Problem 1

Let $A$, $B$, and $C$ be languages over some alphabet $\Sigma$. For each of the following statements, answer “yes” if the statement is always true, and “no” if the statement is not always true. If you answer “no,” provide a counterexample.

(a) $(AB)C \subseteq A(BC)$.
(b) $(AB)C \supseteq A(BC)$.
(c) $A(B \cap C) \subseteq AB \cap AC$.
(d) $A(B \cap C) \supseteq AB \cap AC$.
(e) $A(B \cup C) \subseteq AB \cup AC$.
(f) $A(B \cup C) \supseteq AB \cup AC$.
(g) $A^* \cap B^* \subseteq (A \cap B)^*$.
(h) $A^* \cap B^* \supseteq (A \cap B)^*$.
(i) $A^* \subseteq (A^*)^*$.
(j) $A^* \supseteq (A^*)^*$.
(k) $A^*B^* \subseteq (AB)^*$.
(l) $A^*B^* \supseteq (AB)^*$.

Problem 2

Draw the state diagram of a finite automaton that accepts the language of all strings over \{a, b\} that contain an even number of $a$’s and the number of $b$’s is not divisible by 3. Your finite automaton should not be overly complicated.
Problem 3
Draw the state diagram of a finite automaton that accepts the language of all strings over \{a, b\} that contain exactly one occurrence of the string baba as a substring (containing bababa counts as two occurrences of baba). Your finite automaton should not be overly complicated.

Problem 4
Draw the state diagram of a finite automaton that accepts the language of all strings over \{0, 1, 2, \ldots, 9\} that represent decimal numbers divisible by 3. Leading zeros are not allowed and the empty string should not be accepted. Your finite automaton should not be overly complicated.

Problem 5
Draw the state diagram of a finite automaton that accepts the language \emptyset.

Problem 6
Draw the state diagram of a finite automaton that accepts the language \{\varepsilon\}.

Problem 7
Let \(k\) be a positive integer constant. Let \(L_k\) be the language over \{a, b\} defined as follows:
\[
L_k = \{w \in \{a, b\}^* \mid w \text{ contains at least } k \text{ a’s.}\}
\]
For example, \(L_2\) is the language of all strings over \{a, b\} that contain at least two a’s.

(a) Draw the state diagram of a finite automaton that accepts \(L_5\).

(b) Give the 5-tuple (and specify all its elements, including the transition function) that describes your finite automaton from part (a).

(c) Give a 5-tuple specifying finite automaton \(M_k\) such that \(L(M_k) = L_k\).