Honors Intro to CS Theory—Homework 4
Due: Thursday, October 4, 2012, 4pm

Problem 1
Let $L = \{a^i b^j a^i \mid i, j \geq 0\}$.

(a) Use the pumping lemma to show that $L$ is not regular.

(b) Find infinitely many strings that are pairwise distinguishable by $L$ (i.e., find infinitely many equivalence classes of the indistinguishability by $L$ relation). For every pair of strings show that they are distinguishable by $L$.

Problem 2
(a) Let $B = \{1^k y \mid y \in \{0, 1\}^* \text{ and } y \text{ contains at least } k \text{ 1s, for } k \geq 1\}$. Show that $B$ is a regular language.

(b) Let $B = \{1^k y \mid y \in \{0, 1\}^* \text{ and } y \text{ contains at most } k \text{ 1s, for } k \geq 1\}$. Show that $C$ is not a regular language.

Problem 3
(a) Draw a minimum DFA for $(aa)^*b^*$.

(b) Prove that your DFA is of minimum size. You can use the minimization algorithm from class (see also page 327 in the 3rd edition or page 299 in the 2nd edition of the textbook) or you can use the Myhill-Nerode Theorem.

Problem 4
Let $L = \{w tw \mid w, t \in \{0, 1\}^+\}$. Prove that $L$ is not regular.

Problem 5
For each of the following statements, state whether it is true or false. For the false statements, find a counterexample.

(a) If $L_1 \subseteq \Sigma^*$ is a regular language and $L_2 \subseteq L_1$, then $L_2$ is regular.

(b) If $L_1 \subseteq \Sigma^*$ is a regular language and $L_1 \subseteq L_2$, then $L_2$ is regular.

(c) If $L_1, L_2 \subseteq \Sigma^*$ are regular languages, then $L_1 \cap L_2$ is regular.

(d) If $L_1, L_2 \subseteq \Sigma^*$ and $L_1 \cap L_2$ is regular, then at least one of the languages $L_1$ and $L_2$ is regular.
(e) If $L_1 \subseteq \Sigma^*$ is a regular language and $L_2 \subseteq \Sigma^*$ is not a regular language, then $L_1L_2$ is not regular.

(f) If $L_1, L_2 \subseteq \Sigma^*$ and $L_1L_2$ is not regular, then at least one of the languages $L_1$ and $L_2$ is not regular.

Problem 6

Regular expressions and finite automata and their variations are used in all areas of computer science. Find a nice application, write a short summary (100-200 words), and give a link to a website with further explanation of the application.

Submit the summary and the link in a single html file in the special MyCourses dropbox. Include your name(s) and the title of your application. I will post a merged html of all the applications on the course website.