Problem 1

Let \( M_1 \) and \( M_2 \) be two DFAs, where \( M_i = (Q_i, \Sigma, \delta_i, q_i, F_i) \) for \( i \in \{1, 2\} \). Construct a DFA \( M = (Q, \Sigma, \delta, q_0, F) \) that accepts the language of all strings over \( \Sigma \) that are accepted by exactly one of the automata \( M_1 \) and \( M_2 \), i.e., \( L(M) \) is the symmetric difference of \( L(M_1) \) and \( L(M_2) \).

This exercise consists of two parts:

(a) Give a general construction, i.e., give mathematical definitions of the components \( Q, \delta, q_0, \) and \( F \).

(b) Show how your construction works on the following concrete example: \( M_1 \) is a two-state DFA and \( M_2 \) is a four-state DFA; the automata accept the following languages:

\[
L(M_1) = \{ w \in \{0, 1\}^* | w \text{ contains an odd number of } 1s \} \\
L(M_2) = \{ w \in \{0, 1\}^* | w \text{ contains substring } 001 \}
\]

Draw the state diagrams of \( M_1 \) and \( M_2 \), and the DFA \( M \) that is the result of applying your construction from part (a). Label the states with their names. Do not simplify \( M \).

Problem 2

Consider the following language:

\[
L_1 = (\{0\} \cup \{1\})^* \{0\} \{\{0\} \cup \{1\}\}^*.
\]

This language is the language of all strings over \( \{0, 1\} \) that contain 01 as a substring. Notice that \( L_1 \) is expressed using the regular operations (union, concatenation, and Kleene star), and the languages \( \{0\} \) and \( \{1\} \).

Let \( L_2 \) be the language of all strings over \( \{0, 1\} \) with exactly three or four 0’s. Express \( L_2 \) using the regular operations and the languages \( \{0\}, \{1\}, \{\epsilon\}, \) and \( \emptyset \).

Problem 3

Draw the state diagram of an NFA with at most six states accepting the language of all strings over \( \{0, 1\} \) that contain an even number of 1’s, or contain exactly two 0’s.
Problem 4

Let \( N_A \) and \( N_B \) be two NFAs. Suppose that we want to construct an NFA \( N \) for the union of the languages \( L(N_A) \) and \( L(N_B) \). Suppose we use the following construction: take the start state of \( N_A \) and the start state of \( N_B \) and merge them into a single state, while keeping all transitions of both \( N_A \) and \( N_B \).

(a) Describe this construction mathematically, i.e., formally define the five components of \( N \) in terms of the components of \( N_A \) and \( N_B \).

(b) Is this construction correct? That is, does \( N \) always accept the union of \( L(N_A) \) and \( L(N_B) \)? If yes, informally reason your answer. If not, give a concrete example of \( N_A \) and \( N_B \) and the resulting \( N \) (draw the state diagrams of all three NFAs) such that \( L(N) \neq L(N_A) \cup L(N_B) \).

Problem 5

Let \( N_1 = (\{q_0, q_1, q_2\}, \{a, b\}, \delta_1, q_0, \{q_1\}) \) be an NFA with transition function \( \delta_1 \) given by the following table:

<table>
<thead>
<tr>
<th>State</th>
<th>a</th>
<th>b</th>
<th>( \varepsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_0 )</td>
<td>{q_2}</td>
<td>\emptyset</td>
<td>{q_1}</td>
</tr>
<tr>
<td>( q_1 )</td>
<td>{q_0}</td>
<td>\emptyset</td>
<td>\emptyset</td>
</tr>
<tr>
<td>( q_2 )</td>
<td>{q_1}</td>
<td>{q_1, q_2}</td>
<td>\emptyset</td>
</tr>
</tbody>
</table>

Use the subset construction to construct an equivalent DFA \( M_2 \). Draw the state diagrams of both \( N_1 \) and \( M_2 \). You do not have to draw the unreachable states of \( M_2 \). Do not otherwise simplify \( M_2 \). Label the states with their names.

Problem 6

Let \( M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1) \) be a NFA. Construct an NFA \( N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2) \) that accepts \( L(M_1)^R \), i.e., the reverse of \( L(M_1) \).

(a) Informally describe your construction of \( N_2 \). Examples never hurt.

(b) Formally describe the 5-tuple for \( N_2 \).