Problem 1

Let
\[ \text{EPSTM} = \{ \langle M \rangle \mid M \text{ is a TM that accepts } \varepsilon \} \].

Show that \text{EPSTM} is undecidable by giving a reduction from \text{A_{TM}}. Give your reduction in the pictorial “box-in-a-box” style as we did in class, or, if you prefer, state the reduction using functions instead of boxes, like we did on the board for the \text{A_{TM}} to \text{HALT_{TM}} reduction.

Problem 2

Read ahead about the Post Correspondence Problem (Section 5.2 – or wait until Tuesday). Use the undecidability of the PCP problem to show that the following language is also undecidable:

\[ \text{INTCFG} = \{ \langle G_1, G_2 \rangle \mid G_1, G_2 \text{ are CFG’s and } L(G_1) \cap L(G_2) \neq \emptyset \} \]

In particular, for a given PCP instance, create \( G_1 \) and \( G_2 \) such that \( L(G_1) \cap L(G_2) \) is nonempty if and only if the PCP instance has a solution.

Problem 3

An unrestricted grammar is like a CFG, except that you can now have any string of variables and terminals containing at least one variable on the left-hand side of a rule. Surprisingly, unrestricted grammars generate exactly the Turing-recognizable languages. We will not show this, but we will look at an example.

The following grammar generates the language \( \{a^{2^k} \mid k \geq 1 \} \).

\[
\begin{align*}
S & \rightarrow ACaB \\
Ca & \rightarrow aaC \\
CB & \rightarrow DB \\
CB & \rightarrow E \\
ad & \rightarrow Da \\
AD & \rightarrow AC \\
aE & \rightarrow Ea \\
AE & \rightarrow \varepsilon
\end{align*}
\]

Give a derivation of \textit{aaaaaa}. 

**Problem 4**

Consider the Hamiltonian cycle problem where, for a given undirected graph, one needs to decide if there exists a cycle that goes through every vertex exactly once. Formally:

\[
\text{HAMCYCLE} = \{ \langle G \rangle \mid G \text{ is a graph that contains a cycle going through every vertex} \}.
\]

A similar problem is the Longest cycle problem:

\[
\text{LONGCYCLE} = \{ \langle G, k \rangle \mid G \text{ is a graph that contains a cycle of length } \geq k \text{ (no repeated vertices)} \}.
\]

(a) Imagine that you have a black box (function) that solves LONGCYCLE. How would you use it to solve HAMCYCLE?

(b) Approximate the number of steps you need in addition to the black box calls of LONGCYCLE – in particular, is the number polynomial in the number of vertices and edges?

**Problem 5**

Draw a Venn diagram that shows the relationships between the following classes of languages:

(a) P.

(b) the CFLs.

(c) the decidable (recursive) languages.

(d) the languages whose complement is Turing-recognizable.

(e) the languages whose complement is decidable.

(f) the regular languages.

(g) the Turing-recognizable (recursively enumerable) languages.