \emptyset \\
\text{repeat} \\
\quad M' = M \\
\quad \text{for every rule } A \rightarrow \alpha \in P \text{ such that } A \notin M \text{ and } \alpha \in (\Sigma \cup M)^* \\
\quad M = M \cup \{A\} \\
\text{until } M = M' \\
\]

$L(G)$ is not empty if and only if $S \in M$. 