Homework 2 (due 09/19/07)

Problem 1

Draw the transition diagrams of finite automata for the following languages.

(a) $L_1$ is the language of all strings over $\{0, 1\}$ containing both 00 and 101 as substrings.

(b) $L_2$ is the language of all strings over $\{0, 1\}$ containing 00 as a substring and not containing 111 as a substring.

(c) $L_3 = \{ w \in \{0, 1, 2, \ldots, 9\}^* \mid w$ is a valid number (in decimal notation) divisible by 3$\}$. A valid number which has more than one digit cannot start with a leading 0.

(d) $L_4 = \{ w \in \{0, 1\}^* \mid w$ is a valid binary number divisible by 3$\}$.

Problem 2

Give regular expressions corresponding to the following languages:

(a) The language $L_1$ from Problem 1.

(b) The language of all strings over $\{0, 1\}$ in which the number of 1’s is even.

(c) The language $L_2$ from Problem 1.

Problem 3

Let $k \in \mathbb{N}$ be a nonnegative constant. Let $L_k$ be a language over $\{a, b\}$ defined as follows:

$$L_k = \{ x \in \{a, b\}^* \mid x$ starts with $a^k\}.$$

(a) Draw the transition diagram of a finite automaton for $L_3$.

(b) Give the 5-tuple (and specify all its elements, including the transition function) which describes your FA from part (a).

(c) Give a 5-tuple specifying a finite automaton $M_k$ such that $L(M_k) = L_k$. 