Reductions: if we can reduce (transform) problem A into a problem B, then solving problem B gives solution to problem A.

Example: \( \text{HALT}_{TM} = \{ <M,w> \mid M \text{ is a TM that halts on } w \} \)

Suppose \( \text{HALT}_{TM} \) is decidable.

\[<M,w> \quad \text{HALT}_{TM} \quad <M,w> \]

YES if \( M \) halts on \( w \)

NO if \( M \) does not halt on \( w \)

Run \( M \) on \( w \)

Accept

Reject

YES if \( M \) accepts \( w \)

NO if \( M \) does not accept \( w \)

Thm 5.1: \( \text{HALT}_{TM} \) is undecidable.

Note: \( \text{HALT}_{TM} \) is the halting problem, \( \text{A}_{TM} \) is the acceptance problem.
Thm 5.1: \( E_{TM} \) is undecidable, where

\[
E_{TM} = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) = \emptyset \}
\]

Suppose a decider exists for \( E_{TM} \)

Will construct a decider for \( A_{TM} \) (could have done \( HALT_{TM} \) as well) \( \rightarrow \) a contradiction

\( M_2: \)
1) check the input = w; if no, reject
2) if yes, run M on w

\( \delta \)-func for \( M_2 \) will have to “hard-wire” w into itself and then it will use the \( \delta \)-func of M as a subroutine (again “hard-wiring”)

[Section 5.1]
Thm 5.3: \( \text{REGULAR}_{TM} \) is undecidable, where

\[
\text{REGULAR}_{TM} = \{ <M> \mid M \text{ is a TM and } L(M) \text{ is regular} \}
\]

If we have a decider for \( \text{REGULAR}_{TM} \), we can create a new decider for \( \text{ATM} \).

Create \( M_3 \):

1. Check if input is a palindrome; if yes, accept.
2. If no, run \( M \) on \( w \) and if \( M \) accepts, accept.

Claim:

- If \( M \) accepts \( w \) then \( L(M_3) = \Sigma^* \).
- Else, \( L(M_3) = \{ s \mid s \in \Sigma^*, s \neq w \} \).
Thm 5.4: $EQ_{TM}$ is undecidable, where

$$EQ_{TM} = \{ <M_1, M_2> | M_1, M_2 \text{ are TM's and } L(M_1) = L(M_2) \}$$

Suppose we have a decider for $EQ_{TM}$,

we'll construct a decider for $E_{TM}$. 

Yes if $L(M_1) = L(M_2)$

No otherwise

Yes if $L(M) = \emptyset$

No otherwise
Thm 5.4: $\text{ALL}_{\text{CFG}}$ is undecidable, where

$$\text{ALL}_{\text{CFG}} = \{ <G> \mid G \text{ is a CFG and } L(G) = \Sigma^* \}$$