Problem 1

Give regular expressions for the following languages:

(a) \( L_1 = \{ w \in \{0, 1\}^* \mid w \text{ contains an even number of 1's} \} \)

(b) \( L_2 = \{ w \in \{0, 1\}^* \mid \text{ every odd position in } w \text{ is a 1} \} \)

(c) \( L_3 = \{ w \in \{0, 1, \ldots, 9\}^* \mid w \text{ is a valid decimal number, i.e., no leading zeros are allowed} \} \)

Problem 2

Let \( R = (0 \cup 10)^* \cup (\varepsilon \cup 1)^* \). Use the construction from the proof of Lemma 1.55 to construct an NFA \( N \) such that \( L(N) = L(R) \). Apply the construction literally (do not optimize the resulting NFA – keep all those \( \varepsilon \) arrows in the NFA).

Problem 3

Consider the 2-state DFA recognizing all strings over \( \{0, 1\} \) with an even number of 1’s. Apply the construction from the proof of Lemma 1.60 to construct an equivalent regular expression. In particular:

- Draw the state diagram of the first GNFA.
- After each iteration draw the state diagram of the current GNFA. You may simplify the regular expressions on the transitions.
- State the resulting regular expression.

Problem 4

(a) Exercise 1.25, page 87. Do not forget to define the computation.

(b) Exercise 1.27, page 88.

Problem 5

Problem 1.43, page 90.

Problem 6

Let \( L = \{ a^i b^j c^i \mid i, j \geq 0 \} \). Use closure properties of regular languages to show that \( L \) is not regular. Recall that we know that for every two symbols \( \sigma_1 \neq \sigma_2 \), the language \( \{ \sigma_1^k \sigma_2^k \mid k \geq 0 \} \) is not regular. Do not refer to any other non regular languages in your proof.