Suppose we have dominos of strings, e.g.:

The question: is it possible to arrange the dominos in line (repetitions of dominos are allowed) in such a way so that the top forms the same string as the bottom?
Post Correspondence Problem

Formally, given is a collection $P$ of dominos:

$$P = \{ (t_1, b_1), (t_2, b_2), \ldots, (t_k, b_k) \}$$

A match is a sequence $i_1, i_2, \ldots, i_s$, where $t_{i_1} t_{i_2} \ldots t_{i_s} = b_{i_1} b_{i_2} \ldots b_{i_s}$. The Post Correspondence Problem (PCP) asks if there is a match for $P$.

**Thm 5.15:** PCP is undecidable.
Thm: Ambiguity of CFGs is undecidable.

\[ \text{Ambig}_G = \{ \langle G \rangle \mid G \text{ is CFG and } G \text{ is ambig. i.e. } \exists \text{ string } w \in L(G), \text{ s.t. } w \text{ has } n \text{ different parse trees} \} \]

Create a CFG \( G \):

- \( S \rightarrow X | Y \)
- \( X \rightarrow t_1 x_1 | t_2 x_2 | \ldots | t_k x_k | \varepsilon \)
- \( Y \rightarrow b_1 y_1 | b_2 y_2 | \ldots | b_h y_h | \varepsilon \)

Also add:

- \( X \rightarrow t_2 x_1 | t_3 x_2 | \ldots | t_h x_{h-1} \)
- \( Y \rightarrow b_2 y_1 | b_3 y_2 | \ldots | b_{h-1} y_h \)

Note: \( X \rightarrow X, Y \rightarrow Y \) is incorrect as \( G \) creates ambiguity!

Also note.

- Ambiguity iff PCP instance has a solution.

[Section 5.2]