Turing Machines

- more powerful than PDA's
- what could it have?
Example: \[ A = \{ a^i b^i c^i \mid i \geq 0 \} \]
Example: \[ B = \{ w\#w \mid w \in \{0,1\}^* \} \]
Def 3.3:

A **Turing machine** (TM) is a 7-tuple \((Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})\), where

- \(Q\) is the (finite) set of states
- \(\Sigma\) is the (finite) input alphabet, not containing □
- \(\Gamma\) is the (finite) tape alphabet, \(\square \cup \Sigma \subseteq \Gamma\)
- \(\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}\) is the transition function
- \(q_0 \in Q\)
- \(q_{\text{accept}} \in Q\) is the accept state
- \(q_{\text{reject}} \in Q - \{q_{\text{accept}}\}\) is the reject state
**Computation** of Turing machines

- first we define a **configuration**:

  \[ uqv \]

  - means the tape contains \(uv\), the state is \(q\), and the machine reads the first symbol of \(v\)

- suppose configuration is \(uaqbv\) and \(\delta(q,b)=(p,c,R)\)

We say that \(uaqbv\) **yields**
Computation of Turing machines

- first we define a configuration:
  
uqv - means the tape contains uv, the state is q, and the machine reads the first symbol of v

- start configuration:  \_ q$_0$ w  for input w

- accepting configuration:  u q$_{accept}$ v  for any u, v \in \Gamma^*

- rejecting configuration:  u q$_{reject}$ v

Note: accepting/rejecting configurations are halting
Turing Machines

**Computation** of Turing machines

- first we define a **configuration**:

  \[ uqv \] means the tape contains \( uv \), the state is \( q \), and the machine reads the first symbol of \( v \)

A TM \( M \) **accepts** \( w \) if

\[ \exists \text{ a sequence of config. } c_1, c_2, \ldots, c_e \] where

1) \( c_1 \) is start config.

2) \( c_i+1 \) is yielded from \( c_i \) \( \forall i \in \{1, \ldots, e-1\} \)

3) \( c_e \) is an accept config.

\[ \text{denoted } L(M) \]

The **language of a TM** \( M \) is the set of strings that \( M \) accepts/recognizes.
Def 3.5: A language is **Turing-recognizable** if there is some TM that recognizes it.

Def 3.6: A language is **Turing-decidable** if there is some TM that decides it. That is, for every string \( w \in \Sigma^* \) either \( w \) is accepted or \( w \) is rejected (i.e., no infinite computations allowed.)
Example: \( A = \{ 0^n \mid n=2^k \text{ for some } k \geq 0 \} \)