Turing Machines

- more powerful than PDA's
- what could it have?
Example: $A = \{ a^i b^i c^i \mid i \geq 0 \}$
Example: $B = \{ w#w \mid w \in \{0,1\}^* \}$

Informal description:

1) read $0$, loop through $0/1$ until $#$
   - if $0$, mark it, loop back to the
     first mark below $#$
   - (same with reading $1$), skipping over
     the marks

2) if after the first mark:
   - $0$ or $1$ → repeat 1
   - $#$ then, loop through the
     marks to the right,
     accept $M$, blank $\sqcup$
Turing Machines

Def 3.3:

A Turing machine (TM) is a 7-tuple \((Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})\), where

- \(Q\) is the (finite) set of states
- \(\Sigma\) is the (finite) input alphabet, not containing \(\square\)
- \(\Gamma\) is the (finite) tape alphabet, \(\square \cup \Sigma \subseteq \Gamma\)
- \(\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, S\}\) is the transition function
- \(q_0 \in Q\) is the start state
- \(q_{\text{accept}} \in Q\) is the accept state
- \(q_{\text{reject}} \in Q-\{q_{\text{accept}}\}\) is the reject state
Turing Machines

Computation of Turing machines
- first we define a configuration:
  uqv - means the tape contains uv, the state is q, and
  the machine reads the first symbol of v
- suppose configuration is uaqbv and \( \delta(q,b)=(p,c,R) \)

We say that uaqbv yields uacpv
  \( \delta(q,b)=(p,c,L) \)
  \( \delta(q,b)=(p,c,S) \)
  uapcv
  uapcv
  uapcv
Computation of Turing machines

- first we define a configuration:
  
  \[ uqv \]  - means the tape contains uv, the state is q, and the machine reads the first symbol of v

- start configuration for input x:
  
  \[ q_0x \]

- accepting configuration:
  
  \[ uq_{\text{accept}}v \quad \forall u,v \in \Gamma^* \]

- rejecting configuration:
  
  \[ uq_{\text{reject}}v \quad \forall u,v \in \Gamma^* \]

Note: accepting/rejecting configurations are halting
**Computation** of Turing machines

- first we define a **configuration**:

  
  \[ uvq \]  
  - means the tape contains uv, the state is q, and the machine reads the first symbol of v

A TM \( M \) **accepts** \( w \) if

\[ q_0 w \]  
- eventually yields an accepting configuration

The **language of a TM** \( M \) is the set of strings that \( M \) accepts/recognizes.
Def 3.5: A language is **Turing-recognizable** if there is some TM that recognizes it.

Def 3.6: A language is **Turing-decidable** if there is some TM that decides it.

(i.e. that, for every input, halts -- i.e. no infinite computation)
Example: \( A = \{ 0^n \mid n=2^k \text{ for some } k \geq 0 \} \)

Idea:
keep crossing out every other 0, move to the beginning when done, repeat