Reductions: if we can reduce (transform) problem A into a problem B, then solving problem B gives solution to problem A.

Example: \( \text{HALT}_T^M = \{ <M,w> | M \text{ is a TM that halts on } w \} \)

Thm 5.1: \( \text{HALT}_T^M \) is undecidable.

Note: \( \text{HALT}_T^M \) is the halting problem, \( A_T^M \) is the acceptance problem.
Thm 5.1: $E_{TM}$ is undecidable, where

$$E_{TM} = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) = \emptyset \}$$

Notice: if $M$ does not accept $w$: then $L(Mw) = \emptyset$

if $M$ accepts $w$: then $L(Mw) = \Sigma^*$

Thus, $E_{TM}$ is undecidable
Thm 5.3: $\text{REGULAR}_{\text{TM}}$ is undecidable, where

$\text{REGULAR}_{\text{TM}} = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is regular} \}$
Thm 5.3: $\text{REGULAR}_{TM}$ is undecidable, where

$\text{REGULAR}_{TM} = \{ <M> \mid M \text{ is a TM and } L(M) \text{ is regular} \}$
Thm 5.4: \( EQ_{TM} \) is undecidable, where
\[
EQ_{TM} = \{ <M_1, M_2> \mid M_1, M_2 \text{ are TM's and } L(M_1) = L(M_2) \}
\]
Thm 5.4: $\text{ALL}_{\text{CFG}}$ is undecidable, where

$$\text{ALL}_{\text{CFG}} = \{ \langle G \rangle \mid G \text{ is a CFG and } L(G) = \Sigma^* \}$$
Rice’s Thm [Problem 5.28]:

Let \( p \) be a language property. If \( p \) holds for some but not all languages, then the following language is undecidable:

\[
R = \{ <M> \mid M \text{ is a TM and } L(M) \text{ satisfies } p \}
\]