Suppose we have dominos of strings, e.g.:

\[
\begin{array}{cccc}
\text{b} & \text{a} & \text{ca} & \text{abc} \\
\text{ca} & \text{ab} & \text{a} & \text{c}
\end{array}
\]

The question: is it possible to arrange the dominos in line (repetitions of dominos are allowed) in such a way so that the top forms the same string as the bottom?

\[
\frac{a}{ab} \quad \frac{b}{ca} \quad \frac{ca}{a} \quad \frac{a}{ab} \quad \frac{abc}{c}
\]
Formally, given is a collection $P$ of dominos:

$$P = \{ (t_1, b_1), (t_2, b_2), \ldots, (t_k, b_k) \}$$

A match is a sequence $i_1, i_2, \ldots, i_s$, where $t_{i_1} t_{i_2} \ldots t_{i_s} = b_{i_1} b_{i_2} \ldots b_{i_s}$.

The Post Correspondence Problem (PCP) asks if there is a match for $P$.

**Thm 5.15:** PCP is undecidable.

reduction from $A_{TM}$

the match will correspond

to the sequence of TM config. on $w$

ending in an accept state
First, we’ll consider MPCP where we are looking for instances that have a match that starts with the first domino.

\[
\text{MPCP} = \{ \langle P \rangle \mid P = \{ (t_1, b_1), (t_2, b_2), ..., (t_k, b_k) \} \text{ is a PCP that has match starting with } (t_1, b_1) \}
\]

Claim: PCP is equivalent to MPCP.
Post Correspondence Problem

Thm 5.15: PCP is undecidable.
Post Correspondence Problem

Thm: $EQ_{CFG}$ is undecidable.
Thm: Ambiguity of CFGs is undecidable.

\[ \text{Acceptance problem for ambiguous CFGs (AMB_{CFG})} = \{ \langle G \rangle \mid G \text{ is a CFG and } G \text{ is ambiguous} \} \]