Name: ________________________________

- This exam consists of 5 questions.
- Each question is worth 10 points.
- The exam is worth 40 points.
- Your exam score is the sum of your 4 highest question scores.
1. Suppose that $P \neq NP$ and that the alphabet is $\{a, b, c\}$. Draw a Venn diagram that shows the relationships between the following classes of languages:

(a) NP
(b) P
(c) the DCFLs
(d) the languages that can be accepted by an NFA-A
(e) the languages that can be accepted by a nondeterministic Turing machine with 25 tapes
(f) the languages that can be accepted by a Turing machine that halts on all inputs
(g) the languages that can be generated by an unrestricted grammar
(h) the languages that have an algorithmic solution
(i) the languages whose complement is recursively enumerable
(j) the recursive languages
(k) the recursively enumerable languages
(l) the regular languages
2. (a) Suppose that the alphabet is \{0, 1\}.

For every class of languages in the table, you have to state whether the class is closed under complementation, concatenation, and intersection.

Write “yes” in a box if the class is closed under the operation, write “no” if the class is not closed under the operation, and write “nobody knows” if nobody knows whether or not the class is closed under the operation.

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<th>closed under</th>
<th>Complementation</th>
<th>Concatenation</th>
<th>Intersection</th>
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<tr>
<td>NP</td>
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<td>P</td>
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<td>The context-free languages</td>
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<td>The recursively enumerable languages</td>
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<td>The recursive languages</td>
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(b) For one of the boxes where you answered “no,” give an explicit example that shows why that particular class is not closed under that particular operation. (Clearly state which box you chose.)

(c) For one of the boxes where you answered “yes,” informally argue why that particular class is closed under that particular operation. (Clearly state which box you chose.)
3. Consider the following grammar $G$:

$$ S \rightarrow A \mid 01S \mid 10S \mid 0S1 \mid 1S0 \mid S01 \mid S10 \mid 1S \mid S1 $$

(a) Show that this grammar is ambiguous.

At first glance, it may seem that the language generated by $G$ is

$$ L = \{ x \in \{0, 1\}^* \mid n_0(x) \leq n_1(x) \}. $$

However, this is not true.

(b) For both of the following statements, circle “true” if the statement is true, and circle “false” if the statement is false.

i. $L \subseteq L(G)$  
ii. $L(G) \subseteq L$

(c) Show that $L(G) \neq L$.

(d) Give a context-free grammar that generates $L$. (It suffices to give the rules.)
4. Give the simplified transition diagram of a (deterministic) Turing machine that halts on all inputs and that accepts the language

\[ \{ x \in \{a,b\}^* \mid n_a(x) \neq n_b(x) \} \]

Give a short explanation to convince me that your Turing machine is correct.
5. Let \( L = \{a^i b^k c^i d^k \mid i, k \geq 0\} \).

   (a) Explain informally why \( L \) doesn’t seem to be a context-free language.

   (b) Use the pumping lemma for context-free languages to prove that \( L \) is not context-free.