Problem 1
Let \( L = \{ a^i b^j c^i | i, j \geq 0 \} \). Use closure properties of regular languages to show that \( L \) is not regular. Assume that for every two symbols \( \sigma_1 \neq \sigma_2 \), the language \( \{ \sigma_1^k \sigma_2^k | k \geq 0 \} \) is not regular. Do not refer to any other non-regular languages in your proof.

Problem 2
Let \( L = \{ a^i b^j a^i | i, j \geq 0 \} \).

(a) Use the pumping lemma to show that \( L \) is not regular.

(b) Find infinitely many strings that are pairwise distinguishable by \( L \) (i.e., find infinitely many equivalence classes of the indistinguishability by \( L \) relation). For every pair of strings show that they are distinguishable by \( L \).

Problem 3
Exercise 1.49, page 90.

Problem 4
(a) Draw a minimum DFA for \( (aa)^* b^* \).

(b) Prove that your DFA is of minimum size. You can use the minimization algorithm from class (see also page 299) or you can use the Myhill-Nerode Theorem.

Problem 5
Let \( L = \{ wtw | w, t \in \{0, 1\}^+ \} \). Prove that \( L \) is not regular.

Problem 6
Regular expressions and finite automata and their variations are used in all areas of computer science. Find a nice application, write a short summary (100-200 words), and give a link to a website with further explanation of the application. Submit the summary and the link in a single html file in the special MyCourses dropbox. Include your name(s) and the title of your application. I will post a merged html of all the applications at the course website.