Suppose we have dominos of strings, e.g.:

```
  b  a  ca  abc
  ca ab  a  c
```

The question: is it possible to arrange the dominos in line (repetitions of dominos are allowed) in such a way so that the top forms the same string as the bottom?
Post Correspondence Problem

Formally, given is a collection $P$ of dominos:

$$P = \{ (t_1,b_1), (t_2,b_2), \ldots, t_k,b_k) \}$$

A match is a sequence $i_1,i_2,\ldots,i_s$, where $t_{i_1}t_{i_2}\ldots t_{i_s} = b_{i_1}b_{i_2}\ldots b_{i_s}$.

The Post Correspondence Problem (PCP) asks if there is a match for $P$.

$$\text{PCP} = \{ \langle P \rangle \mid \text{P is a PCP that has a match} \}$$

Thm 5.15: PCP is undecidable.

[Diagram showing a Turing machine and its components, illustrating the process of deciding whether a PCP has a match.]
First, we'll consider MPCP where we are looking for instances that have a match that starts with the first domino.

\[ \text{MPCP} = \{ <P> | P = \{ (t_1,b_1), (t_2,b_2), \ldots, t_k,b_k) \} \] is a PCP that has match starting with \((t_1,b_1)\)

**Claim:** PCP is equivalent to MPCP.

**Part 1:** if have \( <P_1> \) an instance of PCP, we can create \( <P_2> \) an instance of MPCP s.t. \( <P_1> \) has a match iff \( <P_2> \) has a match

**TRY**

**Part 2:** if \( <P_2> \) an instance of MPCP then can create \( <P_1> \) an instance of PCP s.t. (this part used for the then)

\[ P_2 = \{ (\hat{t}_1, \hat{b}_1), \ldots, (\hat{t}_k, \hat{b}_k) \} \] then \[ P_1 = \{ (\# f(\xi_i)\#, \# f(\xi_i)),
\]

\[ f(\hat{\xi}_i)\# \# f(\hat{\xi}_i) \}

\[ \forall i \in \{1, \ldots, k\} \]
Thm 5.15: The Post Correspondence Problem (PCP) is undecidable.

We will show what happens in the green "blob", i.e. for a given pair $<M, w>$ we construct an PCP instance $<P>$ s.t. $M$ accepts $w$ iff $P$ has a (modified) match.

**idea:** $<P>$ will have a match = "computational history"

1. **Start**
   
   $\begin{array}{c}
   # \\
   # q_0 #
   \end{array}$

2. if $\delta(q, a) = (r, b, R)$
   
   $\begin{array}{c}
   q a \\
   b r
   \end{array}$

3. if $\delta(q, a) = (r, b, L)$ then $\forall c \in \Gamma$
   
   $\begin{array}{c}
   c q a \\
   r c b
   \end{array}$

4. copy the rest of the config.: $\forall a \in \Gamma$
   
   $\begin{array}{c}
   a
   \end{array}$

5. copy hash:

   $\begin{array}{c}
   # \\
   #
   \end{array}$

6. when to stop? when we've seen $q_{accept}$, then it's "eat" all neighboring symbols

7. $q_{accept}$

8. $\begin{array}{c}
   \# \\
   \#
   \end{array}$

9. to deal with blanks at the beginning or simply say one-way infinite tape $TM$ is equiv. to two-way infinite tape $TM$